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Wage-Rise Contract and Entry Deterrence: Bertrand and Cournot

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This paper is based on a two-stage model of an incumbent firm and a potential entrant. We consider two cases in terms of strategic relevance between both firms. We also consider both price-setting competition and quantity-setting competition. Therefore, we examine four cases. Each case is correlated with a prior commitment that generates kinks in the reaction curve of the incumbent firm. We then investigate the entry-deterring equilibrium outcomes resulting from the prior commitment of the incumbent firm in all four cases.

Key Words: Entry deterrence; Wage-rise contract; Price-setting model; Quantity-setting model.
JEL Classification Numbers: C72, D21, L13.

1. INTRODUCTION

There are a number of excellent works dealing with strategic commitments that generate kinks in firms' reaction curves, such as Dixit (1980) and Cooper (1986). In them, economists presumed that the strategic behaviour of firms would result in quantitative competition through homogeneous goods or substitute goods in substitutive relationships, and price competition through substitute goods in complementary relationships. However, cases other than these can naturally be considered. Therefore, we classify demand functions into the following two cases: 'complementary goods and strategic complements' and 'complementary goods and strategic substitutes'. We also consider both price-setting and quantity-setting models. Hence, we examine four cases.[1]

[1]On the other hand, Ohnishi (2001) considers the following four cases: quantity-setting competition with 'substitute goods and strategic substitutes' and 'substitute goods and strategic complements' and price-setting competition with 'substitute goods and strategic complements' and 'substitute goods and strategic substitutes'.

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We correlate each of the four cases with a wage-rise-contract policy (henceforth WRCP).[2] WRCP is a promise by the firm that it will announce a certain output level and a wage premium rate, and if it actually produces more than the announced output level, then it will pay each employee a wage premium uniformly. We use a two-stage model of an incumbent firm and a potential entrant. We then investigate the entry-deterring strategies resulting from WRCP in all cases.[3]

In the cases of complementary goods, the incumbent firm's entry-deterring behaviour does not correspond very closely to its profit maximization. Therefore, if the incumbent firm conducts its profit-maximizing behaviour, it will allow the potential entrant to enter and both firms will achieve a duopoly equilibrium.[4] However, our aim is to examine the incumbent firm's entry-deterring strategies. Therefore, the incumbent firm's entry-deterring strategies do not necessary correspond to its profit-maximizing behaviour. We analyse entry-deterring games.

The purpose of this paper is to analyse the entry-deterring strategies resulting from WRCP in the four cases with complementary goods and to show the effectiveness of WRCP as a result of its analyses.

The paper is organized as follows. In Section 2, we present a price-setting model. Section 3 describes WRCP. Section 4 discuses the entry-deterring equilibrium outcomes of the price-setting model. Section 5 presents a quantity-setting model and discuses its entry-deterring equilibrium outcomes. Finally, Section 6 gives a brief conclusion.

2. PRICE-SETTING MODEL

In this section, we describe a two-stage price-setting model. There are two firms: firm 1 (the incumbent firm) and firm 2 (the potential entrant). For the remainder of this paper, when i and j are used to refer to firms in an expression, they should be understood to run from 1 to 2 with $i \neq j$. The two stages of the price-setting model run as follows. In the first stage, firm 1 adopts a countermeasure against firm 2. At the beginning of the second stage, firm 2 observes the countermeasure and decides whether or not to enter the market. In the second stage, if firm 2 enters, both firms achieve a duopoly equilibrium with price-setting. On the other hand, if firm 2 does

[2]For details see Ohnishi (2003).

[3]Ohnishi (2001) examines the entry-deterring equilibrium outcomes resulting from a lifetime-employment-contract policy in the four cases of substitute goods.

[4]In the cases of complementary goods, if the incumbent firm's entry-deterring strategies do not correspond very closely to its profit maximization, it might be seemed that the analysis of its entry-deterring strategies is not very significant. However, if the incumbent firm plans to produce the homogeneous or substitute goods of the potential entrant, the analysis of its entry-deterring strategies is significant.

not enter, firm 1 prevails as a monopoly. Firm 1's countermeasure in the first stage is WRCP, which is described in the following section.

