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Trading Volume and Asset Prices

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Price and quantity are the two fundamental variables in any analysis of market interactions. Yet the study of financial markets has focused primarily on the behavior of asset prices—predictability, volatility, and their relation to economic fundamentals. Far less attention has been devoted to the understanding of quantities such as trading volume. Only recently, there has been a growing body of work to link both price *and* volume to economic fundamentals. In this paper, I attempt to review some of these work within a unified framework. I start by describing an intertemporal asset pricing model that explicitly models investors' trading motives, their optimal portfolio choices and the resulting equilibrium asset prices. I then examine the price-volume implications within the framework of the model. Finally, I discuss the results from the empirical analysis of volume and stock returns based on the data of the U.S. stock market. The theoretical analysis together with its empirical support clearly demonstrate that volume and prices are jointly linked to the economic fundamentals, e.g., the risks of the assets and the investors' attitude toward them. Moreover, the behavior of volume is closely related to the behavior of prices and from which we can learn a great deal about the prices as well as the economic fundamentals.

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1. INTRODUCTION

Fundamentals of the economy drive both the supply and demand of financial assets and their prices. Thus, any asset-pricing model that attempts to establish a link between asset prices and the underlying economic factors also establishes links between prices and quantities such as trading volume. Thus, the construction and empirical implementation of any asset-pricing model should involve both price and quantities as its key elements. Even from a purely empirical perspective, the joint behavior of price and quantities reveals more information about the relation between asset prices and economic factors than prices alone. Yet the asset-pricing literature has centered more on prices and much less on quantities. For example, empirical investigations of the classic asset-pricing models such as the Capital Asset Pricing Model (CAPM) and its intertemporal extensions (ICAPM) have focused exclusively on prices and returns, completely ignoring the information contained in quantities.

In this paper, I review some of the recent work that attempt to link the joint behavior of prices and trading volume with the economic fundamentals. These work demonstrate that even if our main interest is in the behavior of prices, valuable information about price dynamics can be obtained from trading volume. The work I draw from include Campbell, Grossman and Wang (1992), He and Wang (1995), Llorente, Michaely, Saar and Wang (2001), Lo and Wang (2000, 2001a, 2001b) and Wang (1994).

I begin by describing an intertemporal capital asset pricing model of multiple assets in the spirit of Merton's ICAPM, which was first explored by Lo and Wang (2001a, b). The model explicitly captures risk-sharing motives behind investors' asset demands and derive equilibrium asset prices and trading volume. In the model, assets are exposed to two sources of risks: the market risk (the risk associated with the market portfolio) and the risk of changes in market conditions.[1] As a result, investors hold two distinct portfolios of risky assets: the market portfolio and a hedging portfolio. The market portfolio allows them to adjust their exposure to market risk, and the hedging portfolio allows them to hedge the risk of changes in market conditions. In equilibrium, investors trade in only these two portfolios, and the expected asset returns are determined by their exposure to these two risks, i.e., a two-factor linear pricing model holds, where the two factors are the returns on the market portfolio and the hedging portfolio, respectively.

Within the context of this model, I examine the implications of this model on the joint behavior of volume and returns. The model leads to several predictions. First, since investors hold only two portfolios—the market portfolio and the hedging portfolio—they trade in only these port-

[1] One example of changes in market conditions is changes in the investment opportunity set considered by Merton (1973).

folios. This implies that trading volume exhibits an approximate two-factor structure: the first factor arises from their trades in the market portfolio and the second factor arises from their trades in the hedging portfolio. In other words, the cross-section of the volume is driven by two common factors.

Moreover, the factor loading of each asset's trading volume on the hedging-portfolio factor is identical to that asset's portfolio weight in the hedging portfolio. This property of the trading volume of individual assets suggests a way to identify the hedging portfolio from the volume data. After arriving at such a portfolio, we can verify that it is indeed the hedging portfolio: its returns should be the best predictor of future returns on the market portfolio. Furthermore, the return on the hedging portfolio is a risk factor, in addition to the market return. Together, these two factors explain the cross section of asset returns.

In addition, the model predicts a close link between volume and the dynamic behavior of asset returns. In particular, high volume is often generated by the desire of a subset of investors to adjust their asset holdings. Such an adjustment causes adverse price changes in order to induce other investors to accommodate. To the extent that these price changes are unrelated to future cash flows, they will be followed by reverse changes. In other words, price changes associated with high trading volume tend to reverse themselves. As a result, volume has a negative impact on the serial correlation of returns.

Having established the theoretical links between volume and prices, I then discuss several empirical tests of these links based on the volume and return data of securities traded on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX). From the trading volume of individual stocks, we first examine if there is any factor structure in the cross-section of volume. We can then construct the hedging portfolio and its returns. It is found that the hedging-portfolio returns consistently outperform other factors in predicting future returns to the market portfolio. Using the returns to the hedging and market portfolios as two risk factors, a cross-sectional test can be conducted along the lines of Fama and MacBeth (1973). It is shown that the hedging portfolio is comparable to other factors in explaining the cross-sectional variation of expected returns.

The paper proceeds as follows. Section 2 presents an intertemporal equilibrium model of asset prices and trading activity. Section 3 examines the implications of the model on the price-volume relations. Section 4 describes the dataset on which the empirical analysis is based. It also provides some exploratory analysis of the data. Section 5-9 review the empirical evidence on the theoretical implications on the cross-section of volume, the link between volume cross section, the time-series and cross-section of returns, and the dynamic relation between volume and returns. Section 10 provides some concluding remarks.

2. AN INTERTEMPORAL CAPITAL ASSET-PRICING MODEL

In this section, I present an intertemporal equilibrium model of stock trading and pricing with multiple assets and heterogeneous investors. The model is a slight variant of the model considered by Lo and Wang (2001b). Since my purpose here is to draw qualitative implications on the joint behavior of return and volume, the model is kept as parsimonious as possible.

2.1. The Economy

Let us consider an economy defined on a set of discrete dates: $t = 0, 1, 2, \ldots$. There are J risky stocks, each pays a stream of dividends over time. As before, D_{jt} denote the dividend of stock j at date t, $j = 1, \cdots, J$, and $D_t \equiv [D_{1t} \cdots D_{Jt}]$ denote the column vector of dividends. Without loss of generality, in this section the total number of shares outstanding is assumed to be one for each stock.

A stock portfolio can be expressed in terms of its shares of each stock, denoted by $S \equiv [S_1 \ldots S_J]$, where S_j is the number of stock j shares in the portfolio ($j = 1, \ldots, J$). A portfolio of particular importance is the market portfolio, denoted by S_M, which is given by

$$S_M = \iota \tag{1}$$

where ι is a column vector of 1's with length J. $D_{Mt} \equiv \iota^\top D_t$ gives the dividend of the market portfolio, which is the aggregate dividend.

In addition to the stocks, there is also a risk-free bond that yields a constant, positive interest r per time period.

There are I investors in the economy. Each investor is endowed with equal shares of the stocks and no bond. Every period, investor i, $i = 1, \ldots, I$, maximizes his expected utility of the following form:

$$\mathrm{E_t}\left[-e^{-W^i_{t+1} - (\lambda_X X_t + \lambda_Y Y^i_t) D_{Mt+1} - \lambda_Z (1+Z^i_t) X_{t+1}}\right] \tag{2}$$

where W^i_{t+1} is investor i's wealth next period, X_t, Y^i_t, Z^i_t are three one-dimensional state variables, and λ_X, λ_Y, λ_Z are non-negative constants. Apparently, the utility function in (2) is state-dependent. We further assume

$$\sum_{i=1}^{I} Y^i_t = \sum_{i=1}^{I} Z^i_t = 0 \tag{3}$$

where $t = 0, 1, \ldots$.

For simplicity, all the exogenous shocks, D_t, X_t, $\{Y_t^i, Z_t^i, i = 1, \ldots, I\}$, are assumed to be IID over time with zero means. For tractability, D_{t+1} and X_{t+1} are assumed to be jointly normally distributed:

$$u_{t+1} \equiv \begin{pmatrix} D_{t+1} \\ X_{t+1} \end{pmatrix} \overset{d}{\sim} N(\cdot, \sigma) \quad \text{where} \quad \sigma = \begin{pmatrix} \sigma_{DD} & \sigma_{DX} \\ \sigma_{XD} & \sigma_{XX} \end{pmatrix}. \tag{4}$$

Without loss of generality, σ_{DD} is assumed to be positive definite. Different from Lo and Wang (2001b), I do not require that X_t is independent from Y_t^i and Z_t^i $\forall$ i, t. In the discussion of dynamic volume-return relations, potential correlation between X_t and Y_t^i becomes important.

Several features of the model deserves clarification. First, the model assumes that investors have a myopic, but state-dependent utility function in (2). This may seem unusual at the first sight. However, as shown in Wang (1994) and Lo and Wang (2001a), this seemingly unusual utility function can be interpreted as a reduced-form equivalent of a value function from an appropriately specified dynamic optimization problem.

The particular form of the utility function and the normality of distribution for the state variables are assumed for tractability. These assumptions are restrictive. But we hope with some confidence that the qualitative predictions of the model that we explore in this paper are not sensitive to these assumptions.

The model also assumes an exogenous interest rate for the bond without requiring the bond market to clear. This is a modelling choice in order to simplify our analysis and to focus on the stock market. As will become clear later, changes in the interest rate is not important for the issues we examine in this paper.

2.2. Equilibrium

Let $P_t \equiv [P_{1t}; \ldots; P_{Jt}]$ and $S_t^i \equiv [S_{1t}^i; \ldots; S_{Jt}^i]$ be the (column) vectors of (ex-dividend) stock prices and investor i's stock holdings respectively. We now derive the equilibrium of the economy.

DEFINITION 2.1. An equilibrium is given by a price process $\{P_t : t = 0, 1, \ldots\}$ and the investors stock positions $\{S_t^i : i = 1, \ldots, I; t = 0, 1, \ldots\}$ such that:

1. S_t^i solves investor i's optimization problem:

$$S_t^i = \arg\max \quad \mathrm{E}\left[-e^{-W_{t+1}^i - (\lambda_X X_t + \lambda_Y Y_t^i) D_{M\,t+1} - \lambda_Z (1+Z_t^i) X_{t+1}}\right] \tag{5}$$

$$\text{s. t.} \quad W_{t+1}^i = W_t^i + S_t^{i\prime}\left[D_{t+1} + P_{t+1} - (1+r)P_t\right]$$

2. stock market clears:

$$\sum_{i=1}^{I} S_t^i = S_M = \iota. \tag{6}$$

The above definition of equilibrium is standard, except that the bond market does not clear here. As discussed earlier, the interest rate is given exogenously and there is an elastic supply of bonds at that rate.

For $t = 0, 1, \ldots$, let Q_{t+1} denote the vector of excess dollar returns on the stocks:

$$Q_{t+1} \equiv D_{t+1} + P_{t+1} - (1{+}r)P_t. \tag{7}$$

Thus, $Q_{jt+1} = D_{jt+1} + P_{jt+1} - (1 + r)P_{jt}$ gives the dollar return on one share of stock j in excess of its financing cost for period $t + 1$. For the remainder of the paper, we simply refer to Q_{jt+1} as the dollar return of stock j, omitting the qualifier "excess". Dollar return Q_{jt+1} differs from the conventional (excess) return measure R_{jt+1} which is the dollar return normalized by the share price: $R_{jt+1} \equiv Q_{jt+1}/P_{jt}$. We refer to R_{jt+1} simply as the return on stock j in period $t + 1$.

THEOREM 1.
The economy defined above has a unique linear equilibrium in which

$$P_t = -a - bX_t \tag{8}$$

$$S_t^i = \left(I^{-1} - \lambda_Y Y_t^i\right)\iota - \left[\lambda_Z Z_t^i + \lambda_Y (b'\iota) Y_t^i\right](\sigma_{QQ})^{-1}\sigma_{QX} \tag{9}$$

where

$$\begin{aligned}
\sigma_{QQ} &= \sigma_{DD} - (b\,\sigma_{XD} + \sigma_{DX} b') + \sigma_X^2 b\, b' \\
\sigma_{QX} &= \sigma_{DX} - \sigma_X^2 b \\
a &= \frac{1}{r}\left(\bar{\alpha}\sigma_{QQ}\iota + \lambda_Z \sigma_{QX}\right) \\
b &= \lambda_X [(1{+}r) + \lambda_Z \sigma_{XD}\iota)]^{-1} \sigma_{DD}\iota
\end{aligned}$$

and $\bar{\alpha} = 1/I$.

Proof. See Lo and Wang (2001b).

The nature of the equilibrium is intuitive. In the model, an investor's utility function depends not only on his wealth, but also on the stock payoffs directly. In other words, even if he holds no stocks, his utility fluctuates with the payoff of the stocks. Such a “market spirit” affects his demand for the stocks, in addition to the usual factors such as the stocks' expected returns. The market spirit of investor i is measured by $(\lambda_X X_t + \lambda_Y Y_t^i)$. When $(\lambda_X X_t + \lambda_Z Y_t^i)$ is positive, investor i extracts positive utility when the aggregate stock payoff is high. Such a positive “attachment” to the market makes holding stocks less attractive to him. When $(\lambda_X X_t + \lambda_Y Y_t^i)$ is negative, he has a negative attachment to the market, which makes holding stocks more attractive. Such a market spirit at the aggregate level, which is captured by X_t, affects the aggregate stock demand, which in turn affects their equilibrium prices. Given the particular form of the utility function, X_t affects the equilibrium stock prices linearly. The idiosyncratic differences among investors in their market spirit, which are captured by Y_t^i, offset each other at the aggregate level, thus do not affect the equilibrium stock prices. However, they do affect individual investors' stock holdings. As the first term of (9) shows, investors with positive Y_t^i's hold less stocks (they are already happy by just “watching” the stocks paying off).

Since the aggregate utility variable X_t is driving the stock prices, it is also driving the stock returns. In fact, the expected returns on the stocks are changing with X_t (see the discussion in the next section). The form of the utility function further states that the investors utility directly depends on X_t, which fully characterizes the investment opportunities investors face. Such a dependence arises endogenously when investors optimize dynamically. In our setting, however, we assume that investors optimize myopically but insert such a dependence directly into the utility function. This dependence induces investors to care about future investment opportunities when choosing their portfolios. In particular, they prefer those portfolios whose returns can help them to smooth fluctuations in their utility due to changes in investment opportunities. Such a preference gives rise to the hedging component in their asset demand, which is captured by the second term in (9).

3. IMPLICATIONS FOR VOLUME AND RETURNS

Given the intertemporal CAPM defined above, we can derive its implications on the behavior of return and volume. For the stocks, their dollar return vector can be re-expressed as follows:

$$Q_{t+1} = ra + (1+r)bX_t + \tilde{Q}_{t+1} \tag{10}$$

where $\tilde{Q}_{t+1} \equiv D_{t+1} - bZ_{t+1}$ denotes the vector of unexpected dollar returns on the stocks, which are IID over time with zero mean. Equation (10) shows that the expected returns on the stocks change over time. In particular, they are driven by a single state variable X_t.

The investors stock holdings can be expressed in the following form:

$$S_t^i = h_{Mt}^i \iota + h_{Ht}^i S_H \quad \forall\ i = 1, 2, \ldots, I \tag{11}$$

where $h_{Mt}^i \equiv I^{-1} - \lambda_Y Y_t^i$, $h_{Ht}^i \equiv \lambda_Z (b'\iota) Y_t^i - \lambda_Y Z_t^i$, and

$$S^H \equiv (\sigma_{QQ})^{-1} \sigma_{QX}. \tag{12}$$

Equation (11) simply states that three-fund separation holds for the investors' stock portfolios. That is, all investors' portfolios can be viewed as investments in three common funds: the risk-free asset and two stock funds. The two stock funds are the market portfolio ι and the hedging portfolio S_H. Moreover, in the current model, these two portfolios, expressed in terms of stock shares, are constant over time.

The particular structure of the returns and the investors' portfolios lead to several interesting predictions about the behavior of volume and returns. I describe these predictions through a set of propositions.

3.1. The Cross Section of Volume

Given that investors only hold and trade in two stock funds, the market portfolio and the hedging portfolio, the turnover of stock j is given by

$$\tau_{jt} \equiv \frac{1}{2} \sum_{i=1}^{I} \left| \left(h_{Mt}^i - h_{Mt-1}^i\right) + \left(h_{Ht}^i - h_{Ht-1}^i\right) S_j^H \right| \quad \forall\ j = 1, \ldots, J. \tag{13}$$

Let τ_t denote the vector of turnover for all stocks. We have the following proposition on the cross-section of volume:

PROPOSITION 1. *When investors' trading in the hedging portfolio is small relative to their trading in the market portfolio, the two-fund separation in their stock holdings leads to an approximate two-factor structure for stock turnover:*

$$\tau_t \approx \iota F_{Mt} + S^H F_{Ht} \tag{14}$$

where

$$F_{Mt} = \frac{1}{2} \sum_{i=1}^{I} \left| h_{Mt}^i - h_{Mt-1}^i \right| \quad \textit{and}$$

$$F_{Ht} = \frac{1}{2} \sum_{i=1}^{I} \operatorname{sgn}\left(h_{Mt}^i - h_{Mt-1}^i\right) \left(h_{Ht}^i - h_{Ht-1}^i\right).$$

Proof. See Lo and Wang (2000).

In the special case when two-fund separation holds (when $X_t = 0 \ \ \forall\ t$), turnover would have an exact one-factor structure, $\tau_t = \iota F_{Mt}$.

