

Investment Decisions in a New Mixed Market

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The analysis in Fudenberg and Tirole (1983) discusses the perfect equilibria of a continuous-time model of the strategic investment decisions of two profit-maximizing private firms in a new market and suggests that there are perfect equilibria where each firm does not invest to its steady-state reaction curve. This paper examines the perfect equilibria of a continuous-time model of the strategic investment decisions of a social-welfare-maximizing public firm and a profit-maximizing private firm in a new market and shows that there are no perfect equilibria where each firm does not invest to its steady-state reaction curve in the mixed model. © 2006 Peking University Press

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1. INTRODUCTION

The possibility of firms using excess capacity for strategic investment is examined in Wenders (1971), Spence (1977), and Dixit (1979) and is also extended in a two-stage model by Dixit (1980) and in a three-stage model by Ware (1984). Spence (1979) examines strategic investment decisions for private firms in a new industry or market by using a continuous-time asymmetric dynamic model, namely, where leading and following firms exist. He shows that the equilibrium occurs when the leading firm invests as quickly as possible to some capital level and then stops. His result is much like the equilibrium in a static Stackelberg game. Furthermore, Fudenberg and Tirole (1983) establish the existence of a set of perfect equilibria by using Spence's dynamic model and suggest that the steady state of the game is usually on neither firm's steady-state reaction curve; that is, there are early-stopping equilibria where each firm does not invest up to its steady-state reaction curve.¹

¹Fudenberg and Tirole (1986, 1991) and Tirole (1988) explain the continuous-time dynamic model in plain terms.

We examine a continuous-time dynamic model of the strategic investment decisions of a social-welfare-maximizing public firm and a profit-maximizing private firm in a new industry or market. The analysis of mixed market models that incorporate social-welfare maximizing public firms has become increasingly popular in recent years and has been undertaken by many economists. For instance, Vickers and Yarrow (1988), Cremer, Marchand and Thisse (1989), Nett (1993), and Bös (2001) present excellent surveys on mixed market models.

The purpose of the paper is to discuss the equilibrium outcomes of a continuous-time model of the strategic investment decisions of public and private firms in a new industry or market, and show that the equilibrium outcomes of our mixed market model differ from those of Fudenberg and Tirole's pure market model; that is, there are no early-stopping equilibria where each firm does not invest to its steady-state reaction curve in our mixed market model.

The paper is organized as follows. In Section 2, the elements of the continuous-time model are presented. Section 3 contains the equilibrium outcomes of the model. Finally, Section 4 states concluding remarks.

2. THE MODEL

Let us consider a mixed market model with one social-welfare-maximizing public firm (firm 1) and one profit-maximizing firm (firm 2). For the remainder of this paper, when i and j are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with $i \neq j$. Time t is continuous, and the horizon is infinite.

Firm i 's net profit at time t is given by

$$\prod_i(k_1, k_2, a_i) = p(K)k_i - m_i k_i - a_i, \quad (1)$$

where k_i is firm i 's current capital stock, $p(K)$ is price as a function of capital stock ($K = k_1 + k_2$), m_i is firm i 's maintenance cost per unit of capital, and a_i is firm i 's rate of investment in its own capital. We assume that $m_1 > m_2$.² We also assume that $p(K)$ is twice continuously differentiable with $dp/dK < 0$ and $d^2p/dK^2 < 0$. That is, this function is strictly concave.

²The assumption concerning the inefficiency of public sectors is often used in literature studying mixed markets. See, for instance, Cremer, Marchand, and Thisse (1989), Nett (1993), Kenneth and Manfredi (1996), Mujumdar and Pal (1998), Pal (1998), and Matsumura (2003). If firm 1 is more efficient than firm 2, firm 1 will try to maximize social welfare by supplying monopolistically in the market. In this paper, this behavior of firm 1 brings the same result as the pure market model. This assumption is made to avoid such a trivial solution.

