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Debt Contract, Strategic Default, and Optimal Penalties with Judgement Errors*

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We characterize the competitive equilibrium on the credit market when borrowers can strategically default. We assume that the audit is subject of errors of the two types and that lenders cannot commit ex-ante. We determine the penalty, the loan rate, the audit and strategic default probabilities. Borrowers' limited liability is endogenous when "judicial errors" exist, strategic default appears at equilibrium depending on the borrowers' absolute risk aversion. We show that at equilibrium loan contracts exhibit a penalty such that borrowers never strategically default. This is true with IARA and CARA utility function. Finally, we show that with DARA, strategic default may exist.

Key Words: Strategic default; Imperfect audit; Fine; Consumer credit.
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1. INTRODUCTION

Agents typically face adverse selection and moral hazard on credit markets. This problem is particularly severe in consumer credit markets where loans are small in size and are not collateralized (giving few incentives to

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control). Moreover, ex-post, privacy laws often allow consumers not to disclose information on their wealth. Bequests and gifts allow households to modify or conceal their actual wealth. Indeed, if lenders cannot observe the borrower's wealth, the latter will be tempted to strategically default on their debt. Anticipating this, creditors will raise loan rates. This can ultimately lead to the breakdown of the loan market.

The literature on costly state verification claims that the moral hazard problem can be solved if the lender can commit to verify ex-post the borrowers' wealth and/or if the parties can contractually agree on large penalties for those borrowers who strategically default on their debt. In particular, with infinite penalties, the first-best allocation may be achieved. Intuitively, very large penalties provide the right incentive for borrowers to report their financial situation truthfully to the creditor, even if the latter audits only with a very small probability. Consequently, the asymmetric information can be eliminated at a cost which tends to zero as the audit probability becomes sufficiently small. This result was first stated by Becker (1968).

The use of unbounded penalties to induce agents to correctly reveal their information has been subject to many criticism. Stigler (1970) introduces the idea of marginal deterrence according to which crime should be punished taking into account the involved social cost. Polinsky and Shavell (1979) underline the role of risk aversion in limiting penalties, an argument that is particularly relevant in the context of consumer credit where most risks borne by borrowers are uninsurable.

However, most existing models introduce an exogenous upper limit to the penalty that can be imposed to defrauders (Towsend, 1979, Mookerjee and Pn'g, 1989, Border and Sobel, 1987), or assume that there is limited liability on the part of the borrower (Gale and Hellwig, 1985). Under limited liability, as shown by Khalil and Parigi (1998) and Simmons and Garino (2003), strategic default may exist in equilibrium if lenders cannot commit on their audit strategy ex-ante. The existence of strategic default is necessary to induce lenders to audit defaulters. On the contrary, without strategic default, lenders would not want to audit ex-post, which in turn would induce borrowers to strategically default. As a result, there may not exist any equilibrium without strategic default. Then, the exogenous rule of limited liability goes against maximizing welfare in the economy in the absence of commitment. However, not only the universality of the limited liability rule but also homestead or exemptions rules in the US or over-indebtness commissions in France, tell us that there must be something wrong with any model yielding that kind of result.

In this paper, we want to determine the optimal value of the penalty as the result of an endogenous welfare-improving mechanism. In order to do so, we consider a costly state verification model where lenders cannot commit on their auditing strategy and where borrowers are risk averse. Moreover, we introduce the possibility that the audit be subject to errors. More specifically, we assume that the outcome of the audit is a signal imperfectly correlated with the borrower's financial situation.[1] The possibility of "judicial error" provides an argument in favor of limiting penalties, exactly as the one used by the opponents to the death penalty. In this setting, we jointly determine the level of the penalty, the loan rate, the audit probability and the probability of strategic default that will prevail at equilibrium. We show that when the audit technology is imperfect, strategic default may or may not exist in equilibrium depending upon the parameters of the problem and the borrowers' attitude toward risk. This result is one of the main contributions of the paper. This result is in contrast with the one obtained by Khalil and Parigi (1998).

We first examine the case of constant absolute risk aversion, which entails risk neutrality as a special case. We show that the equilibrium loan contract is such that the contractual penalty will be large enough to induce borrowers to never strategically default on their debt. Any other contract with a smaller penalty would be Pareto-dominated. Then we argue that the same result holds when absolute risk aversion is increasing in wealth. Finally, we prove that when the utility function exhibits decreasing absolute risk aversion, strategic default may exist in equilibrium with limited punishment determined by the value of parameters.