In the general case when three-fund separation holds, turnover has an approximate two-factor structure as given in (14). It is important to note that the loading of stock j's turnover on the second factor is proportional to its share weight in the hedging portfolio. Thus, empirically if we can identify the two common factors, F_{Mt} and F_{Ht}, the stocks' loadings on the second factor allow us to identify the hedging portfolio. As discussed below, the hedging portfolio has important properties that allow us to better understand the behavior of returns. Merton (1973) has discussed the properties of hedging portfolios in a continuous-time framework as a characterization of equilibrium. Our discussion here follows Merton in spirit, but is in a discrete-time environment.

3.2. Time Series of Returns and the Hedging Portfolio

By the definition of the hedging portfolio in (12), its current return gives the best forecast of future market return. Let Q_{Mt+1} denote the dollar return on the market portfolio in period $t+1$ and Q_{Ht+1} denote the dollar return on the hedging portfolio. Then,

$$Q_{Mt+1} = \iota' Q_{t+1} \quad \text{and} \quad Q_{Ht+1} = {S_H}' Q_{t+1}. \tag{15}$$

For an arbitrary portfolio S, its dollar return in period t, which is $\mathcal{Q}_t \equiv S'Q_t$, can serve as a predictor for the dollar of the market next period:

$$Q_{Mt+1} = \gamma_0 + \gamma_1 \mathcal{Q}_t + \varepsilon_{Mt+1}.$$

The predictive power of S is measured by the R^2 of the above regression. We can solve for the portfolio that maximizes the R^2. The solution, up to a scaling constant, is the hedging portfolio. Thus, we have the following result:

PROPOSITION 2. *Among the returns of all portfolios, the dollar return of the hedging portfolio, S_H, provides the best forecast for the future dollar return of the market.*

In other words, if we regress the market dollar return on the lagged dollar return of any portfolios, the hedging portfolio gives the highest R^2.

3.3. Cross-Sectional of Returns and the Hedging Portfolio

We now turn to examine the predictions of our model on the cross-section of returns. For expositional simplicity, we introduce some additional notation. Let Q_{pt+1} be the dollar return of a stock or a portfolio (of stocks). $\tilde{Q}_{pt+1} \equiv Q_{pt+1} - \mathrm{E}_t[Q_{pt+1}]$ then denotes its unexpected dollar return and $\bar{Q}_p$ its unconditional mean. Thus, $\tilde{Q}_{Mt+1}$ and $\tilde{Q}_{Ht+1}$ denote, respectively, the unexpected dollar returns on the market portfolio and the hedging portfolio, and

$$\sigma_M^2 \equiv \mathrm{Var}\left[\tilde{Q}_{Mt+1}\right], \quad \sigma_H^2 \equiv \mathrm{Var}\left[\tilde{Q}_{Ht+1}\right], \quad \sigma_{MH} \equiv \mathrm{Cov}\left[\tilde{Q}_{Mt+1}, \tilde{Q}_{Ht+1}\right]$$

denote their conditional variances and covariance. It is easy to show that

$$\sigma_M^2 = \iota' \sigma_{QQ} \iota, \quad \sigma_H^2 = \sigma_{XQ} \left(\sigma_{QQ}\right)^{-1} \sigma_{QX}, \quad \sigma_{MH} = \iota' \sigma_{QX}$$

where σ_{QQ} and σ_{QZ} are given in Theorem 1. Let

$$\bar{Q}_M = \bar{\alpha}\sigma_M^2 + \lambda_Y \sigma_{MH} \qquad \text{and} \qquad \bar{Q}_H = \bar{\alpha}\sigma_{MH} + \lambda_Y \sigma_H^2. \tag{16}$$

From Theorem 1, we have the following result:

PROPOSITION 3. *The stocks' expected returns are given by*

$$\mathrm{E}\left[\tilde{Q}_{t+1} | \tilde{Q}_{Mt+1}, \tilde{Q}_{Ht+1}\right] = \beta_M \tilde{Q}_{Mt+1} + \beta_H \tilde{Q}_{Ht+1} \tag{17a}$$

$$\bar{Q} = \beta_M \bar{Q}_M + \beta_H \bar{Q}_H \tag{17b}$$

where

$$\begin{aligned}(\beta_M, \beta_H) &= \mathrm{Cov}\left[\tilde{Q}_{t+1}, \left(\tilde{Q}_{Mt+1}, \tilde{Q}_{Ht+1}\right)\right] \left\{\mathrm{Var}\left[\left(\tilde{Q}_{Mt+1}, \tilde{Q}_{Ht+1}\right)\right]\right\}^{-1} \\ &= (\sigma_{QM}, \sigma_{QH}) \begin{pmatrix} \sigma_M^2 & \sigma_{MH} \\ \sigma_{MH} & \sigma_H^2 \end{pmatrix}^{-1}\end{aligned}$$

is the vector of the stocks' market betas and hedging betas and $\bar{Q}_M = \bar{\alpha}\sigma_M^2 + \lambda_Y \sigma_{MH}$ *and* $\bar{Q}_H = \bar{\alpha}\sigma_{MH} + \lambda_Y \sigma_H^2$.

Proof. See Lo and Wang (2001b).

In order to develop more intuition about (16), we first consider the special case when $X_t = 0 \ \ \forall\, t$. In this case, returns are IID over time and the hedging portfolio does not appear. The risk of a stock is purely measured by its co-variability with the market portfolio. We have

$$\mathrm{E}\left[\tilde{Q}_{t+1} | \tilde{Q}_{Mt+1}\right] = \beta_M \tilde{Q}_{Mt+1} \tag{18a}$$

$$\bar{Q} = \beta_M \bar{Q}_M \tag{18b}$$

where $\beta_M \equiv \text{Cov}\big[\tilde{Q}_{t+1}, \tilde{Q}_{Mt+1}\big]/\text{Var}\big[\tilde{Q}_{Mt+1}\big] = \sigma_{DD}\iota/(\iota'\sigma_{DD}\iota)$ is the vector of the stocks' market betas and $\bar{Q}_M = \bar{\alpha}\sigma_M^2$ is the market premium. Obviously in this case, the CAPM holds.

In the general case when X_t changes over time, there is an additional risk due to changing market conditions (dynamic risk). Moreover, this risk is represented by the dollar return of the hedging portfolio Q_{Ht}. In this case, the risk of a stock is measured by its risk with respect to the market portfolio *and* its risk with respect to the hedging portfolio. In other words, there are two risk factors, the (contemporaneous) market risk and the (dynamic) risk of changing market conditions. The expected returns of the stocks are then determined by their exposures to these two risks and the associated risk premia.

Thus, a stock's market risk is measured by its beta with respect to the market portfolio and its risk to a changing environment is measured by its beta with respect to the hedging portfolio. The expected dollar return on the market portfolio gives the premium of the market risk and the expected dollar return on the hedging portfolio gives the premium of the dynamic risk. (17) simply states that the premium on a stock is given by the sum of the product of its exposure to each risk and the associated premium.

The pricing relation we obtain in Proposition 3 is in the spirit of Merton's Intertemporal CAPM in a continuous-time framework (Merton, 1971). However, it is important to note that Merton's result is a characterization of the pricing relation under a (class of) proposed price processes and no equilibrium is provided to support these price processes. In contrast, our pricing relation is derived from a dynamic equilibrium model. In this sense, our model provides an particular equilibrium model for which Merton's characterization holds.

If we can identify the hedging portfolio empirically, its return provides the second risk factor. Differences in the stocks' expected returns can then be fully explained by their exposures to the two risks (market risk and dynamic risk), as measured by their market betas and hedging betas. We return to these points in Sections 6-8.

3.4. Dynamic Volume-Return Relation

The discussion above focuses on the implications of ICAPM on the cross-section of volume and its relation with returns. Now, I turn to the time-series relation between volume and returns, and in particular, the joint time-series property of volume and returns. A specific relation to examine is how current volume and returns can forecast future returns. In the context of our model, this relation can be formally expressed by the following conditional expectation: $\text{E}\left[Q_{t+1}|Q_t, \tau_t\right]$.

For parsimony, I consider a simple case of the model when $\lambda_Z = 0$ and $\sigma_{DX} = 0$. In this case, the equilibrium described in Theorem 1 reduces to the following situation:

COROLLARY 1. *When* $\lambda_Z = 0$ *and* $\sigma_{DX} = 0$*, the equilibrium price is* $P_t = -a - bX_t$ *where*

$$a = \frac{\bar{\alpha}}{r}\sigma_{QQ}\iota, \quad b = \frac{\lambda_X}{1+r}\sigma_{DD}\iota, \quad \sigma_{QQ} = (1+\kappa)\sigma_{DD}$$

and the investors' stock holdings are

$$S_t^i = \left(I^{-1} - hY_t^i\right)\iota$$

where $\kappa \equiv \left(\frac{\lambda_X}{1+r}\right)^2 \sigma_X^2 \sigma_{DM}^2$ *and* $h \equiv \lambda_Y\left[1 + (b'\iota)/(1+\kappa)\right]$.

Thus, the investors only trade the market portfolio. Effectively we can treat the market portfolio as a single stock. Thus, for this subsection, we can assume that there is only one stock, which is the market portfolio.

To further simplify the analysis, I let $X_t = Y_t^1 = -Y_t^2$. Thus, X_t and Y_t^i are perfectly correlated (the correlation is 1 with Y_t^1 and -1 with Y_t^2). Of course, we only need partial correlation. The extreme case merely makes the calculations easier.

In this case, the return on the stock and the investors' stock holdings can be expressed as follows:

$$Q_{t+1} = [ra + (1+r)bX_t] + \tilde{Q}_{t+1} \tag{19a}$$

$$S_t^i = \left(\frac{1}{2} - Y_t^i\right) \quad (i = 1, 2) \tag{19b}$$

where $\tilde{Q}_{t+1}$ is normally distributed and uncorrelated with X_t.[2] The turnover of the stock is then

$$\tau_t \equiv \frac{1}{2}\left(|X_t^1 - X_{t-1}^1| + |X_t^2 - X_{t-1}^2|\right) = |X_t - X_{t-1}|. \tag{20}$$

We can now compute the expected return on the stock conditional on the current return and volume. The result is summarized in the following proposition:

[2]From Theorem 1, $(Q_t; X_t)$ is a Gaussian process. $\mathrm{E}[Q_{t+1}|X_t] = ra + (1+r)bX_t$ gives the expectation of Q_{t+1} conditional on X_t and $\tilde{Q}_{t+1}$ is the residual, which is also a normally distributed and independent of X_t.

Proposition 4. *From (19) and (14), we have*

$$\mathrm{E}\left[Q_{t+1}|Q_t,\tau_t\right] = \theta_0+\theta_1 Q_t-\theta_2\tau_t Q_t+\text{higher order terms in } Q_t \text{ and } \tau_t \quad (21)$$

and $\theta_2 \geq 0$.

Proof. See Wang (1994).

In other words, returns accompanied by high volume are more likely to exhibit reversals. Campbell, Grossman and Wang (1992) and Llorente, Michaely, Saar and Wang (2001) have explicitly tested this dynamic relation between volume and returns in the form of (21). We discuss their empirical findings in Section 9.

4. THE DATA

4.1. The Mini-CRSP Dataset

In the remainder of the paper, I discuss the empirical evidence on the price-volume implications discussed above. The empirical analysis is based on the Mini-CRSP dataset used by Lo and Wang (2000). From the University of Chicago's Center for Research in Securities Prices (CRSP) Daily Master File, Lo and Wang have constructed the weekly turnover series for individual NYSE and AMEX securities from July 1962 to December 1996 (1,800 weeks). A weekly horizon is chosen as a compromise between maximizing sample size while minimizing the day-to-day volume and return fluctuations that have less direct economic relevance. In addition, the data is confined to ordinary common shares on the NYSE and AMEX (CRSP sharecodes 10 and 11 only), omitting ADRs, SBIs, REITs, closed-end funds, and other such exotica whose turnover may be difficult to interpret in the usual sense.[3] Also, NASDAQ stocks are omitted altogether since the differences between NASDAQ and the NYSE/AMEX (market structure, market capitalization, etc.) have important implications for the measurement and behavior of volume (see, for example, Atkins and Dyl (1997)), and this should be investigated separately. A more detailed description of the Mini-CRSP dataset is given in Lo and Wang (2000) and Lim, et. al. (1998).

Lo and Wang (2000, 2001a) have provided an extensive exploratory analysis on the Mini-CRSP dataset. In the remainder of this section, I briefly

[3]The bulk of NYSE and AMEX securities are ordinary common shares, hence limiting our sample to securities with sharecodes 10 and 11 is not especially restrictive. For example, on January 2, 1980, the entire NYSE/AMEX universe contained 2,307 securities with sharecode 10, 30 securities with sharecode 11, and 55 securities with sharecodes other than 10 and 11. Ordinary common shares also account for the bulk of the market capitalization of the NYSE and AMEX (excluding ADRs of course).

describe some of their results. In stead of being comprehensive, my objective here is to provide a general sense about various features of the data. More anaysis can be found in their papers.

4.2. Time-Series Properties

The dataset contains between 1,700 and 2,200 individual securities per week over a sample period of 1,800 weeks. Thus, it is difficult to develop simple intuition for the behavior of the entire time-series/cross-section volume data. However, some gross characteristics of trading volume can be observed from the value-weighted and equal-weighted turnover indexes. These indexes are constructed from weekly individual security turnover, where the value-weighted index is re-weighted each week.

Value-weighted and equal-weighted return indexes are also constructed in a similar fashion. These return indexes do not correspond exactly to the time-aggregated CRSP value-weighted and equal-weighted return indexes because we have restricted our universe of securities to ordinary common shares. However, some simple statistical comparisons show that our return indexes and the CRSP return indexes have very similar time series properties. These characteristics are presented in Figure 1 and in Tables 1 and 2. [4]

Summary Statistics

Figure 1a shows that value-weighted turnover has increased dramatically since the mid-1960's, growing from less than 0.20% to over 1% per week. The volatility of value-weighted turnover also increases over this period. However, equal-weighted turnover behaves somewhat differently: Figure 1b shows that it reaches a peak of nearly 2% in 1968, then declines until the 1980's when it returns to a similar level (and goes well beyond it during October 1987). These differences between the value- and equal-weighted indexes suggest that smaller-capitalization companies can have high turnover.

Since turnover is, by definition, an asymmetric measure of trading activity—it cannot be negative—its empirical distribution is naturally skewed. Taking natural logarithms may provide more (visual) information about its behavior and this is done in Figures 1c- 1d. Although a trend is still present, the distribution seems to be less skewed and more stable over time.

[4] Value-weighted and equal-weighted return indexes are also constructed in a similar fashion. These return indexes are based on ordinary common shares, thus they do not correspond exactly to the time-aggregated CRSP value-weighted and equal-weighted return indexes. However, some simple statistical comparisons show that our return indexes and the CRSP return indexes have very similar time series properties.

TABLE 1.

Statistic	τ^{VW}	τ^{EW}	R^{VW}	R^{EW}
Mean	0.78	0.91	0.23	0.32
Std. Dev.	0.48	0.37	1.96	2.21
Skewness	0.66	0.38	−0.41	−0.46
Kurtosis	0.21	−0.09	3.66	6.64
Percentiles:				
Min	0.13	0.24	−15.64	−18.64
5%	0.22	0.37	−3.03	−3.44
10%	0.26	0.44	−2.14	−2.26
25%	0.37	0.59	−0.94	−0.80
50%	0.64	0.91	0.33	0.49
75%	1.19	1.20	1.44	1.53
90%	1.44	1.41	2.37	2.61
95%	1.57	1.55	3.31	3.42
Max	4.06	3.16	8.81	13.68
Autocorrelations:				
ρ_1	91.25	86.73	5.39	25.63
ρ_2	88.59	81.89	−0.21	10.92
ρ_3	87.62	79.30	3.27	9.34
ρ_4	87.44	78.07	−2.03	4.94
ρ_5	87.03	76.47	−2.18	1.11
ρ_6	86.17	74.14	1.70	4.07
ρ_7	87.22	74.16	5.13	1.69
ρ_8	86.57	72.95	−7.15	−5.78
ρ_9	85.92	71.06	2.22	2.54
ρ_{10}	84.63	68.59	−2.34	−2.44
Box-Pierce Q_{10}	13723.0	10525.0	23.0	175.1
	(0.000)	(0.000)	(0.010)	(0.000)

Summary statistics for value-weighted and equal-weighted turnover and return indexes of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for July 1962 to December 1996 (1,800 weeks) and subperiods. Turnover and returns are measured in percent per week and p-values for Box-Pierce statistics are reported in parentheses.

TABLE 2.

