Income Inequality and Economic Growth: A Simple Theoretical Synthesis [*]

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We show that in an AK model of endogenous growth with CRRA specifications for both private and public consumption goods, income inequality exerts theoretically ambiguous effects on the optimal capital tax rate and the economy's growth rate. In a calibrated version of the model, we find that the parameter space can be clearly divided into regions that exhibit a positive or negative relationship between income inequality and economic growth. Hence, our analysis provides a theoretical synthesis on the relationship between inequality and growth and helps bring together some recent results in the literature. © 2005 Peking University Press

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1. INTRODUCTION

In recent years, there has been an extensive literature that explores how distribution of income affects the growth rate of an economy's GDP.[1] For example, Li and Zou (1998) examine the relationship between income inequality and economic growth in an AK model of endogenous growth with distributive conflicts among agents. These authors show that when the household utility function is logarithmic in public consumption and exhibits a higher-than-unity degree of risk aversion in private consumption, a more equal distribution of income leads to a higher rate of capital taxation (through majority voting) and a lower rate of economic growth. That is, income inequality can help generate faster economic growth. On the contrary, an earlier article by Alesina and Rodrik (1994) find a negative relationship between inequality and growth in a similar political-economy model of endogenous growth, but with government spending being used entirely for the purpose of production. [2] Empirically, the existing cross-country evidence is mixed regarding how income inequality affects subsequent economic growth.[3]

In this paper, we extend Li and Zou's analysis in a framework that allows us to further identify model features and parameters that govern the relationship between income inequality and economic growth. Specifically, we postulate CRRA specifications for *both* private and public consumption goods in the period utility function, a formulation that is drawn on recent empirical estimates of McGrattan, Rogerson and Wright (1997) using postwar U.S. data.

With this modification, we obtain two different theoretical results from Li and Zou (1998). First, after solving the dynamic Ramsey problem, we find that the sign of the correlation between income inequality and the optimal capital tax rate is indeterminate because of the additional curvature introduced to the household utility. By contrast, inequality and the tax rate on capital income are negatively related in Li and Zou's model. Second, we show that income inequality exerts an ambiguous effect on the economy's growth rate. The intuition for this result is straightforward. Start with a perfectly egalitarian society in which each household is endowed with the same share of capital stock. When the distribution of

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[1] The direction of causality runs opposite to the Kuznets' Hypothesis which postulates that income inequality first rises and then falls during the course of economic development (Kuznets, 1955).

[2] Persson and Tabellini (1994) obtain the same result as Alesina and Rodrik (1994) in a two-period overlapping generations economy.

[3] See, for example, Alesina and Rodrik (1994), Persson and Tabellini (1994), Clarke (1995), Benabou (1996), Deininger and Squire (1996,1998), Li and Zou (1998), Li, Squire and Zou (1998), Aghion, Caroli and Garcia-Penalosa (1999), Barro (2000), Savvides and Stengos (2000), Forbes (2000), Li, Xu and Zou (2000), Banerjee and Duflo (2000), Li, Xie and Zou (2000), Chen (2003), among many others.

wealth and income becomes unequal, the median voter now could choose the same or a higher/lower tax rate on capital income. It follows that the after-tax return to capital could be unchanged or lower/higher compared to that evaluated at the egalitarian benchmark, which in turn produces the ambiguous relationship between income equality and economic growth.

Given the inconclusive nature of the above theoretical results, we undertake a quantitative investigation. In a calibrated version of our model, we find that the parameter space can be clearly divided into regions that exhibit a positive (as in Li and Zou, 1998) or negative (as in Alesina and Rodrik, 1994) relationship between inequality and growth. In the former case, when the distribution of wealth and income becomes more unequal, the optimal (Ramsey) policy results in a lower tax rate on capital income as households allocate less resources to government consumption in their effort to equalize the marginal utilities between private and public consumption goods. This leads to a higher after-tax return to capital and raises the economy's growth rate. By contrast, in the latter case, a more unequal distribution of wealth and income generates higher capital taxation in a political equilibrium as households increase their consumption of public goods. As a result, the economy's growth rate falls. Overall, our analysis illustrates that a modification of the Li-Zou model can provide theoretical synthesis on the relationship between inequality and growth and help bring together some recent results in the literature.