The paper is organized as follows. In the next section, we present the model. We then describe the lenders and borrowers' strategies at equilibrium and the general features of the equilibrium contracts. In section 3, we characterize the conditions on the borrowers preferences under which the fine that arises as an equilibrium implies no strategic default at equilibrium. Section 4 deals with the case where the borrowers' utility function exhibits decreasing absolute risk aversion. In section 4, we present a numerical example to illustrate our results. The last section concludes the paper.

2. DESCRIPTION OF THE MODEL

We consider an economy of ex-ante identical consumers and two dates. At date $t = 0$, consumers want to purchase a quantity m of a good whose price is normalized to unity. They obtain a private benefit B from consuming the good. Consumers have no cash at date $t = 0$, so they need to borrow an amount m in order to finance their purchase. At date 1, each consumer earns an income $\omega + \widetilde{w}$. We interpret ω as the value of durables earned by

[1] In particular, we consider the risk of error in which the audit states that borrowers can repay whereas they have exogeneous negative shocks on income (as illness or divorce...).

the consumer. We assume that this "income" can be pledged by the lender only through a prosecution.[2] It cannot be used by borrowers to repay either because borrowers do not know how to sell durables or because they are illiquid. The random labor income is denoted $\widetilde{w}$. It is the only source of income directly pledgeable by the lender. It can take two values: $y > 0$ if employed or 0 if unemployed, respectively with probability p and $1 - p$. Consumers are risk-averse with an increasing and concave utility function u for consumption at date 1.

There exists a competitive market for standard debt contracts at date 0. Lenders are risk-neutral and their cost of funds is equal to zero. Lenders do not observe the income shock experienced by borrowers. However, they can audit borrowers' earnings at a cost c. This asymmetric information raises the problem of ex-post moral hazard or strategic default: employed borrowers may want to claim that they are unemployed. Contrary to the standard literature, we assume that lenders can choose the audit technology. They have the choice between three technologies : a perfect one, and two imperfect technologies. The perfect technology is such that the audit signal perfectly reveals the state of nature. In an imperfect technology, the audit signal is imperfectly correlated with the true state of nature. In order to characterize these imperfect technologies, let us consider that the audit signal $\widetilde{s}$ may take value 0 or y. We use the following notation:

$$\alpha = \Pr\left[\widetilde{s} = 0 \mid \widetilde{w} = y\right],$$

and

$$\beta = \Pr\left[\widetilde{s} = y \mid \widetilde{w} = 0\right].$$

Parameter α is the probability that the auditor misleadingly observes that a borrower is unemployed.Conversely, β is the probability that the audit signals that the borrower is employed whereas he is actually unemployed.

The perfect technology is characterized by $\alpha = \beta = 0$. The first imperfect technology is such that $\beta = 0$ and $\alpha > 0$ and the last one is such that $\alpha \geq 0$ and $\beta > 0$. Using Bayes' rule, we obtain

$$\Pr\left[\widetilde{w} = y \mid \widetilde{s} = y\right] = \frac{p(1-\alpha)}{p(1-\alpha) + (1-p)\beta}.$$

Notice that a signal $\widetilde{s} = y$ increases the probability that $\widetilde{w} = y$ only if $\Pr\left[\widetilde{w} = y \mid \widetilde{s} = y\right] > p$, i.e., if $\alpha + \beta \leq 1$. Without loss of generality, we hereafter assume that this inequality holds.[3] Parameters α and β are

[2]In France, durables can only be seized in presence of an usher and after a judicial decision.

[3]Otherwise, the interpretation of signals $a = y$ and $a = 0$ should be reversed.

common knowledge. We assume that the audit signal is observable and verifiable by the lender and the borrower once the audit cost has been paid by the lender.

The timing of the game is as follows: At date 0, the competitive market for loans opens, and loan contracts are signed. At date 1, borrowers observe the realization of income $\widetilde{w}$ and decide whether to report $R = y$ or $R = 0$ to the lender. Lenders thereafter decide whether or not to audit the borrowers' income. If they audit, the signal is observed by the two parties. Finally, monetary payments are made according to the loan contract.

