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A Theory of Co-operatives Based on Rights*

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We investigate the role of cooperatives in the allocation of risk across agents that we call workers and holders of capital. We show that, despite the inalienability of human capital (no forced labor) and limited liability on the part of all agents, financial coalitions can implement Pareto optimal inter temporal risk sharing services for both workers and holders of capital. We specifically show how optimality can be achieved in *worker preferred* equiliria if individual holders of capital collectivize and jointly hire workers, who are paid wages depending on the aggregate output of the coalition. Interestingly, we also provide an example where *capital preferred* equilibria do not provide for optimal risk sharing and re-negotiation proofness.

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"in each village lands are pooled, with each family holding shares in the large-scale enterprise, and wages paid to those who do the work. Such co-operatives, which bear no resemblance to Mao's communes, flourished

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briefly in China in the early 1950's, before farms were collectivized.Many now see that as the golden age for agriculture."

1. INTRODUCTION

Coalitions and other co-operative arrangements have a long and varied history in world economic development[1] . It is by now well understood that there exist a variety of situations to ensure that coalition type arrangements can improve social welfare when compared to competitive financial markets. An important feature of these models is that in these models coalitions arise *endogenously* as the optimal trading arrangement in a constrained environment.

Generally speaking, these models fall into categories that include costly information monitoring/production or transactions costs/imperfect secondary markets, or some combination of both[2] . But as Townsend (1983) has noted, both the welfare properties and welfare prescriptions of these models may be highly sensitive to the nature of the transactions costs or other external "frictions".

The purpose of this paper is to examine the optimality properties of coalitions that arise endogenously in a world of symmetric information and no costs, either indirect or direct. What we do recognize, however, is that each participant still has a minimal number of property rights, which we call inalienable rights. In modern economies these include limited liability, the right to re-negotiate contracts and the right to provide human capital on a strictly voluntary basis.

What we look at in particular is a simple production/exchange economy where there are two types of individuals; people with "human capital" who we call "workers" and those with a current endowment, that we call capi-

[1]The terms coalition, and cooperative will be used interchangeably in the sense that these are situations where a.) individuals contract with "groups" of one or more and b.) these claims are not tradable in a secondary market. Financial market transactions, on the other hand involve those situations where a.) individuals contract with other individuals and b.) these contracts are fully tradable in secondary markets. A labor contract would satisfy condition a.) (bi-lateral trade) in both cases but not b.) precisely *because* of inalienable rights that we discuss in the text.

[2]Early papers of the former type include Diamond (1984), Ramakrishnan and Thakor (1984), Boyd and Presecott (1986) while those utilizing some version of the latter include Townsend (1978) (fixed transactions costs) and Diamond and Dybvig (1983) (imperfect information and imperfect secondary markets). More recently, Diamond and Rajan (2000) can be viewed as a modern type of the former model since the banker is "better" than investors at collecting from borrowers. But at some level, one could make collection skills the same but allow the banker to "punish" the borrower, while investors cannot and get a result similar to Diamond and Rajan. Allen and Gale (2001) on the other hand assume imperfections in the secondary market that essentially requires that *none* of the long-term production process be liquidated at positive value.

talists. The former are assumed to be risk averse while the latter are risk neutral. In this environment we show that if capitalists pool their endowments and hire workers who contribute effort to these coalitions, optimal risk sharing can be achieved despite the inalienable rights of workers and landowners discussed earlier.

Thus, we are lead to conclude that in situations where there is little or no asymmetric information, simple incentive compatible co-operative arrangements may provide for optimal risk sharing. Indeed, we show that even simple bi-lateral labor contracts will suffice for risk sharing purposes in economies with high levels of initial endowments. However, the need for large scale coalitions is shown to be necessary when any individual capitalist has relatively small amounts of current endowment.

There is ample evidence that such co-operative arrangements may in fact be optimal in simple economic settings. Townsend (1994) for example shows that through a combination of small-scale financial intermediaries, crop storage, and family networks, the consumption of villagers in southern India is rendered nearly independent of idiosyncratic risk (one property of optimal risk sharing).

The model developed in this paper is also similar to the type of co-operative arrangements in China in the early to mid 1950's. These coalitions were operated as ones with the following characteristics "Land private-rent paid by co-op for its use but managed centrally by the co-op" (Walker (1965)). This is in contrast to the later (late 1950's) communes whereby there was "communism of all means of production"(Walker, pg. 16).

The remainder of the paper is organized as follows. In section I. we provide an analysis of the economic environment and some preliminary results regarding Pareto optimal risk sharing. Section II contains the main result regarding the optimality of worker preferred co-operatives. In Section III we show that allocations that are preferred by capitalists will generally not provide for optimal risk sharing since they are not immune to re-negotiation demands on the part of workers. Finally, Section III provides conclusions, a discussion of some related research, and suggestions for extensions to the current paper.

2. ECONOMIC FRAMEWORK AND PRELIMINARY RESULTS

Economic Environment

The economy we consider is one with two dates and $2N$ agents. N of the agents are risk neutral and each have an initial endowment of the single consumption good of e. We call these agents capital type (K) agents. The other N agents are also identical, but are strictly risk averse, expected utility maximizers and posses the same time additive preferences given by a continuously differentiable utility function, U, with $U' > 0, U'' < 0$, and $U(0) = 0$[3] .