Statistic	τ^{VW}	τ^{EW}	R^{VW}	R^{EW}	τ^{VW}	τ^{EW}	R^{VW}	R^{EW}
	1962 to 1966 (234 weeks)				*1982 to 1986 (261 weeks)*			
Mean	0.25	0.57	0.23	0.30	1.20	1.11	0.37	0.39
Std. Dev.	0.07	0.21	1.29	1.54	0.30	0.29	2.01	1.93
Skewness	1.02	1.47	−0.35	−0.76	0.28	0.45	0.42	0.32
Kurtosis	0.80	2.04	1.02	2.50	0.14	−0.28	1.33	1.19
	1967 to 1971 (261 weeks)				*1987 to 1991 (261 weeks)*			
Mean	0.40	0.93	0.18	0.32	1.29	1.15	0.29	0.24
Std. Dev.	0.08	0.32	1.89	2.62	0.35	0.27	2.43	2.62
Skewness	0.17	0.57	0.42	0.40	2.20	2.15	−1.51	−2.06
Kurtosis	−0.42	−0.26	1.52	2.19	14.88	12.81	7.85	16.44
	1972 to 1976 (261 weeks)				*1992 to 1996 (261 weeks)*			
Mean	0.37	0.52	0.10	0.19	1.25	1.31	0.27	0.37
Std. Dev.	0.10	0.20	2.39	2.78	0.23	0.22	1.37	1.41
Skewness	0.93	1.44	−0.13	0.41	−0.06	−0.05	−0.38	−0.48
Kurtosis	1.57	2.59	0.35	1.12	−0.21	−0.24	1.00	1.30
	1977 to 1981 (261 weeks)							
Mean	0.62	0.77	0.21	0.44				
Std. Dev.	0.18	0.22	1.97	2.08				
Skewness	0.29	0.62	−0.33	−1.01				
Kurtosis	−0.58	−0.05	0.31	1.72				

Summary statistics for weekly value-weighted and equal-weighted turnover and return indexes of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) for July 1962 to December 1996 (1,800 weeks) and subperiods. Turnover and returns are measured in percent per week and p-values for Box-Pierce statistics are reported in parentheses.

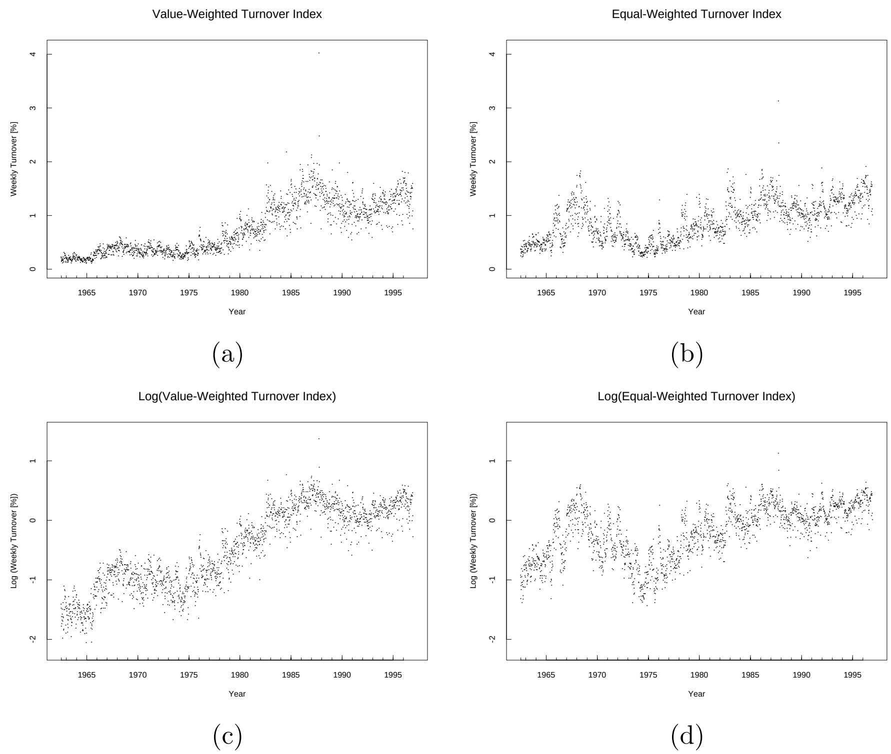

FIG. 1. Weekly Value-Weighted and Equal-Weighted Turnover Indexes, 1962 to 1996.

Table 1 reports various summary statistics for the two indexes over the 1962–1996 sample period, and Table 2 reports similar statistics for five-year subperiods. Over the entire sample the average weekly turnover for the value-weighted and equal-weighted indexes is 0.78% and 0.91%, respectively. The standard deviation of weekly turnover for these two indexes is 0.48% and 0.37%, respectively, yielding a coefficient of variation of 0.62 for the value-weighted turnover index and 0.41 for the equal-weighted turnover index. In contrast, the coefficients of variation for the value-weighted and equal-weighted *returns* indexes are 8.52 and 6.91, respectively. Turnover is not nearly so variable as returns, relative to their means.

Table 2 illustrates the nature of the secular trend in turnover through the five-year subperiod statistics. Average weekly value-weighted and equal-weighted turnover is 0.25% and 0.57%, respectively, in the first subperiod (1962–1966); they grow to 1.25% and 1.31%, respectively, by the last subperiod (1992–1996). At the beginning of the sample, equal-weighted turnover is three to four times more volatile than value-weighted turnover (0.21% versus 0.07% in 1962–1966, 0.32% versus 0.08% in 1967–1971), but by the end of the sample their volatilities are comparable (0.22% versus 0.23% in 1992–1996).

Seasonalities

Tables 3 examines seasonalities in weekly turnover, e.g., quarter-of-the-year, turn-of-the-quarter, and turn-of-the-year effects. It reports regression results for the entire sample period. For brevity, I have omitted the results for the subperiods, which are reported in Lo and Wang (2000). The dependent variable for each regression is either turnover or returns and the independent variables are indicators of the particular seasonality effect. No intercept terms are included in any of these regressions.

Table 3 shows that turnover is relatively stable over quarters—the third quarter has the lowest average turnover, but it differs from the other quarters by less than 0.15% for either turnover index. Turnover tends to be lower at the beginning-of-quarters, beginning-of-years, and end-of-years, but only the end-of-year effect for value-weighted turnover (−0.189%) and the beginning-of-quarter effect for equal-weighted turnover (−0.074) are statistically significant at the 5% level.

Secular Trends

It is well known that turnover is highly persistent. Table 1 shows the first 10 autocorrelations of turnover and returns and the corresponding

TABLE 3.

Regressor	τ^{VW}	τ^{EW}	R^{VW}	R^{EW}
	1962 to 1996 (1,800 weeks)			
Q1	0.842	0.997	0.369	0.706
	(0.025)	(0.019)	(0.102)	(0.112)
Q2	0.791	0.939	0.232	0.217
	(0.024)	(0.018)	(0.097)	(0.107)
Q3	0.741	0.850	0.201	0.245
	(0.023)	(0.018)	(0.095)	(0.105)
Q4	0.807	0.928	0.203	−0.019
	(0.024)	(0.019)	(0.099)	(0.110)
BOQ	−0.062	−0.074	−0.153	−0.070
	(0.042)	(0.032)	(0.171)	(0.189)
EOQ	0.008	−0.010	−0.243	−0.373
	(0.041)	(0.032)	(0.170)	(0.187)
BOY	−0.109	−0.053	0.179	1.962
	(0.086)	(0.067)	(0.355)	(0.392)
EOY	−0.189	−0.085	0.755	1.337
	(0.077)	(0.060)	(0.319)	(0.353)

Seasonality regressions for daily and weekly value-weighted and equal-weighted turnover and return indexes of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) from July 1962 to December 1996. Q1–Q4 are quarterly indicators, BOQ and EOQ are beginning-of-quarter and end-of-quarter indicators, and BOY and EOY are beginning-of-year and end-of-year indicators.

Box-Pierce Q-statistics. Unlike returns, turnover is strongly autocorrelated, with autocorrelations that start at 91.25% and 86.73% for the value-weighted and equal-weighted turnover indexes, respectively, decaying very slowly to 84.63% and 68.59%, respectively, at lag 10. This slow decay suggests some kind of nonstationarity in turnover—perhaps a stochastic trend or *unit root* (see Hamilton (1994), for example). More analysis on the time trend in turnover is provided in Lo and Wang (2000) (see also Andersen (1996) and Gallant, Rossi, and Tauchen (1992)).

For these reasons, many empirical studies of volume use some form of detrending to induce stationarity. This usually involves either taking first differences or estimating the trend and subtracting it from the raw data.

However, Lo and Wang (2000) have shown that different detrending schemes lead to very different statistical properties of the detrended series. In absence of any theoretical justification for a particular detrending scheme and further empirical analysis, only raw turnover is used for most of the empirical analysis in this paper (the two exceptions are the eigenvalue decomposition of the first differences of turnover in Table 7 and the time-series regressions of the dynamic volume-return relation in Table 13 and 15).

To address the problem of the apparent time trend and other nonstationarities in raw turnover, the empirical analysis described in most parts of the paper is conducted within five-year subperiods only (the exploratory data analysis of this section contains entire-sample results primarily for completeness). This is a controversial choice and represents a compromise between imposing sufficient structure to perform meaningful statistical inference and letting the data "speaking for themselves." Readers are referred to Lo and Wang (2000) for more discussions on this issue.

4.3. Cross-Sectional Properties

To develop a sense for cross-sectional differences in turnover over the sample period, we turn our attention from turnover indexes to the turnover of individual securities. Figure 2 provides a compact graphical representation of the cross section of turnover: Figure 2a plots the deciles for the turnover cross-section—nine points, representing the 10-th percentile, the 20-th percentile, and so on—for each of the 1,800 weeks in the sample period; Figure 2b simplifies this by plotting the deciles of the cross section of *average* turnover, averaged within each year; and Figures 2c and 2d plot the same data but on a logarithmic scale.

Figures 2a–b show that while the median turnover (the horizontal bars with vertical sides in Figure 2b) is relatively stable over time—fluctuating

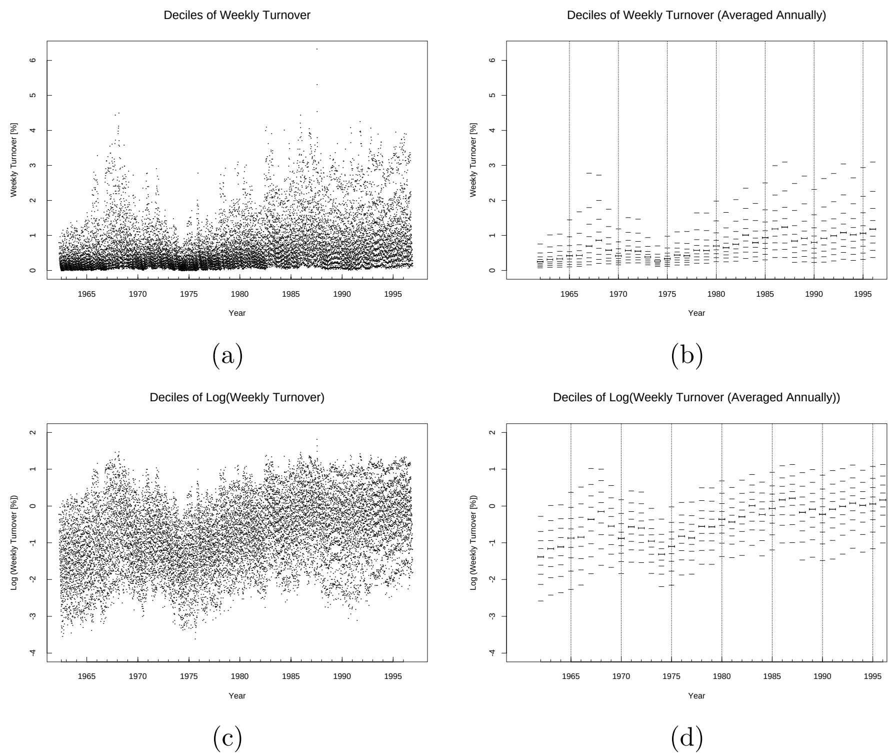

FIG. 2. Deciles of weekly turnover and the natural logarithm of weekly turnover, 1962 to 1996.

between 0.2% and just over 1% over the 1962–1996 sample period—there is considerable variation in the cross-sectional dispersion over time. The range of turnover is relatively narrow in the early 1960's, with 90% of the values falling between 0% and 1.5%, but there is a dramatic increase in the late 1960's, with the 90-th percentile approaching 3% at times. The cross-sectional variation of turnover declines sharply in the mid-1970's and then begins a steady increase until a peak in 1987, followed by a decline and then a gradual increase until 1996.

The logarithmic plots in Figures 2c–d seem to suggest that the cross-sectional distribution of log-turnover is similar over time up to a location parameter. This implies a potentially useful statistical or "reduced-form" description of the cross-sectional distribution of turnover: an identically distributed random variable multiplied by a time-varying scale factor.

To explore the dynamics of the cross section of turnover, we ask the following question: if a stock has high turnover this week, how likely will it continue to be a high-turnover stock next week? Is turnover persistent or are there reversals from one week to the next?

To answer these questions, Table 4 reports the estimated transition probabilities for turnover deciles in adjacent weeks. For example, the first entry of the first row—54.74—implies that 54.74% of the stocks that have turnover in the first decile this week will, on average, still be in the first turnover-decile next week. The next entry—21.51—implies that 21.51% of the stocks in the first turnover-decile this week will, on average, be in the second turnover-decile next week.

These entries indicate some persistence in the cross section of turnover for the extreme deciles, but considerable movement *across* the intermediate deciles. For example, there is only a 18.47% probability that stocks in the fifth decile (40–50%) in one week remain in the fifth decile the next week, and a probability of 12.18% and 11.53% of jumping to the third and seventh deciles, respectively.

For purposes of comparison, Table 5 reports similar transition probabilities estimates for return deciles. Returns are considerably less persistent—indeed, Table 5 provides strong evidence of reversals. For example, stocks in the first return-decile this week have a 19.50% probability of being in the tenth return-decile next week; stocks in the tenth return-decile this week have a 20.49% probability of being in the first return-decile next week.

In summary, the turnover cross-section exhibits considerable variation, some persistence in extreme deciles, and significant movement across intermediate deciles.

TABLE 4.

Turnover Transition		*Next Week Decile*									
		0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
	0–10	54.74	21.51	9.82	5.32	3.17	2.02	1.31	0.93	0.66	0.46
		(0.12)	(0.06)	(0.05)	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)	(0.01)	(0.01)
	10–20	22.12	28.77	19.36	11.48	6.93	4.42	2.95	1.91	1.26	0.75
		(0.06)	(0.10)	(0.06)	(0.05)	(0.05)	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)
	20–30	10.01	20.09	22.37	17.19	11.43	7.50	4.91	3.22	2.05	1.16
		(0.05)	(0.07)	(0.09)	(0.06)	(0.05)	(0.05)	(0.04)	(0.03)	(0.03)	(0.02)
	30–40	5.31	11.92	17.91	19.70	16.21	11.49	7.69	4.97	3.09	1.65
		(0.04)	(0.05)	(0.07)	(0.08)	(0.06)	(0.05)	(0.05)	(0.04)	(0.03)	(0.02)
	40–50	3.15	7.15	12.18	16.81	18.47	15.77	11.53	7.74	4.75	2.40
This		(0.04)	(0.05)	(0.05)	(0.06)	(0.08)	(0.06)	(0.05)	(0.05)	(0.04)	(0.03)
Week	50–60	1.94	4.42	7.82	12.22	16.59	18.37	16.02	11.64	7.33	3.60
		(0.03)	(0.04)	(0.05)	(0.05)	(0.06)	(0.08)	(0.06)	(0.05)	(0.04)	(0.03)
	60–70	1.22	2.79	4.91	8.10	12.41	16.99	19.10	16.84	11.72	5.87
		(0.02)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.07)	(0.06)	(0.05)	(0.04)
	70–80	0.81	1.72	3.05	5.10	8.27	12.73	18.15	21.30	18.69	10.13
		(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.08)	(0.07)	(0.05)
	80–90	0.51	1.04	1.78	2.85	4.58	7.77	13.02	20.78	27.18	20.43
		(0.01)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.05)	(0.07)	(0.09)	(0.06)
	90–100	0.29	0.53	0.79	1.18	1.83	2.97	5.31	10.62	23.28	53.14
		(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.04)	(0.05)	(0.07)	(0.12)

Transition probabilities for weekly turnover deciles (in percents), estimated with weekly turnover of NYSE or AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) from July 1962 to December 1996 (1,800 weeks). Each week all securities with non-missing returns are sorted into turnover deciles and the frequencies of transitions from decile i in one week to decile j in the next week are tabulated for each consecutive pair of weeks and for all (i, j) combinations, $i, j = 1, \ldots, 10$, and then normalized by the number of consecutive pairs of weeks. The number of securities with non-missing returns in any given week varies between 1,700 and 2,200. Standard errors, computed under the assumption of independently and identically distributed transitions, are given in parentheses.

TABLE 5.

Return Transition		*Next Week Decile*									
		0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
	0–10	12.70	8.57	7.20	7.23	7.58	7.77	8.00	9.28	12.13	19.50
		(0.09)	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.07)	(0.11)
	10–20	9.51	9.95	9.60	9.42	9.24	9.44	9.84	10.63	11.38	10.93
		(0.06)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)
	20–30	8.03	9.74	10.43	10.40	10.38	10.51	10.77	10.78	10.33	8.56
		(0.06)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
	30–40	7.60	9.33	10.35	10.85	11.20	11.28	11.22	10.55	9.66	7.90
		(0.06)	(0.05)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.05)	(0.06)
	40–50	7.62	9.07	10.21	10.99	11.70	11.68	11.22	10.38	9.40	7.69
This		(0.07)	(0.05)	(0.06)	(0.07)	(0.08)	(0.07)	(0.07)	(0.06)	(0.05)	(0.06)
Week	50–60	7.43	9.16	10.44	11.11	11.55	11.63	11.29	10.52	9.30	7.52
		(0.07)	(0.05)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.06)	(0.06)
	60–70	7.44	9.61	10.70	11.15	11.17	11.23	11.10	10.45	9.51	7.59
		(0.06)	(0.05)	(0.06)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.06)	(0.05)
	70–80	8.30	10.40	10.88	10.84	10.46	10.40	10.44	10.37	9.78	8.07
		(0.06)	(0.06)	(0.06)	(0.07)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.05)
	80–90	10.92	11.70	10.86	9.93	9.34	9.15	9.30	9.61	9.82	9.32
		(0.07)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
	90–100	20.49	12.39	9.34	8.03	7.28	6.95	6.82	7.38	8.68	12.59
		(0.11)	(0.06)	(0.06)	(0.06)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)	(0.08)

Transition probabilities for weekly return deciles (in percents), estimated with weekly returns of NYSE or AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume) from July 1962 to December 1996 (1,800 weeks). Each week all securities with non-missing returns are sorted into return deciles and the frequencies of transitions from decile i in one week to decile j in the next week are tabulated for each consecutive pair of weeks and for all (i, j) combinations, $i, j = 1, \ldots, 10$, and then normalized by the number of consecutive pairs of weeks. The number of securities with non-missing returns in any given week varies between 1,700 and 2,200. Standard errors, computed under the assumption of independently and identically distributed transitions, are given in parentheses.