Individual strategies are characterized by the probability θ to report $R = 0$ for borrowers with $\widetilde{w} = y$, and by the probability γ for the lender to audit borrowers who reported $R = 0$. We assume in this paper that the lender cannot commit on the probability of audit. Thus, the loan contract does not specify the audit probability γ.[4]

The contract sets monetary transfers contingent on the report of the borrower and the audit signal when available. When there is no audit, the borrower pays $m(1 + r)$ to the lender when he reports $\widetilde{w} = y$, and 0 otherwise. When there is an audit, the borrower pays f, if the audit signal is not congruent with the report, and 0 otherwise. f should be interpreted as a combination of a pecuniary fee and the pecuniary equivalent of a non pecuniary punishment.[5]

3. TYPOLOGY OF EQUILIBRIA

We first examine a perfect Bayesian equilibrium where both parties use mixed strategies and establish hereafter the different conditions that support this equilibrium. We start by analyzing the behavior of borrowers with $\widetilde{w} = y$ who have to decide whether to report $R = 0$ or $R = y$. Let $U_y \equiv u(\omega + y - m(1 + r))$ denote borrowers' utility if they tell the truth. Borrowers will randomize only if they are indifferent between the two strategies. This yield

$$U_y = \gamma \left[(1 - \alpha)\, u(\omega + y - f) + \alpha u(\omega + y) \right] + (1 - \gamma) u\,(\omega + y)\,. \qquad (1)$$

The right-hand-side of this equality measures the risk borne by defrauders. If they strategically default, consumers are audited with probability γ. In this case, they pay a fine f only when the audit is not mistaken. With

[4] We assume that $R = 0$ with probability 1 when $w = 0$. Under our assumptions, this is optimal for the borrower because we assume that the lender cannot reward borrowers when $R = a = y$. Otherwise, when $\alpha > 0$, the borrower with $w = 0$ may be tempted to declare $R = y$ in the hope to get the reward by mistake. This would in turn induce the lender to audit reports $R = y$. This does not seem to be a realistic case. It would be nice however to endogenize this in a more general model. This is left for future research.

[5] As in Gale and Hellwig [1985] for example.

probability $1-\gamma$, they are not audited and pay nothing. Condition (1) can be rewritten as

$$\gamma = \frac{u(\omega + y) - u(\omega + y - m(1+r))}{(1-\alpha)[u(\omega + y) - u(\omega + y - f)]}. \tag{2}$$

This optimality condition for the borrower implies that the probability of audit is decreasing in the fine f, and that it is increasing with the loan rate and with the probability $\alpha = \Pr[\widetilde{s} = 0 \mid \widetilde{w} = y]$. Notice that condition (2) makes sense if and only if f is positive.

We now turn to the audit strategy of the lender. The ex-post incentive of the lender to audit comes from the expectation to collect fines from defrauders. He wants to randomize his audit strategy if he is indifferent between auditing and not auditing. This yields

$$[\delta\beta + (1-\delta)(1-\alpha)] f - c = 0, \tag{3}$$

where δ is the probability that the borrower reporting $R = 0$ tells the truth. The left-hand-side of this equality is the expected revenue of the lender when he audits. The fine f is collected either by mistake from a truthtelling unemployed borrower with probability $\delta\beta$, or from a defrauder with probability $(1-\delta)(1-\alpha)$. The audit cost c is paid with probability 1. Using Bayes rule, we obtain that

$$\delta = \Pr[\widetilde{w} = 0 \mid R = 0] = \frac{1-p}{1-p+\theta p}. \tag{4}$$

Combining conditions (3) and (4) yields

$$\theta = \frac{(1-p)(c - \beta f)}{p[(1-\alpha) f - c]}. \tag{5}$$

Assuming as before that $\alpha + \beta < 1$, we see from equation (5) that the optimality condition for the audit strategy implies that the probability of fraud θ is decreasing in the fine f and in the probability $\beta = \Pr[\widetilde{s} = y \mid \widetilde{w} = 0]$, whereas it is increasing in the probability $\alpha = \Pr[\widetilde{s} = 0 \mid \widetilde{w} = y]$. Notice that when the audit signal is not informative ($\beta = 1 - \alpha$), there is no equilibrium in mixed strategy, since condition (5) would imply that $\theta = -(1-p)/p < 0$. More generally, the θ obtained from condition (5) lies between 0 and 1 if and only if

$$\beta \leq \frac{c}{f} \leq 1 - \alpha \qquad \text{and} \qquad (1-p)\beta + p(1-\alpha) \geq \frac{c}{f}. \tag{6}$$