Assuming symmetry in the number of agents essentially yields a result where bargaining power will depend only on preferences and endowments. Of course, if there are a much larger number of type W agents than type K agents, the allocations discussed below will be biased in favor of capital agents.

Type "W" or worker, agents have no initial endowment of the capital good but are "endowed" with skills that may be used to produce valuable future output. For simplicity, we assume that there is no disutility associated with this provision of effort. Assuming disutility of effort on the part of type "W" agents would not fundamentally alter the outcome unless there is heterogeneity in effort costs. As we discuss in the conclusion, this is an extension that would be of some interest.

We note that type "K" agents hold large balances of liquid, storable, time 0 consumption. Thus, capital agents are well suited to absorbing risk and smoothing consumption of the risk averse "W" agents. Type "W" agents have skills but these skills can only produce future output. "W" agents are in this way similar to entrepreneurs; they have future prospects but no existing capital. Furthermore, since "W" agents are risk averse, they have an incentive to trade some of the future output that they can produce for current consumption[4].

The random cash flow from worker j's effort, $j = 1, 2, \ldots, N$, if they choose to expend it, is given by$\tilde{X}_j$ and accrues at date 1. Payoffs are assumed to be iid across agents of type "W" and each payoff is distributed according to a two point distribution with equal probabilities. In particular, the payoff from agent j's effort is either 0 or $\overline{x}$.

Let $X = \{0, \overline{x}\}^N$ represent all possible realizations of the random vector $\tilde{X} = \tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N)$ and define $\mu = \overline{x}/2$ as the (common) expected value of $\tilde{X}_j, j = 1, \ldots, N$. Finally, define $\bar{m}(x) = \sum x_j/N$ as the average gross cash flow in output state x. The fact that worker efforts generate iid outcomes could be relaxed and the consequences would be similar to assuming heterogeneous effort costs. In particular, workers with better "skills" may, as we discuss later, have an incentive to avoid the co-operative and take their skills elsewhere to use alone (e.g., high skill workers may move to the city).

Agents are restricted to investment/savings plans that induce non-negative consumption at both dates with probability 1. Thus, in our setting, the

[3]For convenience we impose the standard Inada condition $\lim_{x\to 0} U'(x) = \infty$.

[4]This asymmetry of preferences highlights the role of coalitions in allocating risk to those agents most willing to bear it. The results could be reworked to include symmetric risk aversion but our assumption makes clear the idea that those with less current endowment might be more risk averse.

exogenous data for an economy is completely specified by the 4-tuple (U, e, N, μ). The set of economies we consider is the set of 4-tuples where U is strictly concave, e and μ are strictly positive and N is a natural number greater than or equal to 1[5].

Financial Coalitions and Renegotiation

As noted earlier, we assume that type-W agents must voluntarily contribute effort in order to produce output. Hence, they can always threaten to withhold effort contribution if their share of output is not increased. This generates a classical bargaining problem. We model this bargaining problem in a way similar to Hart and Moore (1992).

This approach to bargaining produces allocations resembling those produced in an extensive form game with "exit" options. In particular, we model the problem as one where, given the initial allocations any type-W agent (say (W, i)) is free to propose new sharing rules for future output associated with their specific labor. If her offer is accepted, the new allocation rules replace the old. If the proposal is rejected, a "coin-flip" occurs whereby agent (W, i) or the remaining agents get the total output from the worker's efforts, each with a 1/2 probability. This is equivalent to a bargaining game where, at the cost of losing contracted payments from the other negotiating party and the cash flows to their unique labor effort, each of the negotiating parties can exit the negotiations.

Pareto-optimal risk allocations

Let q represent per agent storage of the consumption good at time 0. Furthermore, for each type t and agent j, let $C^0(t, j)$ be a non-negative scalar representing date 0 consumption. We also denote random future consumption for a given realization of x by $C^1(t, j)(x)$. A Pareto optimal allocation is an allocation with the property that there exist positive weight vectors, $\gamma(t, j)$, such that the allocation solves

$$\begin{aligned}\max \sum_{j=1}^{N} &\gamma(W, j)(U(C^0(W, j)) + E[U(\tilde{C}^1(W, j))]) \\ + \sum_{j=1}^{N} &\gamma(K, j)((C^0(K, j)) + E[(\tilde{C}^1(K, j))]) \end{aligned} \tag{POP}$$

s.t.,

$$\sum_{j=1}^{N} C^1(W, j)(x) + C^1(K, j)(x) = N(\bar{m}(x) + q),\ \forall x \in X \tag{PO1}$$

[5]We also assume that U satisfies the Inada condition discussed earlier.

$$\sum_{j=1}^{N} C^0(W,j) + C^0(K,j) + Nq = Ne \tag{PO2}$$

$$0 \le q \le e \tag{PO3}$$

In the above expressions, $E(.)$ represents expectations relative to the probability measure π, induced by realizations of the random variables $\tilde{X} = (\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N)$. Thus, for all x, $\pi(x) = \pi^0$, where $\pi^0 = 2^{-N}$. The Pareto-optimal allocations are characterized in the next proposition.