It is clear from Figure 2 that turnover varies considerably in the cross section. Lo and Wang (2000) have considered several economically motivated variables, such as expected return, return volatility, dividend yield, size, trading costs, S&P membership, etc., in explaining the cross-section of volume. They found that significant part of the cross-sectional variation can be explained by these variables. For brevity, I omit these results and refer the readers to their paper.

In the following five sections, I discuss the results from the empirical tests of the implications of the intertemporal CAPM on volume and returns.

5. EMPIRICAL TEST OF VOLUME FACTORS

The first set of tests is on the cross-sectional behavior of trading volume. As discussed in Section 3.1, the intertemporal CAPM implies that the volume exhibits an approximate factor structure. In particular, investors only trade in two stock portfolios: the market portfolio and the hedging portfolio. Thus, the volume has two common factors. More generally, there can be more than two portfolios (i.e., the separating funds), which leads to more than two factors. As shown by Lo and Wang (2000), in the case with K separating funds for the stocks, volume has K factors. For the test here, we allow more than two factors and let the data reveals the number of factors.

We can investigate the factor structure by using principal components analysis to decompose the covariance matrix of turnover (see Muirhead (1982) for an exposition of principal components analysis). If turnover is driven by a linear K-factor model, the first K principal components should explain most of the time-series variation in turnover. More formally, if

$$\tau_{jt} = \alpha_j + \delta_1 F_{1t} + \cdots + \delta_K F_{Kt} + \varepsilon_{jt} \tag{22}$$

where $\mathrm{E}[\varepsilon_{jt}\varepsilon_{j't}] = 0$ for any $j \neq j'$, then the covariance matrix Σ of the vector $\tau_t \equiv [\; \tau_{1t} \;\; \cdots \;\; \tau_{Jt} \;]^\top$ can be expressed as

$$\mathrm{Var}[\tau_t] \;\equiv\; \Sigma = \eta\, \Theta\, \eta^\top \tag{23}$$

$$\Theta \;=\; \begin{pmatrix} \theta_1 & 0 & \cdots & 0 \\ 0 & \theta_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & \theta_N \end{pmatrix} \tag{24}$$

where Θ contains the eigenvalues of Σ along its diagonal and η is the matrix of corresponding eigenvectors. Since Σ is a covariance matrix, it is positive semidefinite hence all the eigenvalues are nonnegative. When normalized to sum to one, each eigenvalue can be interpreted as the fraction of the total variance of turnover attributable to the corresponding principal component. If (22) holds, it can be shown that as the size N of the cross section increases without bound, exactly K normalized eigenvalues of Σ approach positive finite limits, and the remaining $N-K$ eigenvalues approach 0 (see, for example, Chamberlain (1983) and Chamberlain and Rothschild (1983)). Therefore, the plausibility of (22), and the value of K, can be gauged by examining the magnitudes of the eigenvalues of Σ.

The only obstacle is the fact that the covariance matrix Σ must be estimated, hence we encounter the well-known problem that the standard estimator

$$\widehat{\Sigma} \equiv \frac{1}{T}\sum_{t=1}^{T}(\tau_t - \bar{\tau})(\tau_t - \bar{\tau})'$$

is singular if the number of securities J in the cross section is larger than the number of time series observations T.[5] Since J is typically much larger than T—for a five-year subperiod T is generally 261 weeks, and J is typically well over 2,000—we need to limit our attention to a smaller subset of stocks. This follows the common practice of forming a small number of portfolios (see Campbell, Lo, and MacKinlay (1997, Chapter 5)), sorted by turnover beta to maximize the dispersion of turnover beta among the portfolios.[6] In particular, within each five-year subperiod, ten turnover-beta-sorted portfolios are formed using betas estimated from the previous five-year subperiod, estimate the covariance matrix $\widehat{\Sigma}$ using 261 time-series observations, and perform a principal-components decomposition on $\widehat{\Sigma}$ (see Lo and Wang (2000) for more details on the procedure). For purposes of comparison and interpretation, a parallel analysis is also performed for returns, using ten return-beta-sorted portfolios. The results are reported in Table 6.

[5] Singularity by itself does not pose any problems for the computation of eigenvalues—this follows from the singular-value decomposition theorem—but it does have implications for the statistical properties of estimated eigenvalues. Preliminary Monte Carlo experiments show that the eigenvalues of a singular estimator of a positive-definite covariance matrix can be severely biased.

[6] The desire to maximize the dispersion of turnover beta is motivated by the same logic used in Black, Jensen, and Scholes (1972): a more dispersed sample provides a more powerful test of a cross-sectional relationship driven by the sorting characteristic.

TABLE 6.

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	Period	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}
Turnover-Beta-Sorted Turnover Portfolios (τ^{VW})											*Return-Beta-Sorted Return Portfolios (R^{VW})*									
85.1	8.5	3.6	1.4	0.8	0.3	0.2	0.1	0.0	0.0	1967 to 1971	85.7	5.9	2.0	1.4	1.4	1.1	0.8	0.7	0.5	0.4
(7.5)	(0.7)	(0.3)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(7.5)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
82.8	7.3	4.9	2.0	1.4	0.8	0.5	0.2	0.1	0.1	1972 to 1976	90.0	3.8	1.8	1.0	0.9	0.7	0.6	0.6	0.4	0.3
(7.3)	(0.6)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)		(7.9)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
83.6	8.6	2.3	2.0	1.2	0.8	0.6	0.4	0.4	0.1	1977 to 1981	85.4	4.8	4.3	1.4	1.3	0.9	0.6	0.5	0.4	0.3
(7.3)	(0.8)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)		(7.5)	(0.4)	(0.4)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
78.9	7.9	3.6	2.9	2.4	1.4	1.3	0.8	0.5	0.4	1982 to 1986	86.6	6.1	2.4	1.6	1.0	0.6	0.5	0.5	0.4	0.3
(6.9)	(0.7)	(0.3)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)		(7.6)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
80.1	6.2	5.2	2.4	1.6	1.3	1.0	1.0	0.8	0.5	1987 to 1991	91.6	2.9	1.7	1.1	0.7	0.6	0.6	0.4	0.3	0.2
(7.0)	(0.5)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)		(8.0)	(0.3)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
71.7	15.6	4.5	2.9	1.8	1.2	0.9	0.8	0.5	0.3	1992 to 1996	72.4	11.6	4.4	3.5	2.2	1.8	1.5	1.1	0.8	0.6
(6.3)	(1.4)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)		(6.3)	(1.0)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)

Eigenvalues $\hat{\theta}_i$, $i = 1, \ldots, 10$ of the covariance matrix of ten out-of-sample-beta-sorted portfolios of weekly turnover and returns of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume)—in percentages (where the eigenvalues are normalized to sum to 100%)—for subperiods of the sample period from July 1962 to December 1996. Turnover portfolios are sorted by out-of-sample turnover betas and return portfolios are sorted by out-of-sample return betas, where the symbols "τ^{VW}" and "R^{VW}" indicate that the betas are computed relative to value-weighted indexes, and "τ^{EW}" and "R^{EW}" indicate that they are are computed relative to equal-weighted indexes. Standard errors for the normalized eigenvalues are given in parentheses and are calculated under the assumption of IID normality.

TABLE 6—*Continued*

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	Period	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}
Turnover-Beta-Sorted Turnover Portfolios (τ^{EW})											*Return-Beta-Sorted Return Portfolios* (R^{EW})									
86.8	7.5	3.0	1.3	0.6	0.5	0.2	0.1	0.1	0.0	1967 to 1971	87.8	4.3	2.2	1.5	1.0	0.9	0.8	0.5	0.5	0.5
(7.6)	(0.7)	(0.3)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)		(7.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
82.8	6.0	5.4	2.9	1.2	1.0	0.4	0.2	0.1	0.0	1972 to 1976	91.6	4.1	0.9	0.8	0.6	0.5	0.4	0.4	0.3	0.3
(7.3)	(0.5)	(0.5)	(0.3)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)		(8.0)	(0.4)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
79.1	8.5	5.4	2.8	1.4	1.0	0.7	0.6	0.3	0.1	1977 to 1981	91.5	3.9	1.4	0.8	0.6	0.5	0.4	0.3	0.3	0.3
(6.9)	(0.7)	(0.5)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)		(8.0)	(0.3)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
78.0	10.4	3.1	2.3	2.0	1.3	1.3	0.8	0.6	0.4	1982 to 1986	88.9	4.4	2.3	1.3	0.7	0.7	0.6	0.5	0.4	0.4
(6.8)	(0.9)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)		(7.8)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
82.5	4.8	3.2	2.4	2.0	1.4	1.3	0.9	0.9	0.6	1987 to 1991	92.7	3.0	1.2	0.7	0.7	0.4	0.4	0.4	0.3	0.2
(7.2)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)		(8.1)	(0.3)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
79.0	8.5	4.9	2.6	1.5	1.1	0.9	0.6	0.5	0.4	1992 to 1996	76.8	10.4	3.9	2.7	1.9	1.1	1.0	0.9	0.7	0.6
(6.9)	(0.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)		(6.7)	(0.9)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.1)

Table 6 contains the principal components decomposition for portfolios sorted on out-of-sample betas, where the betas are estimated in two ways: relative to value-weighted indexes (τ^{VW} and R^{VW}) and equal-weighted indexes (τ^{EW} and R^{EW}).[7] The first principal component typically explains between 70% to 85% of the variation in turnover, and the first two principal components explain almost all of the variation. For example, the upper-left subpanel of Table 6 shows that in the second five-year subperiod (1967–1971), 85.1% of the variation in the turnover of turnover-beta-sorted portfolios (using turnover betas relative to the value-weighted turnover index) is captured by the first principal component, and 93.6% is captured by the first two principal components. Although using betas computed with value-weighted instead of equal-weighted indexes generally yields smaller eigenvalues for the first principal component (and therefore larger values for the remaining principal components) for both turnover and returns, the differences are typically not large.

The importance of the second principal component grows steadily through time for the value-weighted case, reaching a peak of 15.6% in the last subperiod, and the first two principal components account for 87.3% of the variation in turnover in the last subperiod. This is roughly comparable with the return portfolios sorted on value-weighted return-betas—the first principal component is by far the most important, and the importance of the second principal component is most pronounced in the last subperiod. However, the lower left subpanel of Table 6 shows that for turnover portfolios sorted by betas computed against equal-weighted indexes, the second principal component explains approximately the same variation in turnover, varying between 6.0% and 10.4% across the six subperiods.

Of course, one possible explanation for the dominance of the first principal component is the existence of a time trend in turnover. Despite the fact that the analysis is limited to five-year subperiods, within each subperiod there is a certain drift in turnover; might this account for the first principal component? To investigate this conjecture, we perform eigenvalue decompositions for the covariance matrices of the *first differences* of turnover for the 10 turnover portfolios. These results are reported in Table 7 and are consistent with those in Table 6: the first principal component is still the most important, explaining between 60% to 88% of the variation in the first differences of turnover. The second principal component is typically

[7] In particular, the portfolios in a given period are formed by ranking on betas estimated in the immediately preceding subperiod, e.g., the 1992–1996 portfolios were created by sorting on betas estimated in the 1987–1991 subperiod, hence the first subperiod in Table 6 begins in 1967, not 1962.

TABLE 7.

Period	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	$\hat{\theta}_5$	$\hat{\theta}_6$	$\hat{\theta}_7$	$\hat{\theta}_8$	$\hat{\theta}_9$	$\hat{\theta}_{10}$
Out-of-Sample Turnover-Beta-Sorted Turnover-Differences Portfolios (τ^{VW})										
1967 to 1971	82.6	7.1	5.1	2.0	1.6	0.8	0.5	0.1	0.1	0.1
	(7.2)	(0.6)	(0.5)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
1972 to 1976	81.2	6.8	4.7	2.8	2.0	1.0	0.9	0.4	0.2	0.1
	(7.1)	(0.6)	(0.4)	(0.2)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1977 to 1981	85.2	4.5	2.9	2.6	1.6	1.2	0.8	0.5	0.5	0.2
	(7.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1982 to 1986	81.3	5.1	3.5	2.7	2.2	1.7	1.3	0.9	0.7	0.6
	(7.1)	(0.4)	(0.3)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
1987 to 1991	73.1	10.9	4.1	3.0	2.2	1.7	1.6	1.4	1.1	0.9
	(6.4)	(1.0)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)
1992 to 1996	78.4	8.6	4.0	2.8	2.1	1.2	1.0	0.9	0.6	0.4
	(6.9)	(0.8)	(0.4)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
Out-of-Sample Turnover-Beta-Sorted Turnover-Differences Portfolios (τ^{EW})										
1967 to 1971	82.2	8.0	4.5	2.3	1.4	0.7	0.4	0.3	0.1	0.0
	(7.2)	(0.7)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)	(0.0)
1972 to 1976	79.3	7.5	4.8	4.0	1.9	1.3	0.6	0.4	0.2	0.1
	(7.0)	(0.7)	(0.4)	(0.4)	(0.2)	(0.1)	(0.1)	(0.0)	(0.0)	(0.0)
1977 to 1981	80.3	5.3	4.8	3.8	2.0	1.4	1.2	0.7	0.5	0.2
	(7.0)	(0.5)	(0.4)	(0.3)	(0.2)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)
1982 to 1986	82.6	5.0	3.0	2.6	2.0	1.7	1.1	0.9	0.7	0.4
	(7.3)	(0.4)	(0.3)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)
1987 to 1991	77.2	5.5	4.3	2.7	2.5	2.3	1.8	1.6	1.2	1.0
	(6.8)	(0.5)	(0.4)	(0.2)	(0.2)	(0.2)	(0.2)	(0.1)	(0.1)	(0.1)
1992 to 1996	80.4	6.4	4.6	2.6	1.7	1.4	1.1	0.7	0.5	0.4
	(7.1)	(0.6)	(0.4)	(0.2)	(0.1)	(0.1)	(0.1)	(0.1)	(0.0)	(0.0)

Eigenvalues $\hat{\theta}_i$, $i = 1, \ldots, 10$ of the covariance matrix of the first-differences of the weekly turnover of ten out-of-sample-beta-sorted portfolios of NYSE and AMEX ordinary common shares (CRSP share codes 10 and 11, excluding 37 stocks containing Z-errors in reported volume)—in percentages (where the eigenvalues are normalized to sum to 100%)—for subperiods of the sample period from July 1962 to December 1996. Turnover betas are calculated in two ways: with respect to a value-weighted turnover index (τ^{VW}) and an equal-weighted turnover index (τ^{EW}). Standard errors for the normalized eigenvalues are given in parentheses and are calculated under the assumption of IID normality.

responsible for another 5% to 20%. And in one case—in-sample sorting on betas relative to the equal-weighted index during 1987–1991—the third principal component accounts for an additional 10%. These figures suggest that the trend in turnover is unlikely to be the source of the dominant first principal component.

In summary, the results of Tables 6 and 7 indicate that a one-factor model for turnover is a reasonable approximation, at least in the case of turnover-beta-sorted portfolios, and that a two-factor model captures well over 90% of the time-series variation in turnover. This lends some support to the practice of estimating "abnormal" volume by using an event-study style "market model", e.g., Bamber (1986), Jain and Joh (1988), Lakonishok and Smidt (1986), Morse (1980), Richardson, Sefcik, Thompson (1986), Stickel and Verrecchia (1994), and Tkac (1996).

As compelling as these empirical results are from Lo and Wang (2000), it must be kept in mind that there is little statistical inference for the principal components decomposition. In particular, the asymptotic standard errors reported in Tables 6 and 7 were computed under the assumption of IID Gaussian data, hardly appropriate for weekly US stock returns and even less convincing for turnover (see Muirhead (1982, Chapter 9) for further details). Further analysis is needed to assess the statistical significance of these results.