We now turn to the equilibrium condition on the credit market at date 0. We assume Bertrand competition among lenders, which implies that their expected profit per unit of money lent is zero at equilibrium. This yields

$$p(1-\theta)r + [1-p(1-\theta)]\left[-1+\gamma\frac{[(\delta\beta+(1-\delta)(1-\alpha))f-c]}{m}\right] = 0. \quad (7)$$

With probability $p(1-\theta)$, the borrower truthfully reports $R = w = y$, which generates profit r to the lender. Otherwise, the loan is lost for the lender, but some fines may be collected if borrowers' incomes are audited. It yields an expected profit $[\delta\beta + (1-\delta)(1-\alpha)]\, f - c$. Using condition (3), we can rewrite condition (7) as

$$r = \frac{1-p(1-\theta)}{p(1-\theta)}. \quad (8)$$

The equilibrium loan rate is increasing in the probability of fraud. Combining this observation with the property that the probability of fraud is decreasing in the penalty yields the following Lemma.

Lemma 1. *Suppose that $\alpha+\beta$ is less than unity. Then, the break-even loan rate and the incentive-compatible probability of strategic default are decreasing in the penalty f.*

Conditions (2), (5) and (8) determine γ, θ, and r as a function of f. In order to characterize an equilibrium, it must be that the contractible fine f maximizes the expected utility of the borrower subject to the abovementioned constraints:

$$\begin{aligned} \max_f \quad & EU = pu(\omega+y-m(1+r)) \\ & +(1-p)\left[\gamma\beta u(\omega-f)+(1-\gamma\beta)u(\omega)\right] \quad (9) \\ s.t. \quad & (2),(5),(8), \\ & B+EU \geq pu(\omega+y)+(1-p)u(\omega). \quad (10) \end{aligned}$$

With probability p, the income of the borrower is y. Because he is indifferent between reporting $R = 0$ and $R = y$, his utility in that state is $u(\omega + y - m(1 + r))$. With probability $1 - p$, his income is $w = 0$, in which case he reports $R = 0$. With probability $\gamma\beta$, the borrower's income is audited and it mistakenly signals $s = y$, in which case he must pay fine f.

The last inequality in (10) is the participation constraint for borrowers meaning that B the private benefit plus the expected utility obtain through the debt contract is higher than the utility obtained by not consuming. We hereafter assume that B is sufficiently large for this condition to hold.

4. DOES THERE EXIST AN EQUILIBRIUM WITH STRATEGIC DEFAULT?

It is easy to see that there cannot exist any equilibrium in which lenders never audit. By contradiction, if $\gamma = 0$, it would always be optimal for employed borrowers to strategically default on their loan. Because lenders do not audit, they would never recover their funds. This cannot be an equilibrium. However, there may exist equilibria where borrowers never misreport their incomes, but where it is optimal for lenders to audit defaulting borrowers.

In order to characterize these equilibria, we need to determine how the level of the fine f affects the welfare of borrowers once one takes into account all the effects of a change in f on the strategic variables (θ, γ), and on the equilibrium loan rate r. As seen in (9), borrowers ends up with $U_y \equiv u(\omega + y - m(1 + r))$ with probability p, and with $U_0 \equiv \gamma\beta u(\omega - f) + (1 - \gamma\beta)u(\omega)$ otherwise. It is easy to check that U_y is increasing in f. Indeed, from (8), the loan rate r is increasing with the probability of fraud θ. Moreover, from (5), determining if the probability of fraud is decreasing with the fine whenever U_0 is increasing or decreasing in f is more difficult. On the one hand, an increase in f induces a first-order stochastically dominated shift in the distribution of consumption conditional on $\widetilde{w} = 0$, because of the risk of error of the audit technology. On the other hand, a change in f also affects the probability of audit, both directly and indirectly through the change in the loan rate (condition (2)). The global effect of a change in the contractual fine on the borrower's welfare is thus ambiguous. More precisely, two cases may occur depending on the audit technology used by lenders.

4.1. Equilibria when $\beta = 0$

In this section, we assume that lenders choose a technology where the unemployed status of the consumer is always revealed by the audit: $\beta = \Pr[\widetilde{s} = y \mid \widetilde{w} = 0] = 0$. This implies that the utility $U_0 = u(\omega)$ is independent of the level of the fine and thus the only effect of an increase in the fine is to discipline the borrower. A larger f reduces the loan rate r, which in turn increases both $U_y = u(\omega + y - m(1 + r))$ and the expected utility of the borrower. In this case, there is no equilibrium in which both parties use mixed strategies.