PROPOSITION 1. *There exist symmetric Pareto-optimal allocations. Furthermore, all such allocations satisfy the following characterizations:*
(i) there exists $k^* \in [0, \bar{x}]$ such that for all j,

$$C^1(W,j)(x) = \min(\bar{m}(x) + q, k^*), \;\; C^1(K,j)(x) = \max(\bar{m}(x) + q - k^*, 0)$$

(ii) either $\forall j, C^0(W,j) = e$ and $\forall j E[U'(\tilde{C}^1(W,j))] \le U'(C^0(W,j))$, and $q = 0$, or $\forall j, C^0(W,j) = C_0^W < e$ and $\forall j E[U'(\tilde{C}^1(W,j))] = U'(C^0(W,j))$, and $q \in [0, e)$.

Proof. See the appendix. ▌

Notice that Proposition 1 says that, essentially, type W agents get debt claims on total output, while type K agents hold the residual claim. This is a generalization of the original result by Arrow (1974) that the optimal insurance contract between a risk neutral insurer (type K agents) and risk averse "purchasers" of insurance (type W agents) is a debt type contract (i.e., insurance with a deductible). However, as we argue later, it is the lack of *tradability* of the claims that will allow certain manifestations of the co-operative to provide for Pareto optimal solutions[6].

The next result, Lemma 1, provides a further characterization of Pareto optimal allocations under the additional assumption that W-agents display non-increasing absolute risk aversion. In particular, this assumption provides an incentive to defer consumption when faced with risky outcomes.

LEMMA 1. *Let (U, e, μ, N) be an economy in which U exhibits non-increasing absolute risk aversion. In this case, any Pareto optimal allocation has the property that*

$$E[\tilde{C}^1(W,j)] \ge C^0(W,j).$$

[6]If claims could be traded in securities markets first best is generally not achieved. See also footnote 7.

Proof. See the appendix. ▌

We note that there is a net social gain to type K agents absorbing the risk exposure of type W agents. Allocations may differ, however, depending on how the gain is divided between agents in the economy. We consider both cases. For most of the paper we look at equilibrium that just satisfy the individual rationality requirements for type K agents. We call these equilibrium W (or worker) preferred outcomes Later, we consider those Pareto optima that maximize the welfare of type K (capitalist) agents.

It is clear from the set-up that type -K agents can always receive e by simply storing. So individual rationality on the part of type- K agents requires that

$$E[\tilde{C}^1(K,j)] + C^0(K,j) \geq e, \text{ for all } j. \tag{1}$$

The W (or worker) preferred Pareto optimal allocations hold when equation (1) holds as an equality. Conversely, individual rationality on the part of the workers requires that their expected utility from joining the cooperative be no less than if they work alone. In this case we need

$$U(C^0(W,j)) + E[U([\tilde{C}^1(W,j)] \geq E[U(\tilde{X}_j)] = U(\overline{x})/2 = U(2\mu)/2, \tag{2}$$

where the last equality follows from the fact that $\overline{x} = 2\mu$.

We will investigate both equilibria, where equation (2) or equation (1) holds as an equality, in sections III and IV, respectively. We also briefly discuss the differences in the allocations and, importantly, why K preferred equilibrium may not be Pareto efficient because they are not re-negotiation proof.

3. OPTIMALITY OF WORKER PREFERRED CO-OPERATIVES

In this section of the paper we prove our main proposition; that worker preferred co-operatives provide optimal risk sharing and are in fact re-negotiation proof. However, we return to K preferred equilibrium later and compare how the allocations across groups differ depending on whether equation (1) or equation (2) holds as an equality. In particular, we argue that K preferred equilibrium are *not* capable of implementing Pareto optimal risk allocations when there is the possibility for re-negotiation.

The precise nature of the proposed cooperative is as follows. Type K agents pool their holdings of the endowment and become residual claim holders. Workers are provided with a transfer of the current endowment now, as well as being promised a debt-type contract on aggregate output at date 1. This set-up is consistent with the results of proposition 1. In particular, a coalition if uniquely defined by the allocation (k_0, q, w_0, w_1), where k_0 and w_0 are the date 0 allocations for agents of type K and W, respectively. Furthermore, q is per capita storage for the cooperative as a whole. Finally, w_1 represents the promised payment from the coalition to a type-W agent at date 1. Combined with the resourse constraints, these four variables uniquely determine the allocation across agents at both date 0 and date 1. However, we note that each agent of type t still has a private storage/current consumption decision to make once they know the current actual allocation as well as the distribution rule for future payoffs. We now turn to the properties of the W preferred equilibria.

Our first result with regard to W preferred Pareto optima is that they exist and that, if type-W agents display decreasing absolute risk aversion, all W preferred optima provide workers with expected future consumption that is never any smaller than current consumption.