The remainder of this paper is organized as follows. Section 2 describes a modified version of the Li-Zou model. Section 3 analyzes the theoretical relationship between economic growth, tax policy, and distribution of capital ownership in a political equilibrium. Section 4 presents our quantitative results. Section 5 concludes.

2. THE ECONOMY

We incorporate a more generalized utility function into the model of Li and Zou (1998). To facilitate comparison, we follow Li and Zou's notation as much as possible. The economy is populated by a finite number of heterogenous infinitely-lived households indexed by $i = 1, \cdots, N$. Household i maximizes its present discounted lifetime utility

$$U_i = \int_0^\infty \left[\frac{c_i^{1-\theta_1}}{1-\theta_1} + \frac{g^{1-\theta_2}}{1-\theta_2} \right] e^{-\rho t} dt, \ \rho, \ \theta_1 \text{ and} \theta_2 > 0, \qquad (1)$$

where c_i is private consumption and ρ is the discount rate.[4] In addition, households view g, which represents the *aggregate* government spending on public services, as determined outside their control. θ_1 and θ_2 denote the coefficients of relative risk aversion with respect to private and government consumption goods, respectively. It follows that the household utility is logarithmic in private (government) consumption when $\theta_1(\theta_2) = 1$. Notice that our model subsumes the Li-Zou formulation in which $\theta_1 \geq 1$ and $\theta_2 = 1$.

The budget constraint faced by household i is given by

$$\dot{k}_i = (1 - \tau)\, y_i - c_i, \quad k_i(0) > 0 \text{given}, \tag{2}$$

where k_i is household i's capital stock (which does not depreciate as in Li and Zou's framework), and output y_i is produced by

$$y_i = A k_i, \quad A > 0. \tag{3}$$

The variable τ denotes the proportional tax rate applied to capital income, which is taken as given by households. In addition, household i's capital endowment (wealth) share at the initial period is defined as

$$\sigma_i \equiv \frac{k_i(0)}{k(0)}, \quad 0 < \sigma_i < 1, \tag{4}$$

where $k(0) = \sum_{i=1}^{N} k_i(0)$. As a result, households are alike in all aspects except for their beginning endowment of capital stock. Moreover, the household with a high σ is capital-rich, whereas one with a low σ is capital-poor.

The first-order conditions for household i's optimization problem are

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\theta_1}\left[(1 - \tau)A - \rho\right], \tag{5}$$

$$\lim_{t \to \infty} e^{-\rho t} \frac{k_i}{c_i^{\theta_1}} = 0, \tag{6}$$

where (5) shows that the growth rate of consumption is governed by the difference between the after-tax return to capital and the discount rate, and (6) is the transversality condition. Since households take τ as given, and A and ρ are parameters, (5) implies that each household's consumption will grow at the same rate for all t, regardless of its initial relative capital endowment σ_i.

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[4]For ease of notation, the time dependence of all variables is mostly suppressed throughout the paper. For example, c_i represents $c_i(t)$ and so on for other variables.

Finally, the government chooses τ and balances its budget at each point in time. Hence, the instantaneous government budget constraint is

$$g = \tau y = \tau A \sum_{i=1}^{N} k_i = \tau A k, \tag{7}$$

where y is total output and k is the economy's aggregate stock of capital.[5]

3. POLITICAL EQUILIBRIUM

Following Li and Zou (1998), we examine the theoretical relationship between economic growth, tax policy, and distribution of capital ownership in three steps. It turns out that the positive relationship between inequality and growth obtained in the Li-Zou model is *not* robust to the generalized CRRA utility function considered in our framework.