6. EMPIRICAL CONSTRUCTION OF THE HEDGING PORTFOLIO

I now turn to the test of the link between volume cross-section and returns. The first step is to empirically identify the hedging portfolio using the turnover data. From (14), we know that in the two-factor model for turnover in Proposition 1, stock j's loading on the second factor F_{Ht} yields the number of shares (as a fraction of its total number of shares outstanding) of stock j in the hedging portfolio. Thus, we start with an approximate two-factor model for turnover:

$$\tau_{jt} = F_{Mt} + \theta_{Hj} F_{Ht} + \varepsilon_{jt}, \quad j = 1, \ldots, J \tag{25}$$

where F_{Mt} and F_{Ht} are the two factors that generate trading in the market portfolio and the hedging portfolio, respectively, θ_{Hj} is the percentage of shares of stock j in the hedging portfolio (as a percentage of its total number of shares outstanding), and ε_{jt} is the error term, which is assumed to be independent across stocks.[8] If we observe the two common factors, F_{Mt} and F_{Ht}, estimates of individual stock's second factor loadings would allow us to identify the hedging portfolio. Unfortunately, we do not observe the

[8]Cross-sectional independence of the errors is a restrictive assumption. If, for example, there are other common factors in addition to F_{Mt} and F_{Ht}, then ε_{jt} is likely to be correlated across stocks. However, the evidence presented in Section 5 seems to support the two-factor structure, which provides limited justification for our assumption here.

factors. In absence of further theoretical guidance to identify the factors, Lo and Wang (2001b) use two turnover indexes as their proxies: the equally-weighted and share-weighted turnover of the market. Let N_j denote the total number of shares outstanding for stock j and $N \equiv \sum_j N_j$ the total number of shares outstanding of all stocks. The two turnover indexes are

$$\tau_t^{EW} \equiv \frac{1}{J}\sum_{j=1}^{J}\tau_{jt} \approx F_{Mt}+n^{EW}F_{Ht} \qquad \tau_t^{SW} \equiv \sum_{j=1}^{J}\frac{N_j}{N}\tau_{jt} \approx F_{Mt}+n^{SW}F_{Ht} \tag{26}$$

where $n^{EW} = \frac{1}{J}\sum_{j=1}^{J}\theta_{Hj}$ and $n^{SW} = \sum_{j=1}^{J}\frac{N_j}{N}\theta_{Hj}$ are the average percentage of shares of each stock in the hedging portfolio and the percentage of all shares (of all stocks) in the hedging portfolio, respectively.[9] Here, the error terms are omitted in the two indices. Since in (25) are assumed to be independent across stocks, the error terms of the two indexes, which are weighted averages of the error terms of individual stocks, become negligible when the number of stocks is large.

As shown in Lo and Wang (2001b), simple algebra then yields the following relation between individual turnover and the two indexes:

$$\tau_{jt} = \beta_{\tau j}^{SW}\tau_t^{SW} + \beta_{\tau j}^{EW}\tau_t^{EW} + \varepsilon_{jt}, \qquad j = 1, \ldots, J \tag{27a}$$

$$\text{s.t.} \quad \beta_{\tau j}^{EW} + \beta_{\tau j}^{SW} = 1 \tag{27b}$$

$$\sum_{j=1}^{J}\beta_{\tau j}^{EW} = J \tag{27c}$$

where $\beta_{\tau j}^{EW} = \frac{n^{EW}-\theta_{Hj}}{n^{EW}-n^{SW}}$ and $\beta_{\tau j}^{SW} = \frac{\theta_{Hj}-n^{SW}}{n^{EW}-n^{SW}}$.

Estimates of (27) allow us to construct estimates of the hedging portfolio's weights as follows:

$$\widehat{\theta}_{Hj} = (n^{EW} - n^{SW})\widehat{\beta}_{\tau j}^{EW} + n^{SW}. \tag{28}$$

However, there are two remaining parameters, n^{EW} and n^{SW}, that need to be determined.[10] Since the hedging portfolio $\{\theta_{Hj}\}$ is defined only up to a scaling constant, we can set n^{SW} to be 1 and calibrate the remaining parameter $n^{EW} - n^{SW} \equiv \phi$ to the data. This yields the final expression

[9]To avoid degeneracy, we need $N_j \neq N_k$ for some $j \neq k$, which is surely valid empirically.

[10]It should be emphasized that these two remaining degrees of freedom are inherent in the model (25). See Lo and Wang (2001b) for more discussions.

for the J components of the hedging portfolio:

$$\widehat{\theta}_{Hj} = \phi\,\widehat{\beta}^{EW}_{\tau j} + 1. \tag{29}$$

The normalization $n^{SW} = 1$ sets the total number of shares in the portfolio to a positive value. If $\phi = 0$, the portfolio has equal percentage of all the shares of each company, implying that it is the market portfolio. Nonzero values of ϕ represent deviations from the market portfolio.

Lo and Wang (2001b) have estimated (27a)-(27b) for each of the seven five-year subperiods, ignoring the global constraint (27c).[11]

Table 8 reproduces their summary statistics for these constrained regressions. To provide a clearer sense of the dispersion of these regressions, the stocks are first sorted into deciles based on $\{\widehat{\beta}^{EW}_{\tau j}\}$, and then the means and standard deviations of the estimated coefficients $\{\widehat{\beta}^{EW}_{\tau j}\}$ and $\{\widehat{\beta}^{SW}_{\tau j}\}$, their t-statistics, and the $\bar{R}^2$s within each decile are calculated. The t-statistics indicate that the estimated coefficients are generally significant—even in the fifth and sixth deciles, the average t-statistic for $\{\widehat{\beta}^{EW}_{\tau j}\}$ is 4.585 and 6.749, respectively (we would, of course, expect significant t-statistics in the extreme deciles even if the true coefficients were zero, purely from sampling variation). The $\bar{R}^2$s also look impressive, however, they must be interpreted with some caution because of the imposition of the constraint (27), which can yield $\bar{R}^2$ greater than unity and less than zero.[12] Table 8 shows that negative $\bar{R}^2$s appear mainly in the two extreme deciles, except in the last subperiod when they are negative for all the deciles, presumably an indication that the constraint is not consistent with the data in this last subperiod.

Lo and Wang (2001b) have also estimated the unconstrained version of (27). Without the constraint, the $\bar{R}^2$ are well behaved, and of similar magnitude to those in Table 8 that are between 0% and 100%, ranging from 40% to 60%. Clearly the two-factor model of turnover accounts for a significant amount of variation in the weekly turnover of individual stocks. Moreover, except for the last subperiod, the constraint seems to be reasonably consistent with the data (with average p-values well above 5% for all but the extreme deciles in most subperiod). Interested readers are referred to their paper for details.

[11] Because of the large number of individual regressions involved, neglecting the reduction of one dimension should not significantly affect any of the final results.

[12] For example, a negative $\bar{R}^2$ arises when the variance of $\widehat{\beta}^{EW}_{\tau j}\tau^{EW}_t + \widehat{\beta}^{SW}_{\tau j}\tau^{SW}_t$ exceeds the variance of the dependent variable τ_{jt}, which can happen when the constraint (27) is imposed.

TABLE 8.

Decile	Sample Size	$\widehat{\beta}_\tau^{\mathrm{EW}}$ Mean	$\widehat{\beta}_\tau^{\mathrm{EW}}$ S.D.	$t(\widehat{\beta}_\tau^{\mathrm{EW}})$ Mean	$t(\widehat{\beta}_\tau^{\mathrm{EW}})$ S.D.	$\widehat{\beta}_\tau^{\mathrm{SW}}$ Mean	$\widehat{\beta}_\tau^{\mathrm{SW}}$ S.D.	$t(\widehat{\beta}_\tau^{\mathrm{SW}})$ Mean	$t(\widehat{\beta}_\tau^{\mathrm{SW}})$ S.D.	$\overline{R}^2$ (%) Mean	$\overline{R}^2$ (%) S.D.
		July 1962 to December 1966 (234 Weeks)									
1	218	−0.906	0.119	−49.394	19.023	1.906	0.119	103.944	38.755	−2520.4	27817.4
2	219	−0.657	0.069	−26.187	12.805	1.657	0.069	65.488	30.083	56.5	19.5
3	219	−0.432	0.064	−10.917	5.956	1.432	0.064	35.879	17.907	55.0	20.4
4	218	−0.188	0.082	−3.812	2.732	1.188	0.082	22.907	10.555	57.1	17.8
5	219	0.107	0.097	1.273	1.243	0.893	0.097	11.365	4.570	51.5	16.0
6	219	0.494	0.119	4.585	1.943	0.506	0.119	4.847	2.401	50.6	16.5
7	218	0.927	0.145	6.749	2.258	0.073	0.145	0.639	1.190	50.7	15.5
8	219	1.520	0.229	8.229	2.893	−0.520	0.229	−2.714	1.348	49.2	15.4
9	219	2.568	0.434	10.410	3.491	−1.568	0.434	−6.292	2.401	49.4	15.2
10	218	6.563	4.100	11.682	3.880	−5.563	4.100	−9.500	3.332	47.1	15.3
		January 1967 to December 1971 (261 Weeks)									
1	242	−0.783	0.134	−36.725	17.343	1.783	0.134	84.302	38.946	−175.3	976.2
2	243	−0.529	0.056	−18.772	8.459	1.529	0.056	53.969	22.871	58.2	16.1
3	242	−0.315	0.068	−7.905	4.099	1.315	0.068	32.431	13.771	56.4	16.3
4	243	−0.054	0.089	−1.139	1.845	1.054	0.089	18.479	7.855	55.2	14.3
5	242	0.264	0.087	3.269	1.482	0.736	0.087	9.228	3.260	54.1	13.2
6	243	0.623	0.126	6.035	2.217	0.377	0.126	3.723	1.871	53.5	13.4
7	243	1.110	0.154	8.367	2.719	−0.110	0.154	−0.735	1.178	54.4	13.0
8	242	1.782	0.205	10.314	3.151	−0.782	0.205	−4.477	1.630	53.2	13.2
9	243	2.661	0.330	12.249	3.120	−1.661	0.330	−7.609	2.149	54.6	11.0
10	242	5.410	2.540	13.019	4.172	−4.410	2.540	−10.260	3.383	52.6	14.2

Summary statistics for the restricted volume betas using weekly returns and volume data for NYSE and AMEX stocks from 1962 to 1981 in five-year subperiods. Turnover over individual stocks is regressed on the equally-weighted and share- weighted turnover indices, subject to the restriction that the two regression coefficients, $\widehat{\beta}_\tau^{\mathrm{EW}}$ and $\widehat{\beta}_\tau^{\mathrm{SW}}$, must add up to one. The stocks are then sorted into ten deciles by $\widehat{\beta}_\tau^{\mathrm{EW}}$. The summary statistics are then reported for each decile.

TABLE 8—*Continued*

Decile	Sample Size	$\widehat{\beta}_\tau^{\mathrm{EW}}$ Mean	$\widehat{\beta}_\tau^{\mathrm{EW}}$ S.D.	$t(\widehat{\beta}_\tau^{\mathrm{EW}})$ Mean	$t(\widehat{\beta}_\tau^{\mathrm{EW}})$ S.D.	$\widehat{\beta}_\tau^{\mathrm{SW}}$ Mean	$\widehat{\beta}_\tau^{\mathrm{SW}}$ S.D.	$t(\widehat{\beta}_\tau^{\mathrm{SW}})$ Mean	$t(\widehat{\beta}_\tau^{\mathrm{SW}})$ S.D.	$\overline{R}^2$ (%) Mean	$\overline{R}^2$ (%) S.D.
January 1972 to December 1977 (261 Weeks)											
1	262	−2.013	0.845	−13.276	4.901	3.013	0.845	20.755	8.319	−1147.6	5034.9
2	263	−1.069	0.129	−10.986	3.890	2.069	0.129	21.239	7.045	25.4	44.6
3	263	−0.697	0.096	−6.014	2.466	1.697	0.096	14.600	5.619	44.3	27.1
4	263	−0.359	0.105	−2.825	1.444	1.359	0.105	10.608	4.044	50.3	22.8
5	263	0.015	0.114	0.062	0.765	0.985	0.114	6.620	2.466	53.0	19.2
6	263	0.485	0.156	2.577	1.159	0.515	0.156	2.792	1.354	52.8	15.4
7	263	1.084	0.187	4.684	1.801	−0.084	0.187	−0.322	0.870	51.4	14.5
8	263	1.888	0.289	6.827	2.426	−0.888	0.289	−3.180	1.421	52.8	14.2
9	263	3.161	0.501	8.894	3.311	−2.161	0.501	−6.060	2.431	52.5	14.0
10	262	7.770	4.940	11.202	4.447	−6.770	4.940	−9.480	3.965	52.3	13.8
January 1977 to December 1981 (261 Weeks)											
1	242	−3.096	0.347	−22.164	4.591	4.096	0.347	29.341	5.815	−872.7	6958.8
2	243	−2.284	0.192	−15.799	4.883	3.284	0.192	22.701	6.846	32.7	23.6
3	243	−1.654	0.208	−10.524	4.628	2.654	0.208	16.861	7.167	48.9	20.8
4	243	−1.021	0.156	−5.505	2.335	2.021	0.156	10.884	4.304	54.1	18.4
5	243	−0.394	0.189	−1.833	1.180	1.394	0.189	6.387	2.655	55.6	17.1
6	243	0.355	0.250	1.277	1.045	0.645	0.250	2.472	1.438	55.5	16.5
7	243	1.330	0.308	3.864	1.519	−0.330	0.308	−0.894	0.971	53.6	15.7
8	243	2.599	0.457	6.198	2.242	−1.599	0.457	−3.782	1.560	54.5	15.7
9	243	4.913	0.809	8.860	2.983	−3.913	0.809	−7.038	2.487	55.3	14.5
10	242	10.090	4.231	11.202	3.618	−9.090	4.231	−9.980	3.311	55.2	13.4
January 1982 to December 1986 (261 Weeks)											
1	227	−6.968	3.038	−5.636	2.328	7.968	3.038	6.525	2.577	46.6	15.9
2	228	−2.257	0.624	−3.249	1.604	3.257	0.624	4.724	2.199	52.7	20.2
3	228	−0.640	0.380	−1.223	0.967	1.640	0.380	3.180	1.667	45.5	136.9
4	227	0.501	0.283	1.166	0.841	0.499	0.283	1.177	0.903	55.4	22.4
5	228	1.357	0.231	3.540	1.655	−0.357	0.231	−0.954	0.786	41.3	90.7
6	228	2.077	0.201	5.319	2.159	−1.077	0.201	−2.758	1.216	−19.5	686.3
7	227	2.754	0.196	7.402	2.342	−1.754	0.196	−4.710	1.531	28.3	52.8
8	228	3.431	0.201	9.244	2.667	−2.431	0.201	−6.548	1.922	3.2	101.8
9	228	4.168	0.237	11.354	2.905	−3.168	0.237	−8.630	2.248	−163.1	1678.6
10	227	5.399	1.170	14.045	5.229	−4.399	1.170	−11.392	4.405	−348.1	1027.1

TABLE 8—*Continued*

Decile	Sample Size	$\widehat{\beta}_\tau^{\text{EW}}$ Mean	$\widehat{\beta}_\tau^{\text{EW}}$ S.D.	$t(\widehat{\beta}_\tau^{\text{EW}})$ Mean	$t(\widehat{\beta}_\tau^{\text{EW}})$ S.D.	$\widehat{\beta}_\tau^{\text{SW}}$ Mean	$\widehat{\beta}_\tau^{\text{SW}}$ S.D.	$t(\widehat{\beta}_\tau^{\text{SW}})$ Mean	$t(\widehat{\beta}_\tau^{\text{SW}})$ S.D.	$\overline{R}^2$ (%) Mean	$\overline{R}^2$ (%) S.D.
January 1987 to December 1991 (261 Weeks)											
1	216	−8.487	7.040	−7.093	3.763	9.487	7.040	8.082	4.137	50.2	16.8
2	217	−2.866	0.725	−4.616	2.439	3.866	0.725	6.263	3.224	54.8	18.8
3	217	−0.843	0.494	−1.832	1.512	1.843	0.494	4.097	2.537	56.8	21.0
4	217	0.441	0.330	1.196	1.277	0.559	0.330	1.423	1.268	57.0	19.9
5	217	1.502	0.317	4.887	3.062	−0.502	0.317	−1.693	1.583	57.8	18.8
6	217	2.510	0.280	8.434	4.070	−1.510	0.280	−5.074	2.582	51.2	18.7
7	217	3.389	0.234	12.139	4.615	−2.389	0.234	−8.567	3.325	42.2	15.6
8	217	4.157	0.196	15.329	4.607	−3.157	0.196	−11.637	3.513	23.8	19.8
9	217	4.836	0.212	18.370	4.580	−3.836	0.212	−14.572	3.673	−27.0	66.1
10	217	5.743	0.402	21.430	5.101	−4.743	0.402	−17.682	4.229	−921.9	4682.1
January 1992 to December 1996 (261 Weeks)											
1	241	−4.275	2.858	−2.409	1.092	5.275	2.858	3.097	1.342	−423.6	3336.7
2	241	−1.074	0.384	−1.277	0.741	2.074	0.384	2.538	1.369	−147.7	2631.2
3	242	−0.245	0.155	−0.371	0.301	1.245	0.155	1.944	0.899	−14.7	508.2
4	241	0.189	0.100	0.298	0.203	0.811	0.100	1.296	0.534	−135.1	899.3
5	241	0.520	0.098	0.779	0.313	0.480	0.098	0.729	0.330	−1353.9	5755.2
6	242	0.865	0.106	1.226	0.414	0.135	0.106	0.196	0.177	−197.6	669.1
7	241	1.303	0.159	1.725	0.641	−0.303	0.159	−0.400	0.260	−130.3	931.7
8	242	2.022	0.254	2.391	0.824	−1.022	0.254	−1.202	0.480	−58.9	684.5
9	241	3.271	0.498	3.061	1.027	−2.271	0.498	−2.117	0.769	−24.9	225.8
10	241	8.234	9.836	3.844	1.360	−7.234	9.836	−3.237	1.190	−219.9	1145.7

7. THE HEDGING PORTFOLIO RETURN AS A RETURN PREDICTOR

Having constructed the hedging portfolio up to a parameter ϕ, which is to be determined, we can examine its time-series properties as predicted by the model. In particular, in this section, we focus on the degree to which the the hedging portfolio can predict future stock returns, especially the return on the market portfolio. The return of the hedging portfolio are constructed in Section 7.1 by calibrating ϕ, and its forecast power is compared with with other factors in Sections 7.2 and 7.3. The results here are from Lo and Wang (2001b).