LEMMA 2. *In any economy, symmetric W-preferred Pareto optima exist. Further, if type W agents exhibit decreasing absolute risk aversion, in all such Pareto-optimal allocations,*

$$E[\tilde{C}^1(K,j)] \geq E(\tilde{M})/2 = \mu/2, \text{ where } \tilde{M} = \left[\sum_{j=1}^{N} \tilde{X}_j\right]/N.$$

Proof. See the appendix. ▮

PROPOSITION 2. *In any economy (U, e, N, μ) in which type W agents exhibit decreasing absolute risk aversion, coalition structures (k_0, q, w_0, w_1) which satisfy*

$$E(\min[\tilde{M} + q, w_1]) \geq \mu/2$$

are re-negotiation proof.

Proof. See the Appendix ▮

The intuition behind Proposition 2 is that, given equal bargaining power, W-agents can capture no more than half of the returns on their project in post contracting re-negotiations. The cash flows resulting from re-negotiation are also riskier than the debt claim on average aggregate output provided by the coalition. This follows from the fact that the debt claim on aggregate output is the least risky claim on output consistent with limited liability.

Thus, as long as the expected value of the claim received by W-agents at least equals 1/2 the total returns from his/her efforts, W-agents will have no incentive to re-negotiate their contracts even if they are risk neutral. Because they are risk averse, they will strictly lose (in an expected utility sense) from such a re-negotiation. Thus, debt claims on the whole collective's output, w_1, satisfying the conditions of the proposition, are re-negotiation proof.

Our next task is to relate the re-negotiation proof collective structures to the Pareto-optima of the game. To accomplish this, note that, as mentioned earlier, the actual consumption pattern induced by a coalition structure depends on the time 0 per capita consumption of the holders of capital, which we denote by C_0^K, as well as by w_0 and w_1. Knowing these three parameters, and knowing that W-agents will make individually optimal storage/consumption decisions for their initial wage income, allows us to uniquely determine allocations from collective structures as follows: the allocation determined by the collective structure (C_0^K, w_0, w_1) is, for all agents j and types t,

$$\begin{aligned}
&q = \phi(C_0^K, w_0, w_1) + e - C_0^K - w_0\\
&C^0(K,j) = C_0^K\\
&C^0(W,j) = w_0 - \phi(C_0^K, w_0, w_1)\\
&C^1(W,j)(x) = \phi(C_0^K, w_0, w_1) + \min(\bar{m}(x) + e - C_0^K - w_0, w_1)\\
&C^1(K,j)(x) = \max(\bar{m}(x) + e - C_0^K - w_0 - w_1, 0)
\end{aligned}$$

where

$$\begin{aligned}
\phi(C_0^K, w_0, w_1) = &\operatorname{argmax}\{g \in [0, w_0] : U(w_0 - g)\\
&+E[U(g + \min[w_1, \tilde{M} + e - C_0^K - w_0]\}
\end{aligned}$$

The next result is a straightforward consequence of our earlier analysis. In proposition 1 we showed that when agents exhibit decreasing absolute risk aversion, the optimal allocation calls for W-agents to receive a disproportionate share of the expected value of their consumption stream at time 1 (the time at which they must bear risk). This implies, by Proposition 2, that the Pareto-optimal allocations satisfy the sufficient conditions for the optimal allocation to be re-negotiation proof. This result is formalized in Proposition 3.

Proposition 3. *Consider any economy (U, e, μ, N) such that U exhibits decreasing absolute risk aversion. Then there exist coalition structures that induce W-preferred, Pareto-optimal allocations of risk.*

We note that for $e \geq \mu$, coalitions of two, i.e., a bilateral labor contract, will also serve to implement Pareto-optimal allocations of risk. In particular, in this case we have that, for $e \geq \mu$,

$$C^0(W,j) = w_0 = w_1 = C^1(W,j)(x) = w\forall x \tag{3}$$

While this result shows that "cooperatives of 2" can sometimes implement first best risk sharing, we note that any successful implementation would have to satisfy two criteria. First, any individual agent's consumption must depend only on aggregate consumption. Second, type W agents must be prevented from "front loading" i.e., first trade away claims for current consumption, then consume and, finally, renegotiate for additional future cash flows. By providing a *non-tradable* claim on aggregate output, our solution satisfies both criteria. Interestingly, we have shown elsewhere that stock market allocations will not typically satisfy both of these properties[7].

More generally, any alternative allocation mechanism that allows for residual claims on individual worker's output cannot achieve the first best solution, since this implies that individual workers will face idiosyncratic risk; this of course violates one of the properties of Pareto optimality. Indeed, as we discuss next, it will generally not be possible to implement first best risk sharing when one considers K preferred equilibrium with the possibility of re-negotiation since in these allocations workers do end up owning residual claims to their own output[8].

4. SOME REMARKS ON CAPITALIST PREFERRED EQUILIBRIUM

In section 3 of the paper we focused on the optimality of worker preferred co-operatives for risk sharing over time when there is the possibility of re-negotiation for future output from workers. It is of course possible to look at type K preferred allocations that satisfy the properties needed for

[7]For a proof, see Noe and Smith (1994). The result is unambiguously true if workers can "incorporate" However, even with only personal limited liability, stock market allocations cannot generally implement the first best because of the "front loading" problem discussed in the text. For example, if all participants are risk neutral, stock market allocations will never provide for the first best even though autarky will in this case.