3.1. Economic Growth and Tax Policy

Our analysis begins by assuming that the tax rate τ remains constant over time. Using the budget constraint (2) and the transversality condition (6) for household i, it is straightforward to show that k_i and c_i will grow at the same constant rate for all t. In addition, since $k = \sum_{i=1}^{N} k_i$ and $y = Ak$, a steady-state growth path is characterized by

$$\frac{\dot{k}_i}{k_i} = \frac{\dot{c}_i}{c_i} = \frac{\dot{k}}{k} = \frac{\dot{y}}{y} \equiv \gamma = \frac{1}{\theta_1}\left[(1-\tau)A - \rho\right], \tag{8}$$

where the common economy-wide growth rate γ is independent of the initial distribution of capital stock. It follows that σ_i is time-invariant along a balanced growth path. Moreover, (8) implies that the rate of economic growth is negatively related to the tax rate on capital income, namely

$$\frac{\partial \gamma}{\partial \tau} < 0. \tag{9}$$

3.2. Tax Policy and Wealth Distribution

This subsection examines the relationship between household i's preferred tax policy, denoted as τ_i^*, and its beginning share of capital stock σ_i. We consider the dynamic Ramsey problem in which a benevolent government chooses a program of public spending g and distortionary taxes on capital income τ to maximize the discounted utility of household i .

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[5]As a result, household i's income share y_i/y is identical to its wealth (capital) share k_i/k in this economy.

In computing the optimal fiscal policy, the government takes into account the rational responses of private agents, as summarized by (2)-(3), (5)-(6), and (8), and the government budget constraint (7). Substituting these equations into (1) yields the following expression:

$$
\begin{aligned}
U_i = & \ \frac{[k_i(0)]^{1-\theta_1}}{1-\theta_1} \left[\frac{\rho - (1-\tau_i^*)(1-\theta_1)A}{\theta_1} \right]^{-\theta_1} \\
& + \frac{\theta_1 [Ak(0)\tau_i^*]^{1-\theta_2}}{(1-\theta_2)\{\rho\theta_1 - [(1-\tau_i^*)A - \rho](1-\theta_2)\}}.
\end{aligned}
\tag{10}
$$

To ensure that the household utility is bounded, we need to impose further restrictions on θ_1 and θ_2 whereby

$$
\theta_1 > 1 - \frac{\rho}{(1-\tau_i^*)A} \equiv \theta_1^*,
\tag{11}
$$

and

$$
\theta_2 > 1 - \frac{\rho\theta_1}{(1-\tau_i^*)A - \rho} \equiv \theta_2^*.
\tag{12}
$$

Using the definition of σ_i as in (4), the government's first-order condition with respect to τ_i^* is

$$
\begin{aligned}
& A[k(0)\sigma_i]^{1-\theta_1} \left[\frac{\rho - (1-\tau_i^*)(1-\theta_1)A}{\theta_1} \right]^{-\theta_1-1} \\
& + \frac{\theta_1 A^{2-\theta_2} [\tau_i^* k(0)]^{1-\theta_2}}{\{\rho\theta_1 - [(1-\tau_i^*)A - \rho](1-\theta_2)\}^2} \\
& = \frac{\theta_1 (\tau_i^*)^{-\theta_2} [Ak(0)]^{1-\theta_2}}{\rho\theta_1 - [(1-\tau_i^*)A - \rho](1-\theta_2)}.
\end{aligned}
\tag{13}
$$

Since we focus on a steady-state growth path along which σ_i remains fixed, (13) implies that the optimal (Ramsey) policy includes a time-invariant tax rate on capital income. Hence, results from the previous subsection based on a constant τ are consistent with the equilibrium outcome.

Next, we take the total differentiation on (13) and obtain

$$
\frac{\partial \tau_i^*}{\partial \sigma_i} = \frac{\text{Numerator}}{\text{Denominator}},
\tag{14}
$$

where

$$
\text{Numerator} = Ak(0)(1-\theta_1)[k(0)\sigma_i]^{-\theta_1} \left[\frac{\rho - (1-\tau_i^*)(1-\theta_1)A}{\theta_1} \right]^{-\theta_1-1},
$$

and

$$\text{Denominator} = \frac{A^2\left(1-\theta_1^2\right)\left[k(0)\sigma_i\right]^{1-\theta_1}}{\theta_1}\left[\frac{\rho-\left(1-\tau_i^*\right)\left(1-\theta_1\right)A}{\theta_1}\right]^{-\theta_1-2}$$

