A Note on the Relation between Risk Aversion, Intertemporal Substitution and Timing of the Resolution of Uncertainty

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Epstein and Zin (1989) axiomatization allows the distinction between risk aversion and intertemporal substitution. Kreps and Porteus (1978) one introduces the concept of timing of resolution of uncertainty. This paper proposes to generalize the link between these three concepts.

Key Words: Risk aversion; Intertemporal substitution; Timing of the resolution of uncertainty.
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1. INTRODUCTION

In recent research in the areas of finance (see Campbell and Viceira [2002], Garcia, Luger and Renault [2003]) and macroeconomics (for exemple, Weil [1989], Obstfeld [1994], Tallarini [2000], Epaulard and Pommeret [2003]), preferences of agents are characterized by recursive utility functions (Kreps and Porteus [1978], Epstein and Zin [1989]). This class of preferences permits to disentagle risk aversion from intertemporal substitution. Recent studies show that this separation might be important to explain different phenomena. This treatment of preferences is also supported by the empirical studies (Epstein and Zin [1991]).

Otherwise, the class of temporal preferences introduced by Kreps and Porteus [1978] allows the representation of a third concept: the timing of the resolution of uncertainty. The temporal resolution of uncertainty plays a no negligible role on consumption decisions (Blundell and Stoker [1999], Eeckhoudt, Gollier and Treich [2004]). Epstein and Zin [1989], Farmer [1990] and Weil [1990] found a relation between this last concept, risk aversion and intertemporal substitution in the framework of a CES utility and constant relative risk aversion.

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The objective of this note is to underline the relation between risk aversion, intertemporal substitution and preference for the timing of resolution of uncertainty. We show that although recent studies clarify the importance of the different roles playing by risk aversion and intertemporal substitution, there exists a connection between these two concepts via the notion of timing of uncertainty. And this, for any form of utility functions and any form of relative risk aversion.

2. THE MODEL

We consider the Epstein and Zin [1989] model. They introduced a class of recursive preferences over intertemporal consumption lotteries which permits (i) to disantagle the relation between risk aversion and intertemporal substitution, (ii) to explicit the role of preference for the timing of the resolution of uncertainty.

We suppose that there exists a set of probability distributions on future consumptions, denoted by D, and there exists a preference relation on lotteries.

The agents' preferences are represented recursively by

$$\forall t,\ U_t \equiv W\left(c_t, V^{-1}\left(EV\left(\widetilde{U}_{t+1}\right)\right)\right) \tag{1}$$

where $W : R_+^2 \to R$, is increasing, twice differentiable and concave with respect to its two arguments, and is, in Koopmans' [1960] terminology, an aggregator function, $\widetilde{U}_{t+1} : D \to R$, represents the future stochastic utility, $V : R \to R$, is an increasing, twice differentiable and concave function and E is the mathematical expectation conditional on information available at time t.

The equivalent Kreps and Porteus [1979] aggregator, $f : R_+ \times R \to R$, is defined by

$$f\left(c_t, m\right) = V\left[W\left(c_t, V^{-1}\left(m\right)\right)\right] \tag{2}$$

As Johnsen and Donaldson [1985] showed, this representation permits a temporal consistency of preferences. Epstein and Zin [1989] showed that representation (1) (and thus representation (2)) is twofold. First of all, function V represents risk preferences, (the level of certainty equivalent measures the intensity of the risk aversion). Secondly, function W is defined on certain consumption vectors, and thus, it represents intertemporal substitution preferences. At last, we can notice that function U represents the instantanate utility.

3. THE RELATION BETWEEN RISK, INTERTEMPORAL SUBSTITUTION AND TIMING OF THE RESOLUTION OF UNCERTAINTY PREFERENCES

Risk aversion is characterized by the certainty equivalent of future utility. In Expected Utility framework, risk aversion is charaterized by a concave function V. The absolute risk aversion *à la* Arrow [1971] - Pratt [1964] is defined by:

$$\forall h \in R, \quad R_a(h) = -\frac{V''(h)}{V'(h)}. \tag{3}$$

Let us define a function $H : R_+^2 \to R$, $H(c_t, h) \equiv V \circ W(c_t, h)$ with $h \equiv V^{-1}(m)$. Elasticity of substitution between current consumption and future certainty-equivalent utility is defined by:

$$\forall (c_t, h) \in R^+ \times R, \quad e = \frac{\partial \left(\frac{h}{c_t}\right)}{\partial MRS} \times \frac{MRS}{\left(\frac{h}{c_t}\right)} \tag{4}$$

where MRS is the marginal rate of substitution between c_t and h with respect to utility function H.

Following Kihlstrom and Mirman [1974], it is easy to check that this elasticity of substitution can take the following form:

$$\forall (c, h) \in R^+ \times R, \quad e(c, h) = \frac{H_1(c, h)}{h\left(H_{12}(c, h) - H_1(c, h)\frac{H_{22}(c,h)}{H_2(c,h)}\right)} \tag{5}$$

where H_1 and H_2 are, respectively, the first derivative of H with respect to its first and second argument. H_{12} is the second derivative of H with respect to its first and second argument and H_{22} with respect to its second argument.