[8]Interestingly, current agricultural practice in China, as we understand it, is for individual farmers to rent land from the government on a yearly basis for a fixed sum. However, should disaster strike, the government forgoes this rent and may in fact provide some additional subsidy to each. In essence, individual agents issue what look like "income bonds" (pay the coupon if you earn it, but not otherwise) to the government. In return, the government writes an insurance contract that pays off in the case of a natural disaster. So individual landowners/workers are the residual claimant during "good" times, while the government is the residual claimant during "bad" times. It is not clear to us what the optimal contracting scheme would be should a government be added to this model in addition to the two groups discussed in the text.

optimal risk sharing if there was a way to stop workers from re-negotiating. However, as we discuss, these allocations do not provide first best risk sharing since workers end up carrying idiosyncratic risk.

Recall that a K preferred equilibrium will hold if equation (2) holds as an equality, i.e., the utility of the type W agent will be the same whether he/she joins the co-operative or uses his/her labor alone. This implies that

$$U(C^0(W,j)) + E[U([\tilde{C}^1(W,j)] = E[U(\tilde{X}_j)] = U(\bar{x})/2 = U(2\mu)/2 \quad (4)$$

Consider the following as an example of the problems that arise when looking at K preferred equilibrium. Suppose that $e \geq \mu$ so that K type agents can offer riskless contracts to type W workers. Consider the allocation whereby equation (3) holds and solve equation (4) for this $w = w^*$. So w^* solves

$$2U(w^*) = U(\bar{x})/2 = U(2\mu)/2 \quad (5)$$

By the concavity of U we can directly conclude that $w^* < \mu/2$. Now, we have shown in the proof of proposition 2 that any proposal which yields an expected value to a type W agent (call her agent i) that is greater than $\mu/2$ will be rejected by the type K agent. In this case type W agents will be indifferent to re-negotiation since they have a 1/2 chance of getting all of the project's cash flows, which is exactly equal, using (5), to $U(w^*)$, since

$$U(w^*) = U(\bar{x})/4 = U(2\mu)/4 = (1/2)E[U(\tilde{X}_i)] \quad (6)$$

where $(1/2)E[U(\tilde{X}_i)]$ is the expected utility to W agent i if the re-negotiation proposal is rejected and the parties resort to the "coin flip" However, we have shown that any proposal that has an expected value of less than or equal to $\mu/2$ will be accepted by the K type agent. Finally, it is straightforward to show that the expected utility of the type W agent will be strictly greater than that from remaining in the coalition[9]. But this implies that the resulting allocation is not Pareto efficient, since the type W agent will be carrying idiosyncratic risk after re-negotiation is complete.

We could also consider allocations for the case where $e < \mu$, so that type W agents must carry some of the risk, either in the coalition or alone. However, the essential story, that workers can "improve their lot" through re-negotiation, is likely to hold in most cases. In such instances workers

[9]In the example considered here, $w_0 = C^0(W,i) = w^*$. Using equation (A.13), the payoff to agent i from having the re-negotiation proposal accepted is given by $\tilde{C}^1(W,i|y) = \min(f, \tilde{X}_i)$, where $f = \mu$ in this case. So the expected utility is given by $U(\mu)/2$. From the concavity of U and the definition of w^* in equation (6), it immediately follows that $U(\mu)/2 > U(2\mu)/4 = U(w^*)$ so that agent i is better off carrying the idiosyncratic risk of their re-negotiated project. This, of course, violates the requirements for Pareto optimal risk sharing.

end up carrying idiosyncratic risk despite the fact that they have been able to front load some consumption by joining the co-operative[10].

5. SUMMARY AND CONCLUSIONS

In this paper we provide a rationale for co-operatives that, unlike much of the earlier literature, does not assume "exogenous" costs which can be spread over a large number of investors. In particular, we formally establish the Pareto-optimality of these coalitions by simply combining the facts that individuals possess limited liability and have the right to propose re-negotiated contracts with other individuals who hold claims to their effort dependent future cash flows. These two assumptions are what we refer to as inalienable rights.

The intuition for the optimality result obtained here follows from the fact that, under our cooperative structure, workers can be disciplined by other members of the coalition in the sense that they receive only a portion of the total value of their labor effort up front (current wages). In this case, workers find it in their interest to forego proposing new sharing rules over future wages and bonuses since an acceptable (to the coalition) re-negotiated contract will provide them with, at most, an average total wage equal to that of the original agreement.

We have also shown that there exist certain parameters of the model such that simple bilateral labor contracts can achieve the same optimality properties as those obtained by the coalition. This is an avenue that we feel is worth investigating since this result shows that even "coalitions" of two can implement optimal solutions provided that these claims can not be traded in secondary markets.

One limitation of our analysis involves the fact that we have restricted our implementation schemes to settings in which agents are extremely homogeneous. It is an open question as to whether collective implementations of first-best risk sharing arrangements are possible when greater heterogeneity of preferences is allowed, especially if there is some informational asymmetry regarding agent preferences.