7.1. Hedging-Portfolio Returns

To construct the return on the hedging portfolio, let us begin by calculating its dollar value and dollar returns. For stock j to be included in the hedging portfolio in a subperiod, which is referred to as the "testing period", it is required to have volume data for at least one third of the sample in the previous subperiod ($k-1$), which we call the "estimation period". Among the stocks satisfying this criteria, we eliminate those ranked in the top and bottom 0.5% according to their volume betas (or their share weights in the hedging portfolio) to limit the potential impact of outliers (see Lo and Wang (2001a) for the importance of outliers in volume data). The set of stocks that survive these two filters are included in the hedging portfolio with their appropriate share weights. We can then compute its return and dollar return during the subperiod.

This procedure yields the return and the dollar return of the hedging portfolio up to the parameter ϕ, which must be calibrated. Lo and Wang (2001b) further specify ϕ by exploiting a key property of the hedging portfolio: its return is the best forecaster of future market returns (see Section 3.2). They chose ϕ to maximize the R^2 of the following regression

$$R_{Mt+1} = \delta_0 + \delta_1 \{R_{Ht} \text{ or } Q_{Ht}\} + \varepsilon_{Mt+1} \tag{30}$$

where the single regressor is either the return of the hedging portfolio R_{Ht} or its dollar return.

For subperiods 2 to 7, the values for ϕ that give the maximum R^2 are 1.25, 4.75, 1.75, 47, 38, and 0.25, respectively, using R_{Ht} as the predictor. Using Q_{Ht}, the values of ϕ are 1.5, 4.25, 2, 20, 27, and 0.75, respectively. With these values of ϕ in hand, we have fully specified the hedging portfolio, its return and dollar return. Table 9 reports the summary statistics for the return and dollar return on the hedging portfolio.

TABLE 9.

Statistic	Sample Period						
	Entire	67–71	72–76	77–81	82–86	87–91	92–96
	Hedging Portfolio Return R_{Ht}						
Mean	0.013	0.001	0.005	0.007	0.011	0.052	0.003
S.D.	0.198	0.029	0.039	0.045	0.046	0.477	0.013
Skewness	24.092	0.557	0.542	−0.330	0.270	10.200	−0.214
Kurtosis	747.809	1.479	7.597	0.727	1.347	130.476	0.945
ρ_1	0.017	0.199	0.141	0.196	0.125	0.004	−0.165
ρ_2	−0.058	0.018	0.006	0.071	0.036	−0.070	−0.028
ρ_3	0.104	−0.028	−0.036	−0.010	0.073	0.099	−0.003
ρ_4	0.184	0.070	0.043	0.045	−0.113	0.182	−0.010
ρ_5	−0.086	0.114	0.144	−0.026	−0.103	−0.099	−0.025
ρ_6	0.079	−0.003	0.258	−0.089	−0.093	0.072	0.020
ρ_7	0.217	0.037	0.083	−0.031	−0.173	0.218	0.098
ρ_8	−0.098	0.002	−0.124	−0.008	0.006	−0.111	−0.130
ρ_9	0.048	−0.002	−0.008	−0.060	0.011	0.041	0.006
ρ_{10}	−0.044	−0.017	0.174	−0.037	−0.117	−0.055	0.035
	Hedging Portfolio Dollar Return Q_{Ht}						
Mean	2.113	0.072	1.236	2.258	5.589	3.244	0.281
S.D.	16.836	3.639	11.059	21.495	25.423	20.906	1.845
Skewness	0.717	0.210	−0.144	−0.495	−0.080	2.086	0.215
Kurtosis	14.082	−0.085	0.500	2.286	6.537	13.286	2.048
ρ_1	0.164	0.219	0.251	0.200	0.098	0.157	−0.122
ρ_2	0.082	0.014	0.148	0.052	0.125	−0.015	−0.095
ρ_3	0.039	0.003	0.077	0.010	0.071	−0.041	0.037
ρ_4	0.021	0.061	0.084	0.127	−0.037	−0.066	0.014
ρ_5	0.036	0.116	0.102	−0.002	0.051	−0.016	−0.027
ρ_6	−0.010	−0.044	0.127	−0.094	−0.053	0.057	−0.014
ρ_7	−0.006	0.034	0.013	−0.060	−0.014	0.010	0.107
ρ_8	−0.046	0.005	−0.055	−0.028	−0.127	0.016	−0.075
ρ_9	0.027	−0.016	0.045	−0.006	0.047	0.005	−0.006
ρ_{10}	−0.001	−0.030	0.042	0.026	0.014	−0.082	0.031

Summary statistics for the returns and dollar returns of the hedging portfolio constructed from individual stocks' volume data using weekly returns and volume data for NYSE and AMEX stocks from 1962 to 1996.

7.2. Optimal Forecasting Portfolios (OFPs)

Having constructed the return of the hedging portfolio in Section 7.1, we want to compare its forecast power to those of other forecastors. According to Proposition 2, the returns of the hedging portfolio should outperform the returns of any other portfolios in predicting future market returns. Specifically, if we regress R_{Mt} on the lagged return of any arbitrary portfolio p, the $\bar{R}^2$ should be less than that of (30).

It is impractical to compare (30) to all possible portfolios, and uninformative to compare it to random portfolios. Instead, we need only make comparisons to "optimal forecast portfolios", portfolios that are optimal forecasters of R_{Mt}, since by construction, no other portfolios can have higher levels of predictability than these. Lo and Wang (2001b) have provided the following procedure to construct optimal forecasting portfolios (OFPs):

Proposition 5. *Let Γ_0 and Γ_1 denote the contemporaneous and first-order autocovariance matrix of the vector of all returns. For any arbitrary target portfolio q with weights $w_q = (w_{q1}; \ldots; w_{qN})$, define $A \equiv {\Gamma_0}^{-1}\Gamma_1 w_q {w_q}' {\Gamma_1}'$. The optimal forecast portfolio of w_q is given by the normalized eigenvector of A corresponding to its largest eigenvalue.*

Given the large number of stocks in the sample (over 2,000 in each subperiod) and the relatively short time series in each subperiod (261 weekly observations), the standard estimators for Γ_0 and Γ_1 are not viable. However, it is possible to construct OFPs from a much smaller number of "basis portfolios", and then compare the predictive power of these OFPs to the hedging portfolio. As long as the basis portfolios are not too specialized, the $\bar{R}^2$s are likely to be similar to those obtained from the entire universe of all stocks.

Lo and Wang (2001b) have considered several sets of basis portfolios by sorting all the J stocks into K groups of equal numbers ($K \leq J$) according to: market capitalization, market beta, and SIC codes, and then construct value-weighted portfolios within each group.[13] This procedure yields K basis portfolios for which the corresponding Γ_0 and Γ_1 can be estimated using the portfolios' weekly returns within each subperiod. Based on the esti-

[13]It is important that value-weighted portfolios are used here so that the market portfolio, whose return we wish to predict, is a portfolio of these basic portfolios (recall that the target portfolio ω_q to be forecasted is a linear combination of the vector of returns for which Γ_k is the k-th order autocovariance matrix).

mated autocovariance matrices, the OFP can be computed easily according to Proposition 5.

The choice of the number of basis portfolios K faces the following tradeoff: fewer portfolios yield better sampling properties for the covariance matrix estimators, but less desirable properties for the OFP since the predictive power of the OFP is obviously maximized when when $K = J$. As a compromise, for the OFPs based market capitalization and market betas, K is chosen to be 10, 15, 20, and 25. For the OFP based on SIC codes, 13 industry groupings are chosen.

Specifically, for each five-year subperiod in which the forecast power of the hedging portfolio (the testing period), the previous five-year subperiod (the estimation period) to estimate the OFPs. For example, for the OFP based on 10 market-capitalization-sorted portfolios, which is called "CAP10", 10 value-weighted portfolios are constructed each week, one for each market-capitalization decile. Market-capitalization deciles are recomputed each week, and the time series of decile returns form the 10 basis portfolio returns of CAP10, with which we can estimate Γ_0 and Γ_1. To compute the OFP, the weights ω_q of the target portfolio are set to be those of the market portfolio. Since the testing period follows the estimation period, the market capitalization of each group in the last week of the estimation period are used to map the weights of the market portfolio into a 10×1-vector of weights for the 10 basis portfolios. The weights of the OFP for the basis portfolios CAP10 follow immediately from Proposition 5. The same procedure is used to form OFPs for CAP15, CAP20, and CAP25 basis portfolios. To form OFPs for Beta10, Beta15, Beta20 and Beta25, a similar procedure is followed, except that market betas of individual stocks in the estimation period are used to sort them and form the basis portfolios. For the industry portfolios, each week the stocks are sorted into industry groups according to their SIC code as follows:

#	SIC Codes	Description
1	1–14	Agriculture, forest, fishing, mining
2	15–19, 30, 32–34	Construction, basic materials (steel, glass, concrete, etc.)
3	20–21	Food and tobacco
4	22, 23, 25, 31, 39	Textiles, clothing, consumer products
5	24, 26–27	Logging, paper, printing, publishing
6	28	Chemicals
7	29	Petroleum
8	35–36, 38	Machinery and equipment supply, including computers
9	37, 40–47	Transportation-related
10	48–49	Utilities and telecommunications
11	50–59	Wholesale distributors, retail
12	60–69	Financial
13	70–89, 98–99	Recreation, entertainment, services, conglomerates, etc.

The value-weighted returns are then computed for each group, yielding the 13 basis portfolios which are denoted by "SIC13". The autocovariance matrices are then estimated and the OFP constructed according to Proposition 5.

7.3. Forecast Power of the Hedging Portfolio

Table 10 reports the results of the regressions of R_{Mt} on various lagged OFP returns and on the hedging portfolios R_{Ht} and Q_{Ht}. For completeness, the table also included four additional regressions, with lagged value- and equal-weighted CRSP index returns, the logarithm of the reciprocal of lagged market-capitalization, and the lagged three-month constant-maturity Treasury bill return as predictors. Table 10 shows that the hedging portfolios outperforms all of the other competing portfolios in forecasting future market returns in three of the six subperiods (subperiods 2, 4, and 6). In subperiod 3, only one OFP (Beta20) outperforms the hedging portfolio, and in subperiod 5, Beta20 and SIC13's OFPs outperform the hedging portfolio, but only marginally. And in subperiod 7, the equal-weighted CRSP index return outperforms the hedging portfolio.

TABLE 10.

Parameter	Beta10	Beta15	Beta20	Beta25	Cap10	Cap15	Cap20	Cap25	SIC13	R_H	Q_H	$\log(\text{Cap}^{-1})$	VW	EW	TBill
January 1967 to December 1971 (261 Weeks)															
Intercept	0.002	0.002	0.001	0.002	0.001	0.002	0.002	0.002	0.001	0.001	0.172	0.746	0.001	0.001	—
t-Stat	1.330	1.360	1.150	1.430	1.240	1.520	1.400	1.380	0.920	1.270	1.200	2.330	1.240	1.250	—
Slope	0.103	−0.034	−0.153	0.171	−0.262	0.173	−0.039	−0.176	−0.208	0.138	0.154	0.027	0.191	0.092	—
t-Stat	1.810	−0.550	−1.890	1.780	−1.900	1.079	−0.240	−1.070	−2.860	3.460	3.900	2.330	3.130	2.080	—
$\overline{R}^2$	0.013	0.001	0.014	0.012	0.014	0.005	0.000	0.005	0.031	0.045	0.056	0.021	0.037	0.016	—
January 1972 to December 1976 (261 Weeks)															
Intercept	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.103	0.389	0.001	0.001	—
t-Stat	0.650	0.640	0.560	0.670	0.830	0.640	0.730	0.630	0.630	0.820	0.760	1.410	0.700	0.640	—
Slope	0.023	0.204	−0.315	0.079	0.235	0.098	−0.169	0.069	0.040	−0.054	−0.023	0.014	−0.003	0.048	—
t-Stat	0.120	1.150	−2.630	0.580	1.660	0.660	−1.180	0.430	0.430	−1.430	−1.900	1.410	−0.060	0.910	—
$\overline{R}^2$	0.000	0.005	0.026	0.001	0.011	0.002	0.005	0.001	0.001	0.008	0.014	0.008	0.000	0.003	—
January 1977 to December 1981 (261 Weeks)															
Intercept	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.223	0.151	0.002	0.002	—
t-Stat	1.750	1.600	1.800	1.640	1.770	1.760	1.800	1.530	1.749	1.500	1.370	0.720	1.570	1.380	—
Slope	0.007	0.071	0.065	0.033	0.075	0.003	−0.204	−0.186	0.150	0.049	0.013	0.005	0.069	0.080	—
t-Stat	0.040	0.870	0.460	0.510	0.230	0.010	−0.850	−0.990	1.130	1.810	1.760	0.710	1.110	1.370	—
$\overline{R}^2$	0.000	0.003	0.001	0.001	0.000	0.000	0.003	0.004	0.005	0.013	0.012	0.002	0.005	0.007	—

Forecast of weekly market-portfolio returns by lagged weekly returns of the beta-sorted optimal forecast portfolios (OFPs), the market-capitalization-sorted OFP's, the SIC-sorted OFP, the return and dollar return on the hedging portfolio, minus log-market-capitalization, the lagged returns on the CRSP value- and equal-weighted portfolios, and lagged constant-maturity (three-month) Treasury bill rates from 1962 to 1981 in five-year subperiods. The value of ϕ is 1.25 for the return R_H and 1.5 for the dollar return Q_H on the hedging portfolio, respectively.

TABLE 10—*Continued*

Parameter	Beta10	Beta15	Beta20	Beta25	Cap10	Cap15	Cap20	Cap25	SIC13	R_H	Q_H	−Cap	VW	EW	TBill
January 1982 to December 1986 (261 Weeks)															
Intercept	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.003	0.004	0.672	0.179	0.003	0.003	0.010
t-Stat	3.150	3.150	3.130	3.180	3.150	3.160	3.110	3.150	2.640	3.500	3.190	1.130	2.690	2.710	1.860
Slope	−0.006	0.154	−0.309	0.154	−0.105	−0.054	0.142	0.099	−0.203	−0.047	−0.012	0.006	0.068	0.053	−4.053
t-Stat	−0.030	0.910	−1.990	1.180	−0.470	−0.220	0.740	0.530	−1.890	−1.760	−1.490	1.110	1.100	0.820	−1.212
$\overline{R}^2$	0.000	0.003	0.015	0.005	0.001	0.000	0.002	0.001	0.014	0.012	0.009	0.005	0.005	0.003	0.006
January 1987 to December 1991 (261 Weeks)															
Intercept	0.003	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.392	0.559	0.003	0.003	0.10
t-Stat	1.700	1.650	1.770	1.770	1.680	1.730	1.700	1.720	1.800	2.280	2.050	1.460	1.820	1.880	1.098
Slope	0.294	−0.353	0.120	0.130	−0.540	−0.062	0.072	−0.033	0.210	−0.014	−0.023	0.020	0.058	0.032	−5.598
t-Stat	1.580	−2.000	0.680	0.820	−2.320	−0.320	0.320	−0.190	2.320	−4.500	−2.490	1.460	0.930	0.550	−0.810
$\overline{R}^2$	0.010	0.015	0.002	0.003	0.021	0.000	0.000	0.000	0.021	0.073	0.024	0.008	0.003	0.001	0.003
January 1992 to December 1996 (261 Weeks)															
Intercept	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.416	−0.107	0.003	0.003	−0.003
t-Stat	3.170	3.120	3.110	3.060	3.060	3.130	3.120	3.170	3.130	3.700	3.510	−0.780	3.710	4.000	−0.881
Slope	0.118	−0.009	0.090	−0.095	−0.191	−0.040	0.033	−0.074	−0.047	−0.194	−0.153	−0.004	−0.163	−0.192	7.280
t-Stat	1.060	−0.080	0.930	−0.850	−1.090	−0.270	0.240	−0.550	−0.700	−2.910	−2.410	−0.800	−2.710	−3.320	1.661
$\overline{R}^2$	0.004	0.000	0.003	0.003	0.005	0.000	0.000	0.001	0.002	0.032	0.022	0.003	0.028	0.041	0.011

However, comments are in order with regard to the three subperiods in which the hedging portfolios were surpassed by one or two competing portfolios. First, in these three subperiods, the hedging portfolio still outperforms most of the other competing portfolios. Second, there is no consistent winner in these subperiods. Third, the performance of the hedging portfolios are often close to the best performer. Moreover, the best performers in these subperiods performed poorly in the other subperiods, raising the possibility that their performance might be due to sampling variation. In contrast, the hedging portfolios forecasted R_{Mt} consistently in every subperiod. Indeed, among all of the regressors, the hedging portfolios were the most consistent across all six subperiods, a remarkable confirmation of the properties of the model of Sections 2 and 3.2.[14]

8. THE HEDGING-PORTFOLIO RETURN AS A RISK FACTOR

To evaluate the success of the hedging-portfolio return as a risk factor in the cross section of expected returns, Lo and Wang (2001b) implemente a slightly modified version of the well-known regression tests outlined in Fama and MacBeth (1973). The basic approach is the same: form portfolios sorted by an estimated parameter such as market beta coefficients in one time period (the "portfolio-formation period"), estimate betas for those same portfolios in a second non-overlapping time period (the "estimation period"), and perform a cross-sectional regression test for the explanatory power of those betas using the returns of a third non-overlapping time period (the "testing period"). However, in contrast to Fama and MacBeth (1973), weekly instead of monthly returns are used for portfolio-formation, estimation, and testing periods are five years each.