If the elasticity is positive, current consumption and future certainty-equivalent utility are considered as complementary. Since H_{22} is negative, by hypothesis, and H_2 is positive, a sufficient condition is H_{12} positive. Conversely, if the elasticity is negative, they are considered as substitute. We can notice that if H is time additively separable, H_{12} is nil and the elasticity becomes

$$e(c, h) = -\left(h\frac{H_{22}(c, h)}{H_2(c, h)}\right)^{-1} \tag{6}$$

Now, let us define a coefficient

$$\forall (c, h) \in R^+ \times R, \quad MGU(c, h) = -\frac{H_{22}(c, h)}{H_2(c, h)}. \tag{7}$$

This coefficient, positive by assumption, measures the marginal gain of future-period utility when certainty-equivalent utility increases. Using this measure, we can rewrite the elasticity of substitution as

$$e = \frac{1}{h\left(\frac{H_{12}}{H_1} + MGU\right)}.$$

Suppose that the elasticity of substitution, e, varies monotonically with MGU. Then, the more the elasticity is important the smaller MGU is. On the other hand, positivity of e, that is $\frac{H_{12}}{H_1} > \frac{H_{22}}{H_2}$, means that the gain of current-period marginal utility is upper than the loss of future-period marginal utility when certainty-equivalent utility increases. We can notice that it is equivalent to $\frac{W_{12}}{W_1} > \frac{W_{22}}{W_2}$. If certainty-equivalent utility and current consumption are complementary, then an increase in certainty-equivalent utility implies an increase in marginal utility such that the relative variation of current-utility, $\frac{\Delta W_1}{W_1}$, is greater than the relative variation of second-period utility, $\frac{\Delta W_2}{W_2}$.

By now, we suppose that the elasticity of substitution varies monotonically with MGU. We can notice that it is true if function H is time-additive or is a CES function.

Now, let us turn to the the preference for the timing of the resolution of uncertainty. It is characterized by the curvature of function f with respect to m. More precisely, an individual who prefers early resolution of uncertainty (resp. late, is indifferent) is characterized by f convex (resp. concave, linear) with respect to m (see Kreps and Porteus [1978]).

Then, we obtain the following result.

Proposition 1. $\forall\,(c,h) \in R^+ \times R$,

(i) An agent prefers the late resolution of uncertainty if and only if his marginal gain of utility is larger than his absolute risk aversion, $MGU(c,h) > Ra(h)$.

(ii) An agent prefers the early resolution of uncertainty if and only if his marginal gain of utility is smaller than his absolute risk aversion, $MGU(c,h) < Ra(h)$.

(iii) An agent is indifferent toward the timing of the resolution of uncertainty if and only if his marginal gain of utility is equal to his absolute risk aversion, $MGU(c,h) = Ra(h)$.

Proof. Derivating twice the function f with respect to m, we obtain $\frac{\partial f\left(c,V^{-1}(m)\right)}{\partial m} = \frac{\partial H\left(c,V^{-1}(m)\right)}{\partial m} = \frac{H_2\left(c,V^{-1}(m)\right)}{V'(V^{-1}(m))}$ and
$\frac{\partial^2 f}{\partial m^2} = \frac{1}{\left[V'\right]^2} \times \left[H_{22} - H_2 \times \frac{V''}{V'}\right]$. The result comes immediately. ∎

Let us provide the intuition of this result. If someone is very risk averse, he will want to know the realization of the random variable as early as possible and so, he will prefer an early resolution of uncertainty. The risk aversion is then very "great" with respect to MGU which becomes "weak". Marginal utility gain perceived by this individual when the certainty-equivalent utility increases is not "enough important" to compensate risk aversion.

A contrario, if the marginal utility gain is very important regarding to the risk aversion, the individual wishes to wait in order to keep "illusion" about a potential increase in future utility. Then, he prefers the late resolution of uncertainty.

In term of elasticity, if MGU is relatively weak, the expression $\frac{H_{12}}{H_1} + MGU$ could be viewed as "relatively weak" too (or even negative), and the elasticity becomes "relatively important". If the elasticity is positive, in the case of a preference for early resolution, we may obtain $1/e < Rr$, where Rr is the relative risk aversion. This result is easily obtained in the case of a negative elasticity.

In the extrem case of time-additively, $MGU = \frac{1}{e \times h}$ and then, an agent who, for instance, prefers late resolution will be charaterized by $1/e > Rr$. And so, we find the same Epstein and Zin's characterization.

4. CONCLUDED REMARKS

In this paper, we generalized the relation between risk aversion, intertemporal substitution and the timing of the resolution of uncertainty preferences. We did not specifize the form of the utility function and we did not consider a constant relative risk aversion; our result is available for any form of relative risk aversion. A we showed that the three concepts are linked, two of them determinate the third.

Recursive utility models has been mostly used since they permits to disentangle risk aversion and intertemporal substitution. In fact, the result that we obtain shows that there exists a relation between these two concepts *via* the notion of the timing of resolution of uncertainty. Consequently, we have to pay much attention in the interpretation of the role of risk aversion and intertemporal substitution in consumption decision analysis.

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