Thus, when taken as whole, our results suggest that coalitions or collectives will be most effective in situations where participants are fairly homogeneous with respect to information, investment opportunities, and attitudes toward work vs. leisure. For example, with heterogeneous skills in their ability to generate output, some workers may be willing to forego

[10]It is interesting to note that if one assumes such frictions as non-pecuniary costs (in any of its many disguises, e.g., workers receive a broken leg or have a lose of "face" etc if they attempt to re-negotiate) then, trivially, capitalist preferred equilibrium can be implemented. See footnote 2 for a discussion of the use of this (or mathematically equivalent) assumption in the theory of intermediation and co-operatives.

the risk-sharing benefits of the collective in order to extract higher total compensation from other arrangements, including self-employment.

A similar argument could be made in situations where some agents have a very high disutility of effort (i.e., are relatively lazy). Their attempt to "free-ride" on the productive output of other agents could cause the coalition to be unattractive to other members of the group.

As a final example, we note that differences in bargaining power may also cause inefficiencies in the cooperative system. For example, Banerjee, et. al (2001) study sugar cooperatives in India. While organized somewhat differently than the model developed here, these authors show that more powerful (larger) members of the cooperative engage in rent seeking behavior at the expense of less powerful members of the co-operative[11].

A similar situation could arise in instances where there is an excess supply of labor relative to the available demand by owners of the land to be worked. In this case owners of the good in short supply would have additional bargaining power, thereby depressing the wages/bonuses of workers to levels below those associated with first best allocations. However, we note that the ability of workers to employ their skills elsewhere (e.g., in manufacturing) would put some lower bound on how far wage rates could fall in the agricultural co-operatives[12].

In any case, more complex models along these lines may ultimately shed some light on the empirical regularity that there exist a multiplicity of financial contracting schemes and many of them mutual in nature. Moreover, such arrangements continue to exist even as organized markets for transferable bi-lateral exchange have exploded throughout the world. We have simply shown that the co-operative type of arrangement may be optimal in some, but certainly not all, circumstances.

APPENDIX

Proof of Proposition 1. The first order necessary conditions characterizing the Pareto optimal allocations for our problem can be expressed

[11] These co-operatives refine and process the sugar from growers and payouts are supposed to be proportional to raw sugar supplied. But Banerjee, et al argue that larger farmers manage to keep prices too low (relative to the first best allocation). These members can then extract rents from the accumulated surplus in the co-operatives by, for example, donating to causes which increase their prestige and/or financial status in the community.

[12] Mandated minimum wages with worker bonuses left to the discretion of the local co-operatives would be one way of implementing this potentially second best solution.

as follows:

$$\gamma^j U'(C^0(W,j)) - \lambda_1 = 0, \ \forall j \tag{A.1}$$
$$\gamma^j U'(C^1(W,j)(x))\pi^0 - \lambda_2(x) = 0, \ \forall j \text{ and } x \tag{A.2}$$
$$C^0(K,j)(\beta^j - \lambda_1) = 0, (\beta^j - \lambda_1) \leq 0, \ \forall j \tag{A.3}$$
$$C^1(K,j)(x)(\beta^j \pi^0 - \lambda_2(x)) = 0, \beta^j \pi^0 - \lambda_2(x) \leq 0, \ \forall j \text{ and } x \tag{A.4}$$
$$q\left(\sum_{x \in X} \lambda_2(x) - \lambda_1\right) = 0, \left(\sum_{x \in X} \lambda_2(x) - \lambda_1\right) \leq 0 \tag{A.5}$$

Existence follows because the Pareto problem satisfies standard regularity assumptions.

Moreover, symmetry implies that $\gamma^j = \gamma, \beta^j = \beta$, and that there exist scalars $C^d(t), d = 0$ and $d = 1$, such that $C^1(t,j)(x) = C^1(t)(x)$ and $C^0(t,j) = C^0(t)$, $\forall j$ and t.

In order to prove (i), first consider the subset of x's, $x \in X^*$ such that $\beta\pi^0 < \lambda_2(x)$. Then, by (A.4), $C^1(K)(x) = 0$ and by (PO1), $C^1(W)(x) = m(\bar{x}) + q$. Next, consider the complement to X^*, X_c^*. For these x's, $\beta\pi^0 = \lambda_2(x)$. Moreover, if $C^1(W)(x') > C^1(W)(x'')$, then by (A.2), $\lambda_2(x'') > \lambda_2(x')$. But this contradicts the assumption that $\lambda_2(x'') = \lambda_2(x') = \beta\pi^0$, for $x', x'' \in X_c^*$. Thus, for $x \in X_c^*, C^1(W)(x) = k^*$, a constant. Therefore, by (PO1), $C^1(K)(x) = m(\bar{x}) + q - k^*$ for $x \in X_c^*$. It follows that, since $C^d(t)(x) \leq 0$ for $d = 0, 1$ and for all x, it must be the case that for the union of X_c^* and X^*, X, we have that $C^1(K)(x) = \max(m(\bar{x}) + q - k^*, 0)$ and $C^1(W)(x) = \min(m(\bar{x}) + q, k^*)$ for $x \in X$.