Proposition 1. *Suppose that the audit signal is $s = 0$ whenever $w = 0$, i.e., that $\beta = \Pr[\widetilde{s} = y \mid \widetilde{w} = 0] = 0$. Then, there is no equilibrium in mixed strategies.*

Notice that the competitive pressure to raise the penalty has no limit when $\beta = 0$. Remember that when $\beta = 0$, lenders never want to audit when there is no strategic default. The existence of strategic default at equilibrium then depends upon the asymptotic properties of the utility function at low wealth levels. Let us assume that there exists a minimum level of survival $z_{\min}$ such that $u(z)$ tends to $-\infty$ when z tends to $z_{\min}$ from above. Then, as f tends to $\omega + y - z_{\min}$, convicted defrauders would get a utility u tending to $-\infty$. From conditions (2) , lenders and borrowers will reduce γ down to zero. However from condition (5), we know that θ is finite and positive. Then γ cannot tend to zero. There is no equilibrium in mixed strategies.

4.2. Equilibria when $\beta > 0$

In this section, we show that an equilibrium with no strategic default may arise when $\beta > 0$. Indeed, if the contract includes a sufficiently large payment f when $R = 0$ and $\widetilde{s} = y$, auditing defaulting borrowers may be optimal for lenders in spite of the common knowledge that there is no strategic default. The penalty serves in this case as an incentive device for lenders to audit and for borrowers not to misreport their type. Choosing such an imperfect technology may be considered as a commitment to audit.

But, when the audit technology may detect a fraud when the unemployed borrower truthfully reports his status ($\beta > 0$), the borrower's expected utility U_0 conditional to $\widetilde{w} = 0$ is affected by the risk of error. However if U_0 is *increasing* in the penalty f, the increase in the loss due to the error would then be more than compensated by the reduction in the probability to make such an error. In what follows, we show that non-decreasing absolute risk aversion is a sufficient condition for U_0 to be increasing in the fine. This is the case if the probability of audit is sufficiently decreasing with f. The largest possible fine would again be an equilibrium in this case implying no strategic default at equilibrium with $\beta > 0$.

It is important to observe that the risk $(\omega - f, \gamma\beta; \omega, 1-\gamma\beta)$ borne by the unemployed borrower is not very different from the risk $(\omega + y - f, \gamma(1-\alpha); \omega + y, 1-\gamma(1-\alpha))$ borne by the defrauder. This is a useful remark, since we know from condition (1) that the latter risk has a certainty equivalent $\omega + y - m(1+r)$, which is increasing in f from Lemma 1. Suppose that the borrower's utility function exhibits constant absolute risk aversion, which implies that there is no wealth effect. Condition (1) can thus be rewritten as

$$\gamma(1-\alpha)u(\omega - f) + (1-\gamma(1-\alpha))u(\omega) = u(\omega - m(1+r)). \tag{11}$$

Using this equation to eliminate γ from U_0 yields

$$U_0 = u(\omega) + \frac{\beta}{1-\alpha}\left[u(\omega - m(1+r)) - u(\omega)\right] \tag{12}$$

which is decreasing in r and, from Lemma 1, increasing in f. This completes the proof that, under CARA, there is no equilibrium with strategic default. The equilibrium is fully described by the following conditions:

$$\beta f - c = 0, \tag{13}$$

$$pr + (1-p)\left[-1 + \gamma(\beta f - c)\right] = 0, \tag{14}$$

and

$$\gamma\,(1-\alpha)\,u(\omega+y-f)+(1-\gamma(1-\alpha))u\,(\omega+y) \leq u(\omega+y-m(1+r)), \tag{15}$$

or, equivalently,

$$\gamma \geq \gamma_1 = \frac{1}{1-\alpha}\frac{u(\omega+y)-u(\omega+y-mp^{-1})}{u(\omega+y)-u(\omega+y-(c/\beta))}. \tag{16}$$

Condition (13) is the indifference condition for the lender to audit or not. If α is positive, it yields the equilibrium fine $f \equiv f_1 = c/\beta$. Equation (14) is the market-clearing condition, which implies that $1+r = 1/p$. Condition (15) states that employed borrowers don't want to strategically default. This requires that γ be larger than the threshold γ_1 characterized in (16). Such an equilibrium exists if γ_1 is smaller than unity, i.e., if

$$(1-\alpha)u(\omega+y-c) + \alpha u(\omega+y) \leq u(\omega+y-mp^{-1}). \tag{17}$$

If this inequality holds, any solution with $f = f_1$, $r = (1-p)/p$ and $\gamma \in [\gamma_1, 1]$ is an equilibrium with no strategic default. These equilibria, when they exist, can be ranked according to the Pareto criterion, with $\gamma = \gamma_1$ being the preferred one.