The constant cost of one unit of investment is one, capital stocks cannot decrease, and each firm has a constant upper bound on the amount of its capital investment at every time t . Hence, $dk_i/dt = a_i \in [0, \bar{a}_i]$. At time zero, firm 2 enters the market with no capital stock, while firm 1 has an exogenous given capital stock, and each firm can start investing.

Social welfare is defined as the sum of the consumer surplus and total profits of the firms. That is, social welfare at time t is given by

$$W(k_1, k_2, a_1, a_2) = \int_0^{k_1+k_2} p(x)dx - (m_1k_1 + a_1) - (m_2k_2 + a_2). \quad (2)$$

Each firm's net present value of profits is

$$V_i = \int_0^\infty \prod_i (k_1(t), k_2(t), a_i(t)) e^{-\delta t} dt, \quad (3)$$

where $\delta \geq 0$ is the common rate of interest. This is firm 2's objective function.

Firm 1 maximizes the net present value of social welfare, given by

$$WV = \int_0^\infty W(k_1(t), k_2(t), a_1(t), a_2(t)) e^{-\delta t} dt. \quad (4)$$

If δ tends to zero, then firm 1 maximizes time-average social welfare, and firm 2 maximizes its time-average profit.

In this paper, we examine the perfect equilibrium outcomes of a state-space game.³ The state-space game is a game in which both the payoffs and the strategies depend on the history only through the current state. The perfect equilibrium is a strategy combination that induces a Nash equilibrium for the subgame starting from every possible initial state in the state space. Firm i 's steady-state reaction function $R_i(k_j)$ is defined as the locus of points that give the final optimal level of capital k_i for each value of the final level of capital k_j , i.e. $R_1(k_2)$ and $R_2(k_1)$ are $\partial W(R_1(k_2), k_2)/\partial k_1 = \delta$ and $\partial \prod_2(k_1, R_2(k_1))/\partial k_2 = \delta$, respectively. Under our assumptions, the steady-state reaction functions are downward sloping. We assume that both steady-state reaction functions have a unique intersection, which will be the Nash equilibrium of the state-space game.

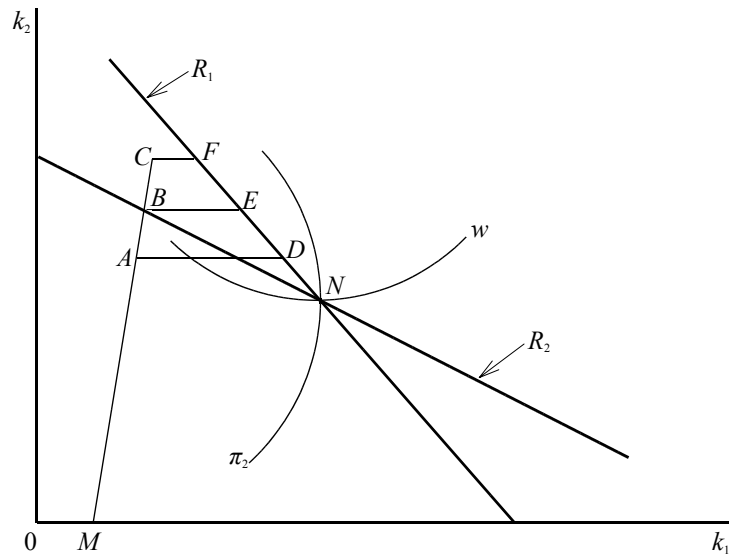
3. EQUILIBRIUM OUTCOMES

In this section, we will discuss the perfect equilibrium outcomes of the continuous-time model. First, we consider the case shown in Figure 1. R_i

³For details of the state-space (steady-state) game with two profit-maximizing private firms, see Fudenberg and Tirole (1983).

is firm i 's steady-state reaction curve, w is the iso-welfare curve, and π_2 is firm 2's iso-profit curve. The iso-welfare curve is drawn as it is in this figure with the assumption of asymmetric costs in favor of firm 2. Spence (1979) and Fudenberg and Tirole (1983) define the industrial growth path (IGP) as a locus on which each firm invests as quickly as possible. Firms are willing to invest as quickly as possible if there are only profit-maximizing firms in a market and the reaction curves are downward sloping. However, in this paper, we examine the case of a mixed market. As understood from this figure, social welfare increases as firm 2 increases its investment. Firm 1 hopes that firm 2 will invest more. Hence, firm 1 does not have the incentive to invest as early as firm 2 does. Therefore, we will not introduce the IGP.