To prove (ii), note that (A.1), (A.2) and (A.5) imply that, in any Pareto optimal allocation, it must be the case that

$$E[U'(C^1(W))] \leq U'(C^0(W)) \tag{A.6}$$

Now (PO2) implies that $C^0(W) \leq e$. First consider the case $C^0(W) = e$. In this case, (PO2) requires that $q = 0$ and $C^0(K) = 0$. Moreover, (A.6) requires that $E[U'(C^1(W))] \leq U'(C^0(W))$. For $C^0(W) < e$, consider the case where $e > q > 0$. Now, (A.5) implies that $\sum_{x \in X} \lambda_2(x) - \lambda_1 = 0$. It then follows immediately from (A.1) and (A.2) that $E[U'(C^1(W))] = U'(C^0(W))$. Finally, consider the case $q = 0$. By (PO2), $C^0(K) > 0$ and $C^0(W) < e$. In this case, (A.3) implies that $\beta = \lambda_1$. But (A.4) implies that $\sum_{x \in X} \lambda_2(x) \geq \beta$. Together these two conditions imply that $\sum_{x \in X} \lambda_2(x) \geq \lambda_1$. Combining this result with (A.5) implies that, for $q = 0, \sum_{x \in X} \lambda_2(x) = \lambda_1$. It again follows from (A.1) and (A.2) that $E[U'(C^1(W))] = U'(C^0(W))$. ▌

Proof of Lemma 1: If $U''' > 0$, then U' is convex. Thus, by Jensen's inequality, $U'(E[\tilde{C}^1(W,j)]) \leq E[U'(\tilde{C}^1(W,j))]$. By Proposition 1 (ii),

$U'(E[\tilde{C}^1(W,j)]) \leq E[U'(C^1(W,j))]$ implies that

$$U'(E[\tilde{C}^1(W,j)]) \leq U'(C^0(W,j)).$$

This implies, because U' is decreasing, that that $E[\tilde{C}^1(W,j)] \geq C^0(W,j)$. ▌

Proof of Lemma 2: The existence of symmetric Pareto optimal allocations follows from the regularity of the optimization problem and the fact that the feasible set is nonempty. The characterization follows because, in any worker preferred allocation, $E[\tilde{C}^1(K,j)] + C^0(K,j) = e, \forall j$. This implies, by the resource conservation constraints and the symmetry of the allocation, that $E[\tilde{C}^1(W,j)] + C^0(W,j) = E(\tilde{M})$. The result is then immediate from Lemma 1. ▌

Proof of Proposition 2: To prove that the coalition structure is renegotiation proof, we need to show that no type W agent has an incentive to renegotiate her contract when she conjectures that no other W agent will renegotiate. If no agents renegotiate, the period 1 cash flow to agent (K,j) is given by $\tilde{C}^1(K,j|n)$, where

$$\tilde{C}^1(K,j|n) = k_0 - C^0(K,j) + \max[\tilde{M} + q - w_1, 0] \tag{A.7}$$

While those for the jth worker can be written as

$$\tilde{C}^1(W,j|n) = w_0 - C^0(W,j) + \min[\tilde{M} + q, w_1] \tag{A.8}$$

Now suppose that a type W agent, agent i, proposes a renegotiated contract. A re-negotiation proposal by the ith worker is a division of the cash flows from the project i between (W,i) and the coalition. By our assumption that the cash flows for each project are supported by $\{0,\bar{x}\}$, the division is uniquely determined by the payment received by agent i when cash flow $\bar{x}$is realized. Let "f" represent this payment. The remaining projects are unaltered. Therefore, the cash flow to the representative type-K agent, if she accepts re-negotiation, is given by

$$\tilde{C}^1(K,j|y) = \tilde{C}^1(K,j,-i) + \max[\tilde{X}_i - f, 0]/N \tag{A.9}$$

where

$$\tilde{C}^1(K,j,-i) = k_0 - C^0(K,j) + \max[\tilde{M}^{-i} + q - w_1 + w_1/N, 0], \tag{A.10}$$

where

$$\tilde{M}^{-i} = \left(\sum_{\substack{k=1 \\ k \neq i}}^{N} \tilde{X}_k \right) /N \tag{A.11}$$

If this proposal is rejected, and nature draws the next proposal, the cash flows to type-K agents will be either $\tilde{C}^1(K, j, -i)$ (agent i gets to choose and $f = \bar{x}$) or $\tilde{C}^1(K, j, -i) + \tilde{X}_i/N$ (the coalition chooses and $f = 0$. Given equal probabilities, the expected cash flows to a representative type K-agent from rejecting the offer is

$$E[\tilde{C}^1(K, j|r)]) = E[\tilde{C}^1(K, j, -i)] + E(\tilde{X}_i)/2N \tag{A.12}$$