$$-\frac{\theta_1\theta_2 A^{1-\theta_2}\left(\tau_i^*\right)^{-\theta_2-1}\left[k(0)\right]^{1-\theta_2}}{\rho\theta_1-\left[\left(1-\tau_i^*\right)A-\rho\right]\left(1-\theta_2\right)}$$

$$-\frac{2\theta_1(1-\theta_2)A^{2-\theta_2}\left(\tau_i^*\right)^{-\theta_2}\left[k(0)\right]^{1-\theta_2}}{\left\{\rho\theta_1-\left[\left(1-\tau_i^*\right)A-\rho\right]\left(1-\theta_2\right)\right\}^2}$$

$$+\frac{2\theta_1(1-\theta_2)A^{3-\theta_2}\left(\tau_i^*\right)^{1-\theta_2}\left[k(0)\right]^{1-\theta_2}}{\left\{\rho\theta_1-\left[\left(1-\tau_i^*\right)A-\rho\right]\left(1-\theta_2\right)\right\}^3}.$$

As in Li and Zou (1998), $\frac{\partial\tau_i^*}{\partial\sigma_i} = 0$ if the household utility (1) is logarithmic in private consumption ($\theta_1 = 1$), for all $\theta_2 > \theta_2^*$. Otherwise, since the Numerator is negative (positive) when $\theta_1 > (<)1$, and the preference parameters θ_1 and θ_2 enter the Denominator in a rather complicated way, the sign of $\frac{\partial\tau_i^*}{\partial\sigma_i}$ is generally ambiguous. By contrast, in Li and Zou's framework with $\theta_2 = 1$, household i's preferred tax rate on capital income is an increasing function of its share of capital endowment ($\frac{\partial\tau_i^*}{\partial\sigma_i} > 0$) as long as $\theta_1 > 1$.

3.3. Economic Growth and Wealth Distribution

As is common in the literature, we postulate that the rate of capital income tax is determined by majority voting. Since voting takes on a single issue and household preferences are single-peaked, the median-voter theorem is applicable in this environment. Therefore, the tax rate selected by the majority rule coincides with the median voter's choice τ_m, which solves

$$A\left[k(0)\sigma_m\right]^{1-\theta_1}\left[\frac{\rho-\left(1-\tau_m\right)\left(1-\theta_1\right)A}{\theta_1}\right]^{-\theta_1-1}$$

$$+\frac{\theta_1 A^{2-\theta_2}\left[\tau_m k(0)\right]^{1-\theta_2}}{\left\{\rho\theta_1-\left[\left(1-\tau_m\right)A-\rho\right]\left(1-\theta_2\right)\right\}^2}$$

$$=\frac{\theta_1\left(\tau_m\right)^{-\theta_2}\left[Ak(0)\right]^{1-\theta_2}}{\rho\theta_1-\left[\left(1-\tau_m\right)A-\rho\right]\left(1-\theta_2\right)}, \tag{15}$$

where σ_m denotes the capital endowment share of the median voter. Notice that it does not matter when or how often voting takes place in that the optimal tax rate remains constant over time, and the distribution of capital

stock is also time-invariant. Furthermore, following the same discussion of the preceding subsection, $\frac{\partial \tau_m}{\partial \sigma_m}$ can be zero, positive or negative.

Using (9), (14) and the chain rule leads to the following relationship between economic growth and wealth distribution in a political equilibrium:

$$\frac{\partial \gamma}{\partial \sigma_m} = \underbrace{\frac{\partial \gamma}{\partial \tau_m}}_{\text{negative}} * \underbrace{\frac{\partial \tau_m}{\partial \sigma_m}}_{\text{zero/positive/ negative}}, \tag{16}$$

whose sign is indeterminate. To understand this result, consider a perfectly egalitarian society in which each household is endowed with the same share of capital stock, $\sigma_m = \sigma_i = \frac{1}{N}$, for all i. When the distribution of wealth and income becomes unequal $(\sigma_m < \frac{1}{N})$, the median voter now could choose the same (when $\theta_1 = 1$) or a higher/lower (when $\theta_1 \neq 1$) tax rate on capital income τ_m. It follows that the after-tax return to capital $(1 - \tau_m)A$ could be unchanged or lower/higher compared to that evaluated at $\sigma_m = \frac{1}{N}$. That is, inequality in wealth and income distribution leads to redistributive tax policies that do not affect or reduce/raise the net return to capital accumulation. As a result, income inequality exerts an ambiguous effect on the economy's growth rate.