FIG. 1. In this case, the investment path stops on firm 1's reaction curve to the northwest of the intersection of both reaction curves.



We will discuss each firm's actual investment paths by using Figure 1. Let M be firm 1's exogenous given capital level. Each firm can start investing from time zero. Social welfare increases as firm 2 increases its investment, and therefore firm 1 hopes that firm 2 will invest more. Firm 1 will not have the incentive to invest as early as firm 2 does. The industry continues to grow along the segment MA in the figure, and firm 2 will stop investing at a point that it finds optimal.

Suppose that point D on firm 1's reaction curve is the Stackelberg point where firm 2 is the leader. If firm 2's capital level associated with D is

below firm 2's reaction curve like point A in the figure, then firm 2 does not invest up to its reaction curve, and stops investing at A . Thereafter, firm 1 unilaterally continues investing. The industry continues to grow along the segment AD in the figure, and firm 1 will stop investing at a point that it finds optimal. Because social welfare decreases if firm 1 invests further, firm 1 will stop at point D on its reaction curve R_1 . Each firm will not have the incentive to invest at point D . Hence, this investment path becomes MAD in the figure. The investment path does not always become MAD , and there are many different paths which lead to D .

Firm 2's profit decreases as the industry grows along AD . Therefore, firm 2 may try to stop firm 1's investment before the investment path reaches D . Even though firm 2 invests further, the best firm 1 can do is to invest to its reaction curve. Since this profit of firm 2 is lower than its profit at D , this behavior of firm 2 does not become a credible threat.

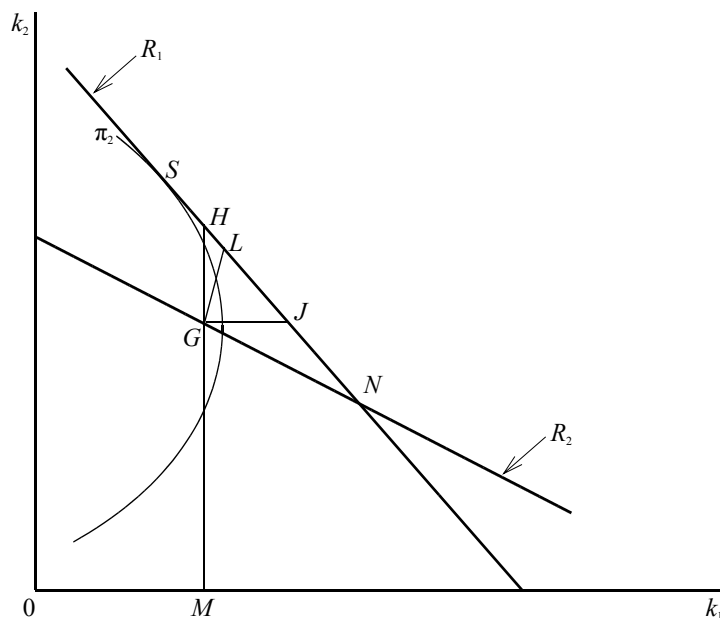
What happens to the investment path if point E on firm 1's reaction curve is the Stackelberg point where firm 2 is the leader? If firm 2's capital level associated with E is on firm 2's reaction curve like point B in the figure, then firm 2 invests up to its reaction curve, and stops investing. Thereafter, firm 1 unilaterally continues investing. The industry continues to grow along the segment BE in the figure, and firm 1 will stop investing at a point that it finds optimal. That is, firm 1 will stop at E on its reaction curve R_1 . Neither firm will have the incentive to invest at E . Hence, this investment path becomes $MABE$ in the figure. The investment path does not always become $MABE$, and there are many different paths which lead to E .