Therefore, the proposal will be accepted if and only if the expected value of the right hand side of equation (A.9) is greater than the right hand side of equation (A.12), or $E[\max(\tilde{X}_i - f, 0)/N)] > E(\tilde{X}_i)/2N$. It follows that any proposal will be accepted by type-K agents if and only if $f < \bar{x}/2 = \mu$. Any $f > \mu$ will be rejected by all agents in the K-coalition. The period 1 consumption of agent (W, j) if an acceptable proposal is made is given by

$$\tilde{C}^1(W, i|y) = \min(f, \tilde{X}_i) + w_0 - C^0(W, i) \tag{A.13}$$

The possible cash flows to agent i when $f > \mu$ are given by $\tilde{X}_i + b_0 - C^0(W, i)$ (agent i wins) or $w_0 - C^0(W, i)$ (coalition wins). We now show that the expected utility to agent i from not renegotiating exceeds that obtained via re-negotiation. We first show that proposals $f \leq \mu$ are dominated. To show this, let $w_1^{'}$ be the solution to

$$E(\min[\tilde{M} + q, w_1^{'}]) = \mu/2 \tag{A.14}$$

Now, $w_1^{'}$ is unique since the left hand side of equation (A.14) is strictly increasing in w_1 and because $E[q + \tilde{M}] \geq E[\tilde{M}]/2 = E(\tilde{X}_i)/2$. By the hypothesis of the proposition, $E(\min[\tilde{M} + q, w_1^{'}]) > E(\tilde{X}_i)/2$. Thus, we have that

$$E(\min[\tilde{M} + q, 0]) < E(\tilde{X}_i)/2 < E(\min[\tilde{M} + q, w_1^{'}]) \tag{A.15}$$

Therefore, there exists a unique $w = w_1^{'} \in (0, w_1]$ which solves equation (A.14).

Now, suppose that the proposal is rejected. The expected utility to agent i from the coin flip is then given by

$$\begin{aligned} E(U[\tilde{C}^1(W, i|r)]) &= E[U(\tilde{X}_i + w_0 - C^0(W, i)) + U(w_0 - C^0(W, i))]/2 \\ &\leq E[U(\tilde{C}^{1\prime\prime}(W, i))], \end{aligned} \tag{A.16}$$

where $(\tilde{C}^{1\prime\prime}(W, i)) = \tilde{X}_i/2 + w_0 - C^0(W, i)$

The last inequality follows from Jensen's inequality and the concavity of U. The consumption associated with not renegotiating is no smaller than

$$(\tilde{C}^{1^{'}}(W, i)) = w_0 - C^0(W, i) + \min[\tilde{M} + q, w_1^{'}] \tag{A.17}$$

It follows from equation (A.7), (A.14), and (A.17) that

$$E[U(\tilde{C}^{1'}(W,i))] \leq E[U(\tilde{C}^{1}(W,i))|n] \tag{A.18}$$

Now, from (A.4), (A.17) and the definition of $\tilde{C}^{1''}(W,i)$, we have that

$$E[\tilde{C}^{1''}(W,i)] = E[\tilde{C}^{1'}(W,i)] \tag{A.19}$$

As shown in Proposition 1, the debt contract maximizes the utility of the risk averse agents over all limited liability claims on output. Thus,

$$E[U(\tilde{C}^{1''}(W,i))] \leq E[U(\tilde{C}^{1'}(W,i))] \tag{A.20}$$

It follows that equations (A.16), (A.18) and (A.20) yield the desired conclusion that agent (W,i) is no better off renegotiating her contract when her re-negotiation proposal is rejected. However, in order to induce proposal acceptance it must be the case that $f \leq \mu$. In this case, the time 1 consumption to agent (W,i) is, from equation (A.13), weakly less than less than $\tilde{C}^{1''}(W,i)$ in all states of the world. It therefore follows from (A.18), (A.19) and (A.20) that $E(U[\tilde{C}^1(W,i|y)]) \leq E(U[\tilde{C}^1(W,i|n)])$. Thus, the coalition structure is re-negotiation proof. ▌

Proof of Proposition 3: Let $(q, C^0(W,j), C^0(K,j), C^1(W,j), C^1(K,j))$ be a W preferred symmetric Pareto optimal allocation. Such an allocation exists by Proposition 1. Pick any j and let $C^0(W,j) = w_0$. Choose $w_1 = k^*$, where k^* is defined in Proposition 1. Let $C_0^K = e - q - w_0$ and note that the W preferred Pareto optimal allocation is induced by (C_0^K, w_0, w_1). This is immediate since $\phi(C_0^K, w_0, w_1) = 0$, where ϕ is defined in the text. We need only show that (C_0^K, w_0, w_1) is re-negotiation proof. Lemma 2 shows that $E[\tilde{C}^1(W,j)] \geq E[\tilde{M}]/2$. By definition,

$$\tilde{C}^1(W,j)(x) = \min[e - C_0^K - w_0 + \bar{m}(x), w_1] \tag{A.21}$$

Thus, $E[\min(e - C_0^K - w_0 + \tilde{M}, w_1)] \geq E[\tilde{M}]/2$, implying that (C_0^K, w_0, w_1) is re-negotiation proof. ▌

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