Firm i's profit function is

$$\pi_i(p_i, p_j) = p_i q_i(p_i, p_j) - c_i(q_i(p_i, p_j)), \tag{1}$$

where $p_i \in [0, \infty)$ is firm i's price, $q_i : \square_+^2 \to \square_+$ is firm i's demand function and $c_i : \square_+^2 \to \square_+$ is firm i's cost function.

Firm i's reaction function $R_i(p_j)$ is defined by

$$R_i(p_j) = \arg \max_{\{p \geq 0\}} (p_i q_i(p_i, p_j) - c_i(q_i(p_i, p_j))). \tag{2}$$

The Bertrand equilibrium is defined as a pair (p_1^B, p_2^B) of price levels, where $p_1^B \in R_1(p_2^B)$ and $p_2^B \in R_2(p_1^B)$.

In this model, we make the following assumptions.

Assumption 1. q_i is twice continuously differentiable with $\partial q_i/\partial p_i < 0$ (downward-sloping demand) and $\partial q_i/\partial p_j < 0$ (complementary goods).

Assumption 2. $\partial^2 \pi_i/\partial p_i^2 < 0$ (convexity).

Assumption 3. If $(R_i(p_j), p_j) \in \square_{++}^2$, then $0 < |R_i'(p_j)| < 1$ (stability condition).

3. WRCP

In this section, we describe WRCP. Firm i employs and dismisses its employees according to the amount of output. That is, the wages of the employees of firm i are originally its variable cost.

In the first stage, if firm 1 adopts WRCP, then it chooses an output level $x_1 \geq 0$ and a wage premium rate $w_1 \geq 0$, and agrees to pay each employee a wage premium uniformly if it actually produces more than x_1.

Therefore, firm 1's cost function changes as follows:

$$c_1(x_1, w_1, p_1, p_2) = \begin{cases} \nu_1 q_1(p_1, p_2) & \text{if } q_1(p_1, p_2) \leq x_1, \\ \nu_1 q_1(p_1, p_2) + (q_1(p_1, p_2) - x_1) w_1 & \text{if } q_1(p_1, p_2) \geq x_1, \end{cases} \tag{3}$$

where $\nu_1 > 0$ is firm 1's constant marginal cost for output. Firm 1's marginal cost exhibits a discontinuity at $q_1 = x_1$.

On the other hand, firm 2's cost function is

$$c_2(p_1, p_2) = \nu_2 q_2(p_1, p_2) + f_2, \tag{4}$$

where $\nu_2 > 0$ is firm 2's constant marginal cost for output and $f_2 > 0$ is firm 2's fixed set-up cost.

If firm 1's marginal cost for output is constantly equal to ν_1, then its reaction function is defined by

$$R_1^{\nu}(p_2) = \arg \max_{\{p_1 \geq 0\}} (p_1 q_1(p_1, p_2) - \nu_1 q_1(p_1, p_2)). \quad (5)$$

On the other hand, if firm 1's marginal cost for output is constantly equal to $\nu_1 + w_1$, then its reaction function is defined by

$$R_1^{\nu+w}(p_2) = \arg \max_{\{p_1 \geq 0\}} (p_1 q_1(p_1, p_2) - (\nu_1 + w_1) q_1(p_1, p_2)). \quad (6)$$

Therefore, if firm 1 adopts WRCP, then its reaction function changes as follows:

$$R_1(p_2) = \begin{cases} R_1^{\nu}(p_2) & \text{if } q_1(p_1, p_2) < x_1, \\ x_1 & \text{if } q_1(p_1, p_2) = x_1, \\ R_1^{\nu+w}(p_2) & \text{if } q_1(p_1, p_2) > x_1. \end{cases} \quad (7)$$

Firm 1's reaction curve is kinked at the level equal to $q_1 = x_1$ by the strategy that it adopted in the first stage.

4. ENTRY-DETERRING EQUILIBRIUM OUTCOMES

First, we make the following assumption.

Assumption 4. Firm 2 enters the market if and only if its post-entry profit is positive.