What happens to the investment path if point F is the Stackelberg point and firm 2's capital level associated with F is above firm 2's reaction curve like point C in the figure? At point B on firm 2's reaction curve, if firm 2 invests further, its profit will decrease. However, because social welfare increases if firm 1 continues to invest, firm 1 will continue to do so. Therefore, because firm 2's profit of point F exceeds its profit of point E , firm 2 will resume investment. Social welfare increases as firm 2 increases its investment, and therefore firm 1 hopes that firm 2 will invest more. Hence, firm 2 can continue investing up to its capital level point C associated with F , and stop investing at C . Thereafter, firm 1 unilaterally continues to invest and stops investing at point F on its reaction curve. This investment path becomes $MABCF$ in the figure. The investment path does not always become $MABCF$, and there are many different paths which lead to F .

Second, we consider the case shown in Figure 2. In this case, firm 1's first capital stock level M is larger than firm 1's capital stock level associated with the Stackelberg point S on firm 1's reaction curve where firm 2 is the leader. Since capital stocks cannot decrease, we can see intuitively that

the equilibrium does not occur at the Stackelberg point S . Therefore, the investment path is as follows. Social welfare increases as firm 2 increases its investment. From time zero, firm 2 unilaterally invests and the state reaches point G on firm 2's reaction curve. Firm 2 will lose the incentive to invest at point G . At point G , because social welfare increases if firm 1 invests whether firm 2 invests or not, firm 1 will start investing. If firm 1 unilaterally continues to invest, the state reaches point J on its reaction curve. At point J , because social welfare decreases if firm 1 invests further, the best firm 1 can do is to stop. Here, we can see that firm 2's profit of each point except point J on the segment SJ exceeds its profit of J . Since capital stocks cannot decrease, the equilibrium will never occur at any point northwest of point H . Therefore, at point G , if firm 1 invests, firm 2 will resume investment. Firm 2 invests as quickly as possible, and stops investing if point L on the segment HJ is reached. Neither firm will have the incentive to invest at point L . Hence, this investment path becomes MGL in the figure.

FIG. 2. In this case, the investment path stops at a point on the segment HJ like point L .

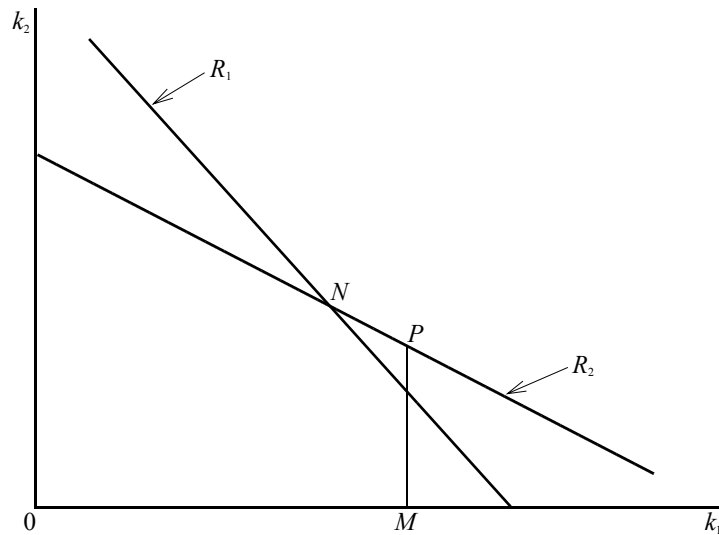


Firm 2's profit decreases as the industry grows along GL . Therefore, firm 2 may try to stop firm 1's investment before the investment path reaches L . Even though firm 2 stops investing, the best firm 1 can do is to invest

slowly to its reaction curve. Since this profit of firm 2 is lower than its profit at L , this behavior of firm 2 does not become a credible threat.