Finally, if condition (17) is not satisfied, it is easy to verify that there exists an equilibrium with no strategic default, systematic auditing and a penalty f_2 larger than f_1 such that

$$(1-\alpha)u(\omega+y-f_2) + \alpha u(\omega+y) = u(\omega+y-mp^{-1}). \tag{18}$$

Notice that this case covers the situation where borrowers are also risk neutral, i.e., when the constant absolute risk aversion tends to zero.

We generalize this result by considering preferences that do not exhibit constant absolute risk aversion. Our result is summarized in the following Proposition.

PROPOSITION 2. *Suppose that the borrower's utility function exhibits non-decreasing absolute risk aversion (iARA), i.e.,* $[-u''(z)/u'(z)]' \geq 0$.

Then, there is no equilibrium with strategic default. The equilibrium penalty equals $f_1 = c$ if condition (17) is satisfied, otherwise f equals f_2 as defined by equation (18).

Proof. We just prove that U_0 is increasing in f when there exists some strategic default. Following the same argument as in the CARA case examined above would then yield the result. Differentiating condition (1) with respect to f yields

$$\begin{aligned} & \gamma'(f)(1-\alpha)\left[u(\omega+y-f)-u(\omega+y)\right] \\ = \ & \gamma(f)(1-\alpha)u'(\omega+y-f)+U_y'(f). \end{aligned} \tag{19}$$

Fully differentiating U_0 with respect to f would be nonnegative if and only if

$$\gamma'(f)\beta\left[u(\omega-f)-u(\omega)\right] \geq \gamma(f)\beta u'(\omega-f). \tag{20}$$

Eliminating γ' from this inequality by using condition (19) yields

$$\frac{u(\omega)-u(\omega-f)}{u(\omega+y)-u(\omega+y-f)} \geq \frac{u'(\omega-f)}{u'(\omega+y-f)+\frac{1-\alpha}{\beta\gamma}U_y'(f)}. \tag{21}$$

From Lemma 1, we know that U_y is increasing in f. This implies that inequality (21) would automatically be satisfied if

$$\frac{u(\omega)-u(\omega-f)}{u(\omega+y)-u(\omega+y-f)} \geq \frac{u'(\omega-f)}{u'(\omega+y-f)},$$

or, equivalently, because f is positive,

$$\frac{u(\omega)-u(\omega-f)}{u'(\omega-f)} \geq \frac{u(\omega+y)-u(\omega+y-f)}{u'(\omega+y-f)}. \tag{22}$$

Using Lemma 2 in the appendix concludes the proof as it states that condition (22) holds under IARA ▌

The underlying intuition of this result is that judgement errors induce lenders to audit even when they know there is no strategic default at equilibrium. One may argue that this way of viewing the errors as a committing device or an incentive scheme is not satisfactory because it relies on the fact that the lender knows that the only borrowers that are punished are honest. However, the contract is accepted by borrowers because it increases their expected utility. The imperfect audit technology with $\beta > 0$ is better than the audit technology with $\beta = 0$. The penalty f can be interpreted as a transfer borrowers have to pay with a given probability in the bad state of the world. Even if this contract may lead borrowers to be worse off in a given state, they are better off in expectation. This contract Pareto-dominates all others.

Under the alternative conditions of Propositions 1 and 2, the competitive pressure to offer contracts with an increasing penalty when $R = 0$ and $s = y$ implies that there is no strategic default at equilibrium. The underlying assumptions leading to this result are quite restrictive. The assumption in Proposition 1 that there is no risk of error to punish unemployed borrowers is not realistic. And the assumption in Proposition 2 about absolute risk aversion is in contradiction with the well-documented fact that absolute risk aversion is decreasing. For example, it is well established that wealthier investors purchase more risky assets, or that consumers with riskier future incomes raise their precautionary savings.