Specifically, Lo and Wang (2001b) run the following bivariate regression for each security in the portfolio-formation period, using only those securities that exist in all three periods:

$$R_{jt} = \alpha_j + \beta_j^M R_{Mt} \ + \ \beta_j^H R_{Ht} + \varepsilon_{it} \tag{31}$$

where R_{Mt} is the return on the CRSP value-weighted index and R_{Ht} is the return on the hedging portfolio. Using the estimated coefficients $\{\widehat{\beta}_i^M\}$

[14] On the other hand, the results in Table 10 is tempered by the fact that the OFPs are only as good as the basis portfolios from which they are constructed. Increasing the number of basis portfolios should, in principle, increase the predictive power of the OFP. However, as the number of basis portfolios increases, the estimation errors in the autocovariance estimators $\widehat{\gamma}_0$ and $\widehat{\gamma}_1$ also increase for a fixed set of time series observations, hence the impact on the predictive power of the OFP is not clear.

and $\{\widehat{\beta}_i^H\}$, the individual stocks are double sorted in the estimation period, creating 100 portfolios corresponding to the deciles of the estimated market and hedging-portfolio betas. The two betas for each of these 100 portfolios are re-estimated in the estimation period, and they are used as regressors in the testing period in the following cross-sectional regression:

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^H + \varepsilon_{pt} \qquad (32)$$

where R_{pt} is the equal-weighted portfolio return for securities in portfolio p, $p = 1, \ldots, 100$, constructed from the double-sorted rankings of the portfolio-estimation period, and $\widehat{\beta}_{pt}^M$ and $\widehat{\beta}_{pt}^H$ are the market and hedging-portfolio returns, respectively, of portfolio p obtained from the estimation period. This cross-sectional regression is estimated for each of the 261 weeks in the five-year testing period, yielding a time series of coefficients $\{\widehat{\gamma}_{0t}\}$, $\{\widehat{\gamma}_{1t}\}$, and $\{\widehat{\gamma}_{2t}\}$. Summary statistics for these coefficients and their diagnostics are then reported, and this entire procedure is repeated by incrementing the portfolio-formation, estimation, and testing periods by five years. The same analysis is performed for the hedge-portfolio dollar-return series $\{Q_{Ht}\}$.

Because weekly instead of monthly data is used, it may be difficult to compare our results to other cross-sectional tests in the extant literature, e.g., Fama and French (1992). Therefore, Lo and Wang (2001b) apply the same procedure to two other benchmark models: the standard CAPM in which R_{Mt} is the only regressor in (31) and 100 market-beta-sorted portfolios constructed, and a two-factor model in which the hedging-portfolio return factor is replaced by a "small-minus-big capitalization" or "SMB" portfolio return factor as in Fama and French (1993).[15] Table 11 reports the the correlations between the different portfolio return factors, returns on CRSP value- and equal-weighted portfolios, return and dollar return on the hedging portfolio, returns on the SMB portfolio, and the two turnover indices. Here, for brevity, only the results for the whole sample are reported. Similar results for each of the subperiods can be found in Lo and Wang (2001b).

Table 12 summarizes the results of all of these cross-sectional regression tests for each of the five testing periods from 1972 to 1996. In the first subpanel, corresponding to the first testing period from 1972 to 1976, there is little evidence in support of the CAPM or any of the two-factor models

[15] Specifically, the SMB portfolio return is constructed by taking the difference of the value-weighted returns of securities with market capitalization below and above the median market capitalization at the start of the five-year subperiod.

TABLE 11.

	R_{VWt}	R_{EWt}	R_{Ht}	Q_{Ht}	R_{SMBt}	R_{OFPt}	τ_t^{EW}	τ_t^{SW}
	July 1962 to December 1996 (1,800 Weeks)							
R_{VWt}	100.0	88.7	−13.2	15.6	14.0	−26.9	10.6	8.1
R_{EWt}	88.7	100.0	−15.8	4.6	53.5	−25.3	12.6	5.5
R_{Ht}	−13.2	−15.8	100.0	40.3	−10.7	−11.0	14.9	16.8
Q_{Ht}	15.6	4.6	40.3	100.0	−13.3	−6.7	7.5	9.9
R_{SMBt}	14.0	53.5	−10.7	−13.3	100.0	−4.8	4.6	−5.8
R_{OFPt}	−26.9	−25.3	−11.0	−6.7	−4.8	100.0	−4.9	−2.4
τ_t^{EW}	10.6	12.6	14.9	7.5	4.6	−4.9	100.0	86.2
τ_t^{SW}	8.1	5.5	16.8	9.9	−5.8	−2.4	86.2	100.0

Correlation matrix for weekly returns on the CRSP value-weighted index (R_{VWt}), the CRSP equal-weighted index (R_{EWt}), the hedging-portfolio return (R_{Ht}), the hedging-portfolio dollar-return (Q_{Ht}), the return of the small-minus-big capitalization stocks portfolio (R_{SMBt}), the return return R_{OFPt} of the optimal-forecast portfolio (OFP) for the set of 25 market-beta-sorted basis portfolios, and the equal-weighted and share-weighted turnover indices (τ_t^{EW} and τ_t^{SW}), using CRSP weekly returns and volume data for NYSE and AMEX stocks from 1962 to 1996.

estimated. For example, the first three rows show that the time-series average of the market-beta coefficients, $\{\widehat{\gamma}_{1t}\}$, is 0.000, with a t-statistic of 0.348 and an average $\bar{R}^2$ of 10.0%.[16] When the hedging-portfolio beta $\widehat{\beta}_t^H$ is added to the regression, the $\bar{R}^2$ does increase to 14.3% but the average of the coefficients $\{\hat{\gamma}_{2t}\}$ is −0.002 with a t-statistic of −0.820. The average market-beta coefficient is still insignificant, but it has now switched sign. The results for the two-factor model with the hedging-portfolio dollar-return factor and the two-factor model with the SMB factor are similar.

In the second testing period, both specifications with the hedging-portfolio factor exhibit statistically significant means for the hedging-portfolio betas, with average coefficients and t-statistics of −0.012 and −3.712 for the hedging-portfolio return factor and −1.564 and −4.140 for the hedging-portfolio dollar-return factor, respectively. In contrast, the market-beta coefficients are not significant in either of these specifications, and are also of the wrong sign. The only other specification with a significant mean coefficient is the two-factor model with SMB as the second factor, with an average coefficient of 0.299 for the SMB factor and a t-statistic of 4.433.

[16]The t-statistic is computed under the assumption of independently and identically distributed coefficients $\{\gamma_{1t}\}$, which may not be appropriate. However, since this has become the standard method for reporting the results of these cross-sectional regression tests, we follow this convention to make our results comparable to those in the literature.

TABLE 12.

Model	Statistic	$\widehat{\gamma}_{0t}$	$\widehat{\gamma}_{1t}$	$\widehat{\gamma}_{2t}$	$\bar{R}^2$ (%)
January 1972 to December 1976 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \epsilon_{pt}$	Mean:	0.002	0.000		10.0
	S.D.:	0.015	0.021		10.9
	t-Stat:	1.639	0.348		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \varepsilon_{pt}$	Mean:	0.004	−0.002	−0.002	14.3
($\phi = 1.25$)	S.D.:	0.035	0.035	0.037	10.9
	t-Stat:	2.040	−1.047	−0.820	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \varepsilon_{pt}$	Mean:	0.004	−0.002	−0.104	15.5
($\phi = 1.50$)	S.D.:	0.032	0.034	3.797	10.9
	t-Stat:	2.162	−1.081	−0.442	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \varepsilon_{pt}$	Mean:	0.001	0.000	0.063	12.1
	S.D.:	0.014	0.024	1.142	10.8
	t-Stat:	1.424	0.217	0.898	
January 1977 to December 1981 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \epsilon_{pt}$	Mean:	0.001	0.003		11.7
	S.D.:	0.011	0.022		12.8
	t-Stat:	1.166	2.566		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \varepsilon_{pt}$	Mean:	0.003	−0.001	−0.012	13.1
($\phi = 4.75$)	S.D.:	0.014	0.020	0.051	12.4
	t-Stat:	3.748	−0.902	−3.712	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \varepsilon_{pt}$	Mean:	0.003	−0.001	−1.564	12.5
($\phi = 4.25$)	S.D.:	0.013	0.020	6.104	12.2
	t-Stat:	3.910	−0.754	−4.140	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \varepsilon_{pt}$	Mean:	0.001	0.000	0.299	14.9
	S.D.:	0.011	0.017	1.088	13.4
	t-Stat:	2.251	−0.164	4.433	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{OFP} + \varepsilon_{pt}$	Mean:	0.003	0.001	0.001	14.1
	S.D.:	0.018	0.023	0.036	11.6
	t-Stat:	2.735	0.843	0.632	

Cross-sectional regression tests of various linear factor models along the lines of Fama and MacBeth (1973) using weekly returns for NYSE and AMEX stocks from 1962 to 1996, five-year subperiods for the portfolio-formation, estimation, and testing periods, and 100 portfolios in the cross-sectional regressions each week. The five linear-factor models are: the standard CAPM ($\widehat{\beta}_p^M$), and four two-factor models in which the first factor is the market beta and the second factors are, respectively, the hedging portfolio return beta ($\widehat{\beta}_p^{HR}$), the hedging portfolio dollar-return beta ($\widehat{\beta}_p^{HQ}$), the beta of a small-minus-big cap portfolio return ($\widehat{\beta}_p^{SMB}$), and the beta of the optimal forecast portfolio based on a set of 25 market-beta-sorted basis portfolios ($\widehat{\beta}_p^{OFP}$).

TABLE 12—*Continued*

Model	Statistic	$\widehat{\gamma}_{0t}$	$\widehat{\gamma}_{1t}$	$\widehat{\gamma}_{2t}$	$\bar{R}^2$ (%)
January 1982 to December 1986 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \epsilon_{pt}$	Mean:	0.006	−0.001		9.4
	S.D.:	0.011	0.019		11.1
	t-Stat:	8.169	−1.044		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \varepsilon_{pt}$	Mean:	0.006	−0.001	−0.006	9.6
($\phi = 1.75$)	S.D.:	0.011	0.020	0.055	9.4
	t-Stat:	8.390	−0.780	−1.732	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \varepsilon_{pt}$	Mean:	0.006	−0.002	−0.740	10.4
($\phi = 2.00$)	S.D.:	0.011	0.019	19.874	9.5
	t-Stat:	8.360	−1.297	−0.602	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \varepsilon_{pt}$	Mean:	0.005	−0.002	0.038	10.0
	S.D.:	0.012	0.019	1.154	8.4
	t-Stat:	7.451	−1.264	0.531	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{OFP} + \varepsilon_{pt}$	Mean:	0.005	−0.001	0.000	11.7
	S.D.:	0.011	0.020	0.021	10.8
	t-Stat:	7.545	−0.818	0.199	
January 1987 to December 1991 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \varepsilon_{pt}$	Mean:	0.002	0.000		5.9
	S.D.:	0.013	0.023		8.7
	t-Stat:	2.649	0.204		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \varepsilon_{pt}$	Mean:	0.002	0.000	0.000	5.4
($\phi = 47$)	S.D.:	0.016	0.019	0.060	6.1
	t-Stat:	2.254	0.105	0.132	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \varepsilon_{pt}$	Mean:	0.002	0.000	0.189	6.0
($\phi = 20$)	S.D.:	0.016	0.019	18.194	6.7
	t-Stat:	2.434	−0.147	0.168	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \varepsilon_{pt}$	Mean:	0.003	0.000	−0.075	7.8
	S.D.:	0.014	0.020	1.235	8.2
	t-Stat:	3.101	0.158	−0.979	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{OFP} + \varepsilon_{pt}$	Mean:	0.003	−0.001	0.000	6.4
	S.D.:	0.015	0.021	0.021	7.3
	t-Stat:	2.731	−0.385	−0.234	

TABLE 12—*Continued*

Model	Statistic	$\widehat{\gamma}_{0t}$	$\widehat{\gamma}_{1t}$	$\widehat{\gamma}_{2t}$	$\bar{R}^2$ (%)
January 1992 to December 1996 (261 Weeks)					
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \varepsilon_{pt}$	Mean:	0.002	0.001		5.7
	S.D.:	0.013	0.020		7.7
	t-Stat:	2.679	1.178		
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HR} + \varepsilon_{pt}$	Mean:	0.002	0.001	−0.004	6.9
($\phi = 38$)	S.D.:	0.013	0.020	0.091	6.8
	t-Stat:	2.785	1.164	−0.650	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{HQ} + \varepsilon_{pt}$	Mean:	0.003	0.000	−1.584	6.2
($\phi = 27$)	S.D.:	0.015	0.022	12.992	6.6
	t-Stat:	3.279	−0.178	−1.970	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{SMB} + \varepsilon_{pt}$	Mean:	0.002	0.001	0.154	6.7
	S.D.:	0.015	0.019	1.157	7.0
	t-Stat:	1.653	0.861	2.147	
$R_{pt} = \gamma_{0t} + \gamma_{1t}\widehat{\beta}_p^M + \gamma_{2t}\widehat{\beta}_p^{OFP} + \varepsilon_{pt}$	Mean:	0.001	0.002	0.002	7.9
	S.D.:	0.016	0.020	0.015	7.4
	t-Stat:	0.895	1.236	2.407	

For the three remaining test periods, the only specifications with any statistically significant factors is the two-factor models in the 1992–1996 testing period. However, the $\bar{R}^2$s in the last two testing periods are substantially lower than in the earlier periods, perhaps reflecting the greater volatility of equity returns in recent years.

Overall, the results do not provide overwhelming support for any factor in explaining the cross-sectional variation of expected returns. There is, of course, the ubiquitous problem of lack of power in these cross-sectional regression tests, hence we should not be surprised that no single factor stands out (see, for example, MacKinlay (1987, 1994)). However, the point estimates of the cross-sectional regressions show that the hedging-portfolio factor is comparable in magnitude and in performance to other commonly proposed factors.

9. EMPIRICAL VOLUME-RETURN DYNAMICS

In this section, I review the empirical support for the implications of our model for the joint behavior of returns and volume as discussed in Section 3.4.

Many authors have explored the dynamic volume-return relation (21). For example, Campbell, Grossman and Wang (1993) (CGW) present a formal model to derive it and provided supporting evidence based on market indices and aggregate volume measures (see also LeBaron (1992)). Antoniewicz (1993) and Stickel and Verrecchia (1994) examine the same relation using pooled returns and volume of individual stocks and find conflicting evidence. Wang (1994) develops a model to incorporate the effect of information asymmetry on volume-return relations, which provides a framework to explain the difference between market indices and individual stocks. Llorente, Michaely, Saar and Wang (2001) (LMSW) sharpened the results of Wang (1994) and empirically examined the cross-sectional variations in the volume-return relation predicted by the model. Their results reconcile the previous empirical evidence on the volume-return relation.

I first describe the test of (21) using volume and return on market indices. I then turn to the tests using individual securities. In these tests, we need to further consider the effect of information asymmetry on the dynamic volume-return relation, which is not captured in the model presented in Section 2. I briefly describe these effects, relying on Wang (1994), and then discuss the empirical evidence.

9.1. Empirical Evidence from Market Indices

In testing the dynamic volume-return relation, we need to reconsider the time interval at which the the relation is most prominent. This in part depends on the dynamics of the state variables, especially X_t, as shown in Wang (1994). It also depends on practical factors concerning the data such as zero daily volume for many small stocks. CGW and LMSW show that the result becomes weaker with longer intervals. For this reason, in this section we will use daily data on turnover and returns for the tests. The sample period is again from June 1962 to December 1996 as in the previous sections.

Another deviation in the empirical strategy from the previous sections is regarding detrending. In order to make the results comparable to those in CGW and LMSW, both using a moving average detrending, I follow the same procedure here. In particular, the detrended turnover is defined by

$$\tilde{\tau}_t^i = \log \tau_t^i - \frac{1}{250} \sum_{s=-250}^{-1} \log \tau_{t+s}^i \tag{33}$$

where $i = EW, VW$ for equal- and value-weighted indices, respectively.

Following Campbell, Grossman and Wang (1993), the following equation is estimated using daily return and detrended turnover of the two market indices:

$$R_{t+1}^i = \theta_0^i + \left(\sum_{d=1}^{5} \theta_{1d}^i \delta_{d,t} + \theta_2^i \tilde{\tau}_t^i \right) R_t^i + \varepsilon_{t+1}^i \tag{34}$$

where R_t^i denotes the rate of return on index i and $\delta_{d,t}$ is a dummy variable for each day of a week. For example, $\delta_{1,t}$ is 1 if day t is a Monday and 0 otherwise.

It should be pointed out that there is some discrepancy between the variables proposed in the theoretical model and those used in the empirical test. In particular, the model considers excess dollar returns per share and turnover while the empirical analysis considers returns per dollar and detrended log-turnover. The difference between the theoretical and corresponding empirical variables is mainly a matter of normalization. At the daily frequency that we focus on, the relation among these variables should not be very sensitive to the normalization used here.