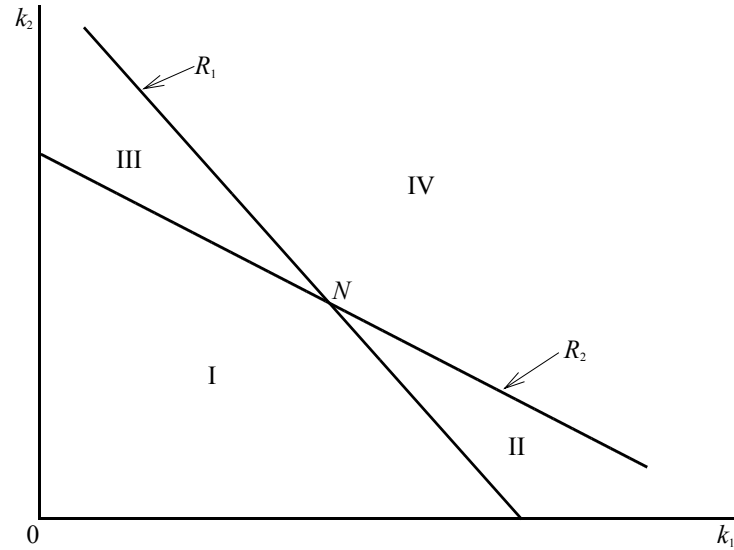
Third, we consider the case shown in Figure 3. In this case, firm 1's first capital stock level M is equal to or larger than firm 1's capital stock level associated with the intersection N of both reaction curves. Since capital stocks cannot decrease, the equilibrium will never occur at any point to the left of N . Firm 2 can increase its own profit by investing, and therefore it will invest. Since firm 1 hopes that firm 2 will invest more, if firm 2 invests, then the best firm 1 can do is not to invest. Firm 2 keeps investing up to point P on its reaction curve and then stops. Neither firm will have the incentive to invest at point P . This investment path becomes MP in the figure. Social welfare increases as firm 2 increases its investment, and thus the incentive by which firm 2's investment is stopped before the investment path reaches P does not happen to firm 1.

FIG. 3. In this case, firm 2 unilaterally continues investing from point M to point P .



From above discussions, we can see that there are no early-stopping equilibria in the continuous-time mixed market model. The following proposition states the result of the model.

PROPOSITION 1. *In the continuous-time mixed market model, one can construct perfect equilibrium strategies such that the equilibrium path stops at the intersection of both reaction curves, on firm 1's reaction curve to the northwest of the intersection of both reaction curves or on firm 2's reaction curve to the southeast of the intersection of both reaction curves.*

FIG. 4. The state space is divided into four regions.

Proof. We divide the state space into four regions as depicted in Figure 4: Region I is the set below both reaction curves; Region II is the set not below R_1 and below R_2 ; Region III is the set below R_1 and not below R_2 ; and Region IV is the set not below either reaction curve.

First, we show each firm's strategy in Region I. Since $\prod_2(k_1, k_2, a_2)$ is assumed to be concave in k_2 , firm 2 wishes to be as close to its reaction curve as possible. Therefore, if firm 1 does not invest, the best firm 2 can do is to invest. Since $W(k_1, k_2, a_1, a_2)$ is assumed to be concave in k_1 , firm 1 also wishes to be as close to its reaction curve as possible. Therefore, if firm 2 does not invest, the best firm 1 can do is to invest. Firm 2's profit decreases as firm 1 increases its investment, and therefore firm 2 tries to stop firm 1 from investing or to invest earlier than firm 1 does. Both social welfare and firm 2's profit increase as firm 2 increases its investment. Therefore, if firm 2 continues to invest, firm 1 may stop investing, and firm 1 does not have the incentive to stop firm 2 from investing. However, if firm 2 stops investing, the best firm 1 can do is to invest. In Region I, since at least one firm continues to invest, the state will reach from Region I to either Region II, Region III or Region IV.