5. SMALL PENALTIES AND STRATEGIC DEFAULT AT EQUILIBRIUM

When there is a risk to penalize unemployed borrowers and when absolute risk aversion is decreasing, imposing large penalties may not be feasible. To see this, consider again the assumption that there exists a minimum level of subsistence $z_{\min}$ such that $u(z_{\min}) = -\infty$. Then, when $\beta > 0$, the solution with $f = \omega + y - z_{\min}$ is not feasible. For such a penalty, it would imply a level of consumption $\omega - f = z_{\min} - y$ smaller than $z_{\min}$ for unemployed borrowers that have been audited with $s = y$.

The penalty is limited by the willingness to reduce the effect of errors on borrowers' expected utility.

PROPOSITION 3. *Suppose that the borrower's utility function exhibits decreasing absolute risk aversion. Then, the fine f must be lower than $\omega - z_{\min}$ and there exist cases involving strategic default.*

Proof. Assume that $f = \omega - z_{\min}$. Then by replacing f in (2), we obtain $\gamma = \dfrac{u(\omega + y) - u(\omega + y - m(1+r))}{(1-\alpha)\left[u(\omega + y) - u(y + z_{\min})\right]}$. Due to the asymptotic properties of u, γ is small but finite. Replacing f in the expression of the expected utility that borrowers receive we obtain

$$EU = pu(\omega + y - m(1+r)) + (1-p)\left[\gamma\beta u(z_{\min}) + (1-\gamma\beta)u(\omega)\right].$$

But $u(z_{\min}) = -\infty$ by construction, the expected utility is equal to $-\infty$. This implies a contradiction with the participation constraint. Then the fine $f = \omega - z_{\min}$ cannot be an equilibrium. ∎

As previously said, an optimal contract has to verify the following program:

$$\begin{array}{lrcl} \max\limits_{f} & EU & = & pu(\omega + y - m(1+r)) + (1-p)\left[\gamma\beta u(\omega - f) + (1-\gamma\beta)u(\omega)\right] \\ & & s.t. & \begin{array}{l} (2),(5),(8), \\ B + EU \geq pu(\omega + y) + (1-p)u(\omega). \end{array} \end{array}$$

Assuming that the participation constraint is always verified, the first order condition of this program is

$$\begin{array}{l} \dfrac{dr}{df}u'(\omega + y - m(1+r))\left[p + \frac{(1-p)\beta[u(\omega)-u(\omega-f)]}{(1-\alpha)[u(\omega+y)-u(\omega+y-f)]}\right] \\ -(1-p)\beta\gamma\left[u'(\omega - f) - \frac{[u(\omega)-u(\omega-f)]}{[u(\omega+y)-u(\omega+y-f)]}u'(\omega + y - f)\right] = 0. \end{array} \tag{23}$$

If an optimum exists, the optimal penalty to be enforced to a borrower who strategically defaults should verify this condition. Under the assumption that u is DARA, it may be the case that this condition is verified for a fine such that the strategic default probability is positive. Moreover, it may be the case that the optimal penalty is softer than limited liability. However this cannot be proved directly.

6. NUMERICAL EXAMPLE

When u is DARA, no analytical solution arises from (23). The following example provides some interesting insights on changes in the probability of strategic default, optimal penalties and audit probability. We calibrate the model with a logarithmic consumer: $u(x) = \ln(10 + x)$. We assume that α and β are strictly positive and such that $\alpha + \beta < 1$. The fixed parameter values we take are: $\omega = 20$ and $y = 10$ in the good state, $p = 0.98$[6] and $\alpha = 0.05$. We are interested in variations of the following parameters: the audit error α and the audit cost c. In this example, $z_{\min}$ is equal to zero corresponding to a maximal penalty f^* of 30.

First, we consider variations of the audit error. In order to isolate the effect of this parameter, we fix the value of $c = 2$ and $m = 5$. One may think that the value of m is large considering the value of the income $\omega + y$ but it is often the case in consumer credit.[7] The effects of the variations of β on the optimal fine and the audit probability are summarized by the figures in the appendix. Figure 1 shows that the fine is decreasing in β.

[6]This probability corresponds to the average repayment probability observed by an Italian company specialized in consumer credit.

[7]See "Consumer credit evidence from Italian microdata" R. Alessie, S. Hochguertel, G. Weber [2001] for more information on data.