The results are reported in Table 13. For the whole sample, the estimate of θ_2 is negative and significant. For the seven subperiods, the estimate of θ_2 is always negative and significant for most subperiods. The results here are qualitatively the same as those reported in CGW. Both set of results provide supporting evidence to the theoretical prediction of the model on the dynamic volume-return relation.

There are some quantitative differences between the results here and the results reported in CGW. They come from two sources. One is that a longer sample period is used here. Another is that equal- or value-weighed turnover indices are used here while CGW used the total number of shares traded in the market as a measure of aggregate volume.

9.2. The Impact of Asymmetric Information

Several authors have also tested (21) using volume and return data on individual securities. For example, CGW also used the largest 32 stocks in addition to market indices and found similar results. However, Antoniewicz (1993) and Stickel and Verrecchia (1994) used a large sample of stocks. Based on pooled regressions, they obtained positive estimates for θ_2.

Interestingly, this difference was predicted by theoretical model of Wang (1994). His model incorporates two trading motives: risk sharing and speculation based on private information. He shows that while trading for risk sharing tend to generate negatively serial correlation in returns conditional on high volume (hence a negative θ_2), trading on private information tend to generate positively serial correlation in returns conditional on high volume (hence a positive θ_2).

TABLE 13.

Subperiod	Index	θ_2	θ_0	θ_{11}	θ_{12}	θ_{13}	θ_{14}	θ_{15}
1	EW	−0.438	0.000	0.218	0.299	0.394	0.583	0.374
		(0.103)	(0.000)	(0.060)	(0.076)	(0.068)	(0.076)	(0.074)
	VW	−0.466	0.000	0.128	0.372	0.284	0.426	0.202
		(0.117)	(0.000)	(0.068)	(0.078)	(0.070)	(0.085)	(0.087)
2	EW	−0.313	0.001	0.330	0.224	0.539	0.500	0.649
		(0.081)	(0.000)	(0.048)	(0.062)	(0.056)	(0.060)	(0.065)
	VW	−0.425	0.000	0.272	0.215	0.429	0.414	0.563
		(0.106)	(0.000)	(0.052)	(0.064)	(0.056)	(0.064)	(0.069)
3	EW	−0.178	0.000	0.366	0.564	0.443	0.462	0.620
		(0.079)	(0.000)	(0.051)	(0.061)	(0.056)	(0.057)	(0.059)
	VW	−0.413	0.000	0.180	0.385	0.262	0.366	0.415
		(0.106)	(0.000)	(0.057)	(0.063)	(0.059)	(0.064)	(0.063)
4	EW	−0.373	0.001	0.244	0.391	0.393	0.388	0.526
		(0.085)	(0.000)	(0.056)	(0.063)	(0.066)	(0.066)	(0.061)
	VW	−0.334	0.000	−0.021	0.209	0.182	0.376	0.479
		(0.115)	(0.000)	(0.055)	(0.064)	(0.066)	(0.068)	(0.068)
5	EW	−0.134	0.001	0.221	0.282	0.418	0.435	0.264
		(0.087)	(0.000)	(0.057)	(0.063)	(0.065)	(0.064)	(0.067)
	VW	−0.061	0.000	0.024	0.084	0.185	0.302	0.187
		(0.102)	(0.000)	(0.061)	(0.063)	(0.069)	(0.070)	(0.075)
6	EW	−0.233	0.000	0.445	−0.175	0.040	0.489	0.776
		(0.060)	(0.000)	(0.053)	(0.075)	(0.068)	(0.072)	(0.063)
	VW	0.098	0.001	−0.172	0.169	−0.112	0.131	0.488
		(0.065)	(0.000)	(0.061)	(0.077)	(0.075)	(0.072)	(0.065)
7	EW	−0.231	0.001	0.200	0.415	0.271	0.317	0.233
		(0.137)	(0.000)	(0.060)	(0.061)	(0.062)	(0.063)	(0.061)
	VW	−0.383	0.001	0.002	0.251	0.200	0.143	0.156
		(0.141)	(0.000)	(0.061)	(0.064)	(0.072)	(0.065)	(0.061)
Total	EW	−0.271	0.000	0.350	0.286	0.364	0.462	0.567
		(0.027)	(0.000)	(0.020)	(0.025)	(0.023)	(0.024)	(0.024)
	VW	−0.161	0.000	0.057	0.221	0.175	0.286	0.404
		(0.030)	(0.000)	(0.022)	(0.025)	(0.025)	(0.026)	(0.026)

Impact of volume on the serial correlation of market returns. We use equal- or value-weighted turnover and returns as measures of volume and return indices, respectively. For each index, we measure the impact of volume on the serial correlation of returns by the parameter θ_2 from the regression: $R^i_{t+1} = \theta^i_0 + \left(\sum_{d=1}^{5} \theta^i_{1d} + \theta^i_2 \tilde{\tau}^i_t\right) R_{jt} + \epsilon^i_{t+1}$, where $i = EW, VW$ denote equal- or value-weighted volume and return indices, R^i_t is the daily return of index i and $\tilde{\tau}^i_t$ is its daily detrended log-turnover. We report the estimate for each parameter and its standard errors (in the parenthesis).

The dynamic equilibrium model of Section 2 does not consider the possibility of information asymmetry, which is an important aspect of financial markets. To fully examine the impact of information asymmetry on trading and pricing and in particular on the dynamic volume-return relation is beyond the scope of this paper. However, to extend the model of Section 2 to include certain form of information asymmetry among investors is feasible. The single-asset case is given in Wang (1994) and LMSW. The multi-asset case was presented in an earlier version of LMSW under the myopic assumption and also in Wu and Zhou (2001).

Instead of describing these formal models here, I summarize their qualitative predictions on the dynamic volume-return relation and provide the economic intuition behind these predictions. Interested readers are referred to these papers for more details.

Without information asymmetry, returns accompanied with higher volume are more likely to reverse themselves, as Proposition 4 states. The intuition behind this result is simple: Suppose (a subset of) investors sell stocks to adjust their risk profile in response to exogenous shocks to their risk exposure or preferences (e.g., their exposure to the non-financial risk), stock prices must decrease to attract other investors to take the other side of the trade. Consequently, we have a negative current return accompanied by high volume. Since the expectation of future stock dividends has not changed, the decrease in current prices corresponds to an increase in expected future returns.

In the presence of information asymmetry, especially when some investors have superior information about the stocks relative to other investors, the dynamic volume-return relation can be different. Suppose, for example, better informed investors reduce their stock positions in response to negative private information about future dividends. Current prices have to decrease to attract other investors to buy. However, prices will not drop by the amount that fully reflects the negative information since the market, in general, is not informationally efficient (in our model, due to incompleteness). As a result, they drop further later as more of the negative information gets impounded into prices (through additional trading or public revelation). In this case, we have negative current returns accompanied by high volume, followed by lower returns later.

Of course, investors may still trade for non-informational reasons (e.g., hedging their non-financial risk), in which case the opposite is true, as discussed before. The net effect, i.e., the expected future return conditioned on current return and volume, depends on the relative importance of asymmetric information. Wang (1994) has shown that in the presence of severe information asymmetry, it is possible that θ_2 becomes negative. Under the simplifying assumption that private information is short-lived and investors are myopic, LMSW further show that θ_2 decreases monotonically with the degree of information asymmetry. The differences in the degree of information asymmetry can give rise to differences in the dynamic volume-return relation across stocks.

9.3. Empirical Evidence from Individual Securities

We now discuss the empirical results presented in LMSW in testing (21) for individual stocks. From the CRSP dataset, they have chosen the sample to be stocks traded on the NYSE and AMEX between January 1, 1983 and December 31, 1992.[17] Stocks with more than 20 consecutive days of missing values or more than 20 consecutive days without a single trade are excluded from the sample. The characteristics of the sample are given in Table 14, which reports the summary statistics of the average market capitalization, average daily share volume, average daily turnover and average price for the whole sample and for the five quintiles.

For the stocks in the sample, the return series is the daily returns from CRSP, and turnover is used as a measure of trading volume. To be consistent with the existing literature, LMSW measure turnover in logs and detrend the resulting series. Since stocks often have zero daily trading volume, we add a small constant (0.00000255) to the turnover before taking logs.[18] The resulting series is detrended by subtracting a 200-trading-day moving average:

$$\tilde{\tau}_{jt} = \log\left(\tau_{jt} + 0.00000255\right) - \frac{1}{200}\sum_{s=-200}^{-1} \log\left(\tau_{jt+s} + 0.00000255\right). \quad (35)$$

The time window for moving average is slightly narrower than the 250-day window used in CGW and in Section 9.1. But their results are not sensitive to this choice.

LMSW estimate the following equation:

$$R_{jt+1} = \theta_0 + \theta_1 R_{jt} + \theta_2 \tilde{\tau}_{jt} R_{jt} + \epsilon_{jt+1} \quad (36)$$

where R_{jt} is the rate of return on stock j at day t. Their results are reproduced in Table 15.

[17] In the published version of LMSW, the sample period has been updated to January 1, 1993 to December 31, 1998. But the results remain qualitatively the same.

[18] The value of the constant is chosen to maximize the normality of the distribution of daily trading volume. See Richardson, Sefcik and Thompson (1986), Cready and Ramanan (1991), and Ajinkya and Jain (1989) for an explanation.

TABLE 14.

Sample Quintiles	AvgCap ($\times\$10^6$)	AvgTrd ($\times100$)	AvgTurn (%)	AvgPrc ($)
Mean (Q1)	26.53	73	0.162	9.46
Median	23.01	42	0.139	7.94
Std. Dev.	16.54	93	0.121	6.63
Obs	(222)	(222)	(222)	(222)
Mean (Q2)	119.97	241	0.216	17.39
Median	115.12	145	0.202	16.62
Std. Dev.	40.18	306	0.126	8.05
Obs	(222)	(222)	(222)	(222)
Mean (Q3)	401.96	477	0.233	26.33
Median	384.57	311	0.194	25.16
Std. Dev.	141.47	481	0.139	12.32
Obs	(222)	(222)	(222)	(222)
Mean (Q4)	1210.95	1181	0.269	33.46
Median	1150.49	953	0.258	31.90
Std. Dev.	378.44	930	0.128	14.51
Obs	(221)	(221)	(221)	(221)
Mean (Q5)	6553.68	3426	0.280	49.38
Median	4020.11	2739	0.259	44.65
Std. Dev.	8032.98	2623	0.133	28.35
Obs	(221)	(221)	(221)	(221)
Mean(Sample)	1658.61	1077	0.232	27.18
Median	383.10	366	0.212	24.76
Std. Dev.	4359.62	1768	0.136	21.03
Obs	(1108)	(1108)	(1108)	(1108)

Summary statistics for the 1,108 firms used in Llorente, Michaely, Saar, and Wang (1999). To be included in the sample, a firm had to have a stock traded on the NYSE or AMEX for the entire sample period (1983–1992) with return and volume information in the CRSP database. 'AvgCap' is the average end-of-year market capitalization, 'AvgTurn' is the average daily turnover, 'AvgTrd' is the average number of shares traded daily, and 'AvgPrc' is the average stock price over the sample period.

TABLE 15.

	θ_0 $(\# < 0)$	θ_1 $(\# < 0)$	$-\theta_2$ $(\# < 0)$	t_0 $(\lvert\#\rvert > 1.64)$	t_1 $(\lvert\#\rvert > 1.64)$	$-t_2$ $(\lvert\#\rvert > 1.64)$	R^2 (%)	AvgCap $(\times\$10^6)$
Q1	0.000935	−0.104318	0.032001	0.899	−4.557	2.712	2.766	26.53
$(n=222)$	(29)	(171)	(32)	(32)	(172)	(154)		
Q2	0.000448	0.003522	0.015588	0.994	−0.178	0.995	1.313	119.97
$(n=222)$	(36)	(101)	(85)	(55)	(155)	(107)		
Q3	0.000593	0.041883	−0.005034	1.582	1.599	−0.217	0.879	401.96
$(n=222)$	(13)	(63)	(131)	(100)	(160)	(84)		
Q4	0.000602	0.055762	−0.016220	1.683	2.255	−0.677	0.800	1210.95
$(n=221)$	(7)	(44)	(140)	(109)	(157)	(88)		
Q5	0.000696	0.048459	−0.019830	1.956	1.968	−0.700	0.581	6553.68
$(n=221)$	(1)	(42)	(137)	(144)	(137)	(84)		

Impact of volume on the autocorrelation of stock returns, by average-market-capitalization quintiles (where the average is computed over the entire sample period for each stock). For each stock, we measure the impact of volume on the autocorrelation of stock returns by the parameter θ_2 from the regression: $R_{jt+1} = \theta_0 + \theta_1 R_{jt} + \theta_2 \tilde{\tau}_{jt} R_{jt} + \epsilon_{jt+1}$ where R_{jt} is the daily return of stock j and $\tilde{\tau}_{jt}$ is its daily detrended log-turnover. We report the mean value of each parameter for five size quintiles. The number of negative parameters as well as the number of statistically significant (at the 10% level) parameters are also noted, and t-statistics are reported in parentheses (t_k denotes the t-statistic for θ_k, where $k = 0, 1, 2$).

The results in Table 15 show that for stocks in the largest three quintiles, the average of their estimates for θ_2 is positive, as our model predicts. This is consistent with the results of CGW and LeBaron (1992) based on market indices or large stocks. However, for the smallest two quintiles, the average of the θ_2 estimate is negative. As LMSW suggest, if market capitalization can be used an negative proxy for the degree of information asymmetry about a stock, the decrease in the value of θ_2 as market capitalization decreases is consistent with the above theoretical discussion. The negative θ_2 estimates for many small stocks are consistent with the negative θ_2 Antoniewicz (1993) reported based on pooled individual returns and volume.

What we conclude from the discussion in this section is that our theoretical model leads to important implications about the joint behavior of volume and returns. In particular, its implication on the dynamic volume-return relation is generally supported by the empirical findings. However, the model ignores many other factors in the market, such as the existence of private information, frictions, and a rich set of trading motives. These factors can be important in developing a more complete understanding the behavior of volume and its relation with returns. For the specific dynamic volume-return relations examine by LMSW, for example, the existence of information asymmetry is crucial. In this sense, the model here is merely a starting point, from where we can build a more complete model.

10. CONCLUSIONS

Trading volume is an important aspect of the economic interactions among different investors in the financial market. Both volume and prices are driven by underlying economic forces, and thus convey important information about the workings of the market. Although the literature on financial markets has focused on analyzing the behavior of returns based on simplifying assumptions about the market such as allocational and informational efficiency, we need to develop a more realistic framework to understand the empirical characteristics of prices and volume.

In this article, I have summarized some of the recent work in achieving this goal. By deriving an explicit link between economic fundamentals and the dynamic properties of asset returns and volume, it is shown that interactions between prices and quantities in equilibrium yield a rich set of implications for any asset-pricing model. Indeed, by exploiting the relation between prices and volume in a dynamic equilibrium model, we can identify and construct the hedging portfolio that all investors use to hedge against changes in market conditions. Moreover, the empirical analysis shows that this hedging portfolio has considerable forecast power in predicting future returns of the market portfolio—a property of the true hedging portfolio—and its abilities to explain cross-sectional variation in expected returns is comparable to other popular risk factors such as market betas and the Fama and French (1993) SMB factor.

Although the model presented here is parsimonious in order to focus attention on the essential features of risk-sharing and trading activity, it underscores the general point that quantities, together with prices, should be an integral part of any analysis of asset markets, both theoretically and empirically.

An important direction for future research is to model the rich set of trading motives investors may have and the specific aspects of the actual trading process. Trading motives may arise from richer risk-sharing and risk-management needs (e.g., Grossman and Vila (1992), Grossman and Zhou (1993, 1996)), tax considerations (e.g., Michaely and Vila (1995, 1996) and Michaely, Vila and Wang (1996)), and differences in information and views regarding the prospects of the assets as well as the market (e.g., He and Wang (1995), Kyle (1985) and Wang (1994)). Given a specific trading motive, the actual trading strategy can depend on factors affecting the actual execution such as liquidity and market impact (e.g., Chan and Lakonishok (1995), Kyle (1985) and Vayanos (1998)).

In addition, the presence of market frictions such as transactions costs can influence investors' trading strategies (e.g., Constantinides (1986)) and the level of trading volume. They are particularly important in understanding the level of volume we observe as well as the time trend in the data. Despite the many market microstructure studies that relate trading behavior to market-making activities and the price-discovery mechanism,[19] the seemingly high level of volume in financial markets has often been considered puzzling from a rational asset-pricing perspective (see, for example, Ross, 1989). Some have even argued that additional trading frictions or "sand in the gears" ought to be introduced in the form of a transactions tax to discourage high-frequency trading.[20] Yet in the absence of transactions costs, most dynamic equilibrium models will show that it is quite rational and efficient for trading volume to be *infinite* when the information flow to the market is continuous, i.e., a diffusion. An equilibrium model with fixed

[19]See, for example, Admati and Pfleiderer (1988), Bagehot (1971), Easley and O'Hara (1987), Foster and Viswanathan (1990), Kyle (1985), and Wang (1994).

[20]See, for example, Stiglitz (1989), Summers and Summers (1990a,b), and Tobin (1984).

transactions costs, e.g., Lo, Mamaysky, and Wang (2001), may reconcile these two disparate views on trading volume.

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