Assumption 4 means that firm 2 does not enter the market if its total revenues do not exceed its total costs in the post-entry equilibrium. Therefore, firm 2's reaction function is defined by

$$R_2(p_1) = \begin{cases} \arg \max_{\{p_2 \geq 0\}} (p_2 q_2(p_1, p_2) - \nu_2 q_2(p_1, p_2)) & \text{if } \pi_2(p_1, p_2) > 0, \\ 0 & \text{if } \pi_2(p_1, p_2) \leq 0. \end{cases} \quad (8)$$

The entry-deterring equilibrium is as follows. If firm 1 does not adopt WRCP, the equilibrium in the second stage occurs at the Bertrand point $p_1^B, p_2^B)$, where $\pi_2(p_1^B, p_2^B) > 0$. Therefore, in the first stage, firm 1 chooses $x_1^* = q_1^*(p_1^*, R_2(p_1^*))$ and w_1^* corresponding to $\pi_2(p_1^*, R_2(p_1^*)) = 0$ and adopts WRCP. In the second stage, firm 2's post-entry equilibrium is decided in a Bertrand fashion and its post-entry profit is zero. Thus, firm 2 does not enter the market and firm 1 acts as a monopolist.

In this section, we consider the following two cases.

Case 1: $dR_i/dp_j > 0$.

Case 2: $dR_i/dp_j < 0$.

In Cases 1 and 2, firm 1's entry-deterring behaviour does not correspond very closely to its second-stage profit maximization. Therefore, if firm 1 conducts its second-stage profit-maximizing behaviour, it will allow firm 2 to enter and both firms will achieve a duopoly equilibrium with price-setting. However, if firm 1 plans to produce the homogeneous or substitute goods of firm 2, the analysis of its entry-deterring strategies is significant. In this paper, we analyse firm 1's entry-deterring strategies. Hence, firm 1's entry-deterring strategies do not necessary correspond to its second-stage profit-maximizing behaviour.

We discuss the equilibrium outcomes resulting from WRCP in Cases 1 and 2. Case 1 (resp. Case 2) is the case of strategic complements (resp. substitutes) in which goods are complements. In Case 1 (resp. Case 2), firm i's reaction curve is upward-sloping (resp. downward-sloping) because of strategic complements (resp. substitutes). Firm i's profit increases (resp. decreases) with the fall (resp. rise) of firm j's price on firm i's reaction curve.

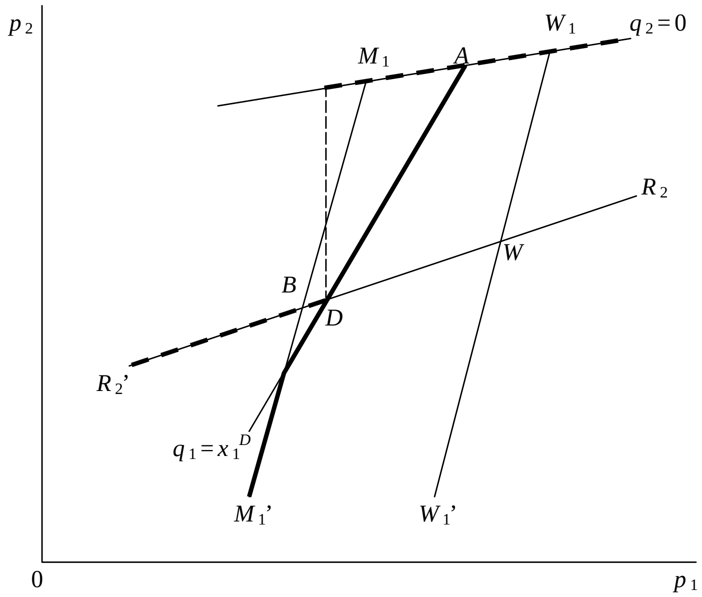

FIG. 1. Case 1: $\partial q_i/\partial p_j < 0$ and $\partial R_i/\partial p_j > 0$

Figures 1 and 2 illustrate Cases 1 and 2, respectively. M_1M_1' is firm 1's reaction curve when the marginal cost is constantly equal to ν_1, W_1W_1' is firm 1's reaction curve when the marginal cost is constantly equal to ν_1+w_1, and R_2R_2' is firm 2's reaction curve when the marginal cost is constantly equal to ν_2. We suppose that R_2R_2' meets M_1M_1' at $B = (p_1^B, p_2^B)$ and W_1W_1' at $W = (p_1^W, p_2^W)$ as shown in Figure 1 (Figure 2). Furthermore, we suppose $D = (p_1^D, p_2^D)$ as a point on firm 2's reaction curve, where