Intuitively, the more likely lenders enforce unfair penalties the less they want to penalize. Moreover, for any positive value of β, we obtain that the penalty is lower than ω. The penalty is then softer than required by limited liability which would imply $f^* = \omega$. An increase in β implies an increase in the audit probability as shown in figure 2. Since the audit becomes less efficient, the lender needs to audit more in order to induce the right incentives. This increase in the audit probability induces the successful borrower not to strategically default since this would increase the probability to be fined. Then the probability of strategic default is decreasing when β increases. Moreover, at some threshold, the probability of strategic default becomes equal to zero. For values of β smaller than this threshold, we can see that it is optimal to tolerate strategic default in equilibrium.

Equation (8) shows that the interest rate is completely defined by the strategic default probability. It is straightforward to show that the interest rate is an increasing function of the strategic default probability. The intuition of this result is simple. If the proportion of employed workers who strategically default increases, then lenders have to increase the loan rate paid by honest borrowers in order to earn non-negative profits.

Now, let us assume that $\beta = 0.02$. Consider the effects of an increase of the audit cost c. Figure 4 shows that the audit probability is decreasing with c. The intuition is simple: when auditing becomes more costly, lenders prefer to reduce the audit frequency. They arbitrage between the audit cost and the potential gain that auditing implies. Figure 5 shows that the equilibrium fine is increasing in the audit cost. This is due to the fact that since auditing becomes more costly, lenders audit less frequently in order to reduce the expected audit cost. But lenders have to increase the fine. Otherwise, borrowers will increase their strategic default probability which can lead to a market failure. However, the changes in the lenders' auditing strategy (γ, f) are not sufficient to reduce strategic default.

It is intuitive that the probability of strategic default is increasing in the audit cost. As this increase leads to less attractive auditing conditions for lenders, borrowers will expect a lower probability to be fined. Even if the fine may be higher than the previous one, defaulting may still be more interesting than before. As in the previous case, the interest rate is fully determined by the probability of strategic default. Since the interest rate is an increasing function of this probability, it is obvious that the interest rate increases with the audit cost since θ increases. An increase in the audit cost reduces the gain to audit. The more costly the audit, the less efficient the contract. As shown in the previous section, the fine is always lower than the maximal punishment under DARA.

7. CONCLUSION

We consider a model of costly state verification in credit markets. In particular, we focus on consumer credit markets that exhibit specific features: borrowers are risk-averse and lenders are unable to commit on auditing strategies. This is due both to consumer protection and to high costs of audit with respect to the size of consumption loans. In general, borrowers may be unable to repay their loan if they face an adverse shock on their income. In such a setting, borrowers may decide to default even if they are in the good state of nature. In this context, lenders can decide to audit defaulting consumers and to fine them in case of an audit confirming the fraud. We characterize credit contracts that appear in equilibrium when the fine is not exogenously given but arises as an outcome of the game.

We also consider the case where the audit technology is imperfect. By imperfect we mean both that the audit can mistakenly reveal that a defaulter is in a good situation whereas he is in a bad one and that a borrower who strategically defaults is not found. We show in such a case of imperfect audit that, if the expected utility exhibits non-decreasing absolute risk aversion, there is no strategic default at equilibrium. However, if borrowers' expected utility exhibits decreasing absolute risk aversion, it may the case that strategic default arises at equilibrium. Nevertheless, the optimal fine is always finite.

APPENDIX A

LEMMA 2. *Consider function* $F(w) = \frac{u(\omega)-u(\omega-f)}{u'(\omega-f)}$, *with* $f > 0$. *Function F is increasing (resp. decreasing) when u exhibits DARA (resp. iARA).*

Proof.

Let us assume that u is DARA, implying that $A(c) = -\frac{u''(c)}{u'(c)}$ is decreasing in c. F is decreasing if

$$X = [u'(w) - u'(w-f)]\, u'(w-f) - [u(w) - u(w-f)]\, u''(w-f)$$

is positive. We have that

$$X = \int_{-f}^{0} [u''(w+x)u'(w-f) - u'(w+x)u''(w-f)]\, dx$$

which in turn is equal to

$$X = u'(w-f) \int_{-f}^{0} u'(w+x) \left[A(w-f) - A(w+x)\right] dx$$

Because $w-f$ is smaller than $w+x$ for all $x \in [-f, 0]$ and because A is decreasing, we obtain that $A(w-f) - A(w+x)$ is positive over the interval of integration. Thus, X is positive. The symmetric proof holds in the case of iARA. ▌

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