4. QUANTITATIVE RESULTS

Given the inconclusive nature of the above theoretical analysis, we undertake a quantitative investigation of $\dfrac{\partial \tau_m}{\partial \sigma_m}$ in a calibrated version of our model. In particular, we are interested in examining whether the positive relationship between inequality and growth that Li and Zou (1998) have found is robust to the additional curvature incorporated into the household utility function. In our benchmark specification, the economy's beginning aggregate capital stock is normalized to unity, $k(0) = 1$, and equally distributed among households, thus $k_m(0) = k_i(0) = \frac{1}{N}$, for all i. Moreover, the discount rate ρ is chosen to be 0.025, the technology parameter A is set to be 0.5, and the number of households in the economy N is fixed at 5. Finally, we adopt the tax rate on capital income $\tau_m = 0.5$, which is equal to the average level observed in the post-Korean war U.S. economy (see Cooley and Hansen, 1992).[6]

It turns out that given the selected values of ρ, A and τ_m, the lower bound of θ_1 described in (11) is $\theta_1^* = 0.9$. Therefore, our quantitative results are reported in three cases as follows:

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[6]We obtain qualitatively similar results with different values of N as well as various combinations of ρ, A and τ_m that ensure a positive rate of economic growth (see equation 8).

(i) $\theta_1 = 1$. As discussed earlier, $\frac{\partial \tau_m}{\partial \sigma_m} = 0$ in this configuration, for all $\theta_2 > \theta_2^*$. It follows that $\frac{\partial \gamma}{\partial \sigma_m} = 0$, i.e., income inequality does not affect economic growth when the household utility is logarithmic in private consumption.

(ii) $\theta_1 > 1$. Recall that $\frac{\partial \tau_m}{\partial \sigma_m} > 0$ in Li and Zou (1998) provided $\theta_1 > 1$ and $\theta_2 = 1$. By contrast, Figure 1 illustrates that in our model, $\frac{\partial \tau_m}{\partial \sigma_m} < 0$ when $1 < \theta_1 \leq 9$, and θ_2 ranges between θ_2^* (not included) and a downward-sloping curve (included).[7] Notice that all feasible θ_2's in this "Negative" region are less than 1. On the other hand, Li and Zou's finding of a positive $\frac{\partial \tau_m}{\partial \sigma_m}$ holds true whenever θ_2 lies above the dividing boundary.

(iii) $\theta_1^* < \theta_1 < 1$. This is the setting that Li and Zou (1998) have *not* investigated. Figure 2 shows that similar to case (ii), the sign of $\frac{\partial \tau_m}{\partial \sigma_m}$ is governed by a dividing boundary. Specifically, $\frac{\partial \tau_m}{\partial \sigma_m} < 0$ along and above this boundary, whereas $\frac{\partial \tau_m}{\partial \sigma_m} > 0$ when θ_1 and θ_2 fall in the "Positive" area. Notice that in contrast to case (ii), all feasible θ_2's in the "Negative" region of Figure 2 are greater than 1.