Second, we show each firm's strategy in Region II. Since $\prod_2(k_1, k_2, a_2)$ is assumed to be concave in k_2 , firm 2 wishes to be as close to its reaction curve as possible. Therefore, if firm 1 does not invest, the best firm 2

can do is to invest. On the other hand, firm 1 maximizes social welfare. Given k_1 , an increase in k_2 increases social welfare. From the assumption of strategic substitutes, an increase in k_1 decreases firm 2's profit maximizing investment level. Firm 1 knows that since $\prod_2(k_1, k_2, a_2)$ is assumed to be concave in k_2 , firm 2 will invest up to a point on its reaction curve. Hence, firm 1, which maximizes social welfare, never invests in this region. In Region II, since firm 2 continues to invest, the state will reach from Region II to Region IV.

Third, we show each firm's strategy in Region III. Since firm 2 wishes to be as close to its reaction curve as possible, if firm 1 does not invest, the best firm 2 can do is not to invest. However, firm 1 also wishes to be as close to its reaction curve as possible. Therefore, if firm 2 does not invest, the best firm 1 can do is to invest, and firm 2's profit decreases as firm 1 increases its investment. Hence, if firm 1 invests, firm 2 may invest too. If firm 2 invests, the best firm 1 can do is not to invest. However, if firm 1 does not invest, firm 2 does not either, and since this state is the worst for firm 1, firm 1 will invest. In Region III, since at least firm 1 continues to invest, the state will reach from Region III to Region IV.

Fourth, we show each firm's strategy in Region IV. Once the state is in this region, it can never leave. Each firm wishes to be as close to its own reaction curve as possible. If only firm 2 or both firms continue to invest, then firm 2's profit will decrease. Hence, firm 2 does not invest. If only firm 1 continues to invest, then social welfare will decrease, and therefore firm 1 does not invest either. Each firm's best response to the other firm's strategy at any point of this region is not to invest. Consequently, each firm's optimization problem at any point in this region, given the other firm's strategy, induces a Nash strategy at any point of this region. Thus, the strategies are in perfect equilibrium, and the result follows. ■

4. CONCLUDING REMARKS

We examined the equilibrium outcomes of the continuous-time dynamic model between a social-welfare-maximizing public firm and a profit-maximizing private firm in a new industry or market. Fudenberg and Tirole (1983) suggest that if two profit-maximizing private firms compete in a continuous-time model, there are early-stopping equilibria where each firm does not invest to its reaction curve. On the other hand, we showed that there are no early-stopping equilibria in the continuous-time model of a mixed market. Furthermore, we found that behaving like a private firm is not the optimal role for the public firm; that is, the public firm invests less than the private firm, and this behavior of the public firm enhances social welfare.

Most studies using strategic investment focus on capital costs, such as the construction of factories and the installation of machine equipment, while the following two works focus on labor costs. Ohnishi (2001) proposes lifetime employment contracts as a strategic commitment and shows their effectiveness by examining entry deterrence. Furthermore, Ohnishi (2002) shows concretely in what kinds of cases lifetime employment contracts as a strategic commitment are effective by using a linear quantity-setting model. If the firm legally enters into a lifetime employment contract with its employees, then its wage cost sinks and its marginal cost decreases. If we use labor costs, the model is as follows. Each firm employs new graduates of universities, colleges, etc. as its employees at each time, legally enters into lifetime employment contracts with them, and expands its scale. Therefore, it is assumed that there is an upper bound in the number of employees that each firm can employ newly at each time. Of course, both capital costs and labor costs can be used as a strategic investment. Even if only capital costs, only labor costs or both are used as a strategic investment, the equilibrium outcomes of the model will become the same. However, although industries that use factories and machines are limited to some degree, it is thought that the range of the adjustment broadens, because almost all industries need people.

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