FIG. 1. $\partial \tau_m / \partial \sigma_m$ when $\theta_1 > 1$
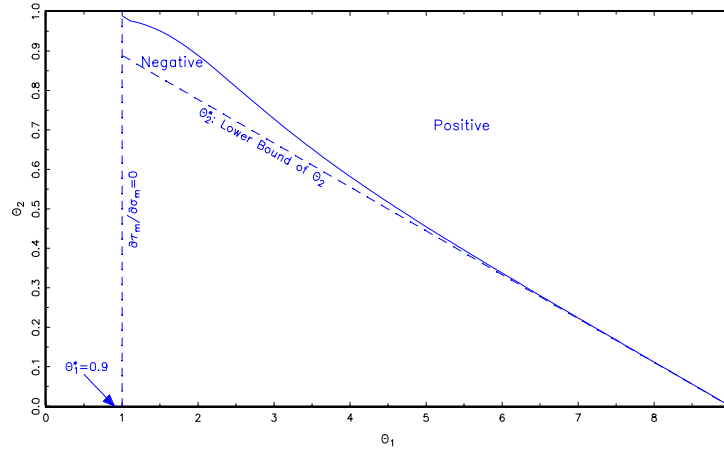
To summarize, $\frac{\partial \tau_m}{\partial \sigma_m} > 0$ in the "Positive" regions of Figures 1 and 2. In this case, as in Li and Zou (1998), income inequality is helpful to economic growth whereby $\frac{\partial \gamma}{\partial \sigma_m} < 0$ (see equation 16). The intuition for this result is straightforward. When the distribution of wealth and income becomes unequal (i.e., when σ_m falls below $\frac{1}{N}$), the median voter will be endowed with smaller capital stock. Hence, majority voting results in a lower tax

[7]The maximum value of $\theta_1 = 9$ is chosen such that the lower bound of θ_2, given by (12), is $\theta_2^* = 0$.
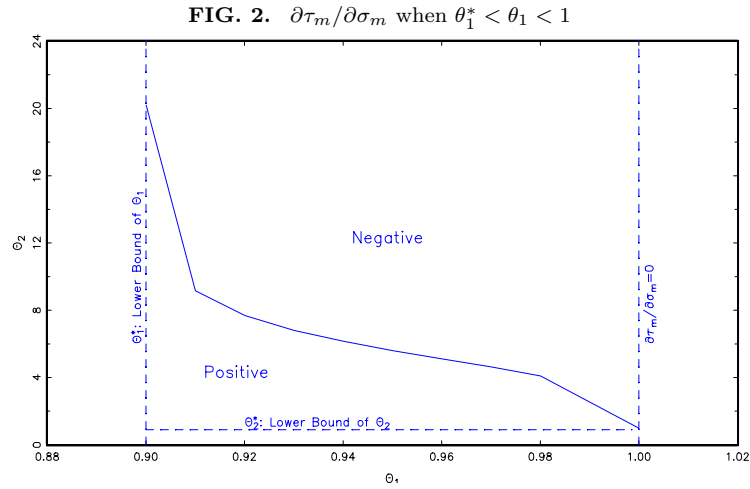
FIG. 2. $\partial\tau_m/\partial\sigma_m$ when $\theta_1^* < \theta_1 < 1$

rate on capital income as households allocate less resources to government consumption in their effort to equalize the marginal utilities between private and public consumption goods. It follows that the after-tax return to capital becomes higher, which in turn raises the economy's growth rate. However, our quantitative analysis shows that this positive relationship between inequality and growth is not a robust result. In particular, $\frac{\partial\tau_m}{\partial\sigma_m} < 0$ when feasible combinations of θ_1 and θ_2 are in the "Negative" regions (including the dividing boundaries) of Figures 1 and 2. That is, a more unequal distribution of wealth and income leads to higher capital taxation in a political equilibrium as households now increase their consumption on public goods. In this case, as in Alesina and Rodrik (1994), income inequality is harmful for economic growth whereby $\frac{\partial\gamma}{\partial\sigma_m} > 0$. Our model thus has the versatility to produce theoretical results obtained in previous studies.

5. CONCLUSION

We have explored the interrelations between income inequality, optimal tax policy and economic growth in an AK model of endogenous growth with CRRA specifications for both private and public consumption goods. Our theoretical analysis shows that with a more generalized utility function, the effect that income inequality exerts on economic growth can be zero, positive or negative. On the other hand, our quantitative analysis allows us to clearly identify parameter combinations that produce a positive or negative relationship between inequality and growth. Overall, this

paper synthesizes the theoretical relationship between income inequality and economic growth that has been found in the existing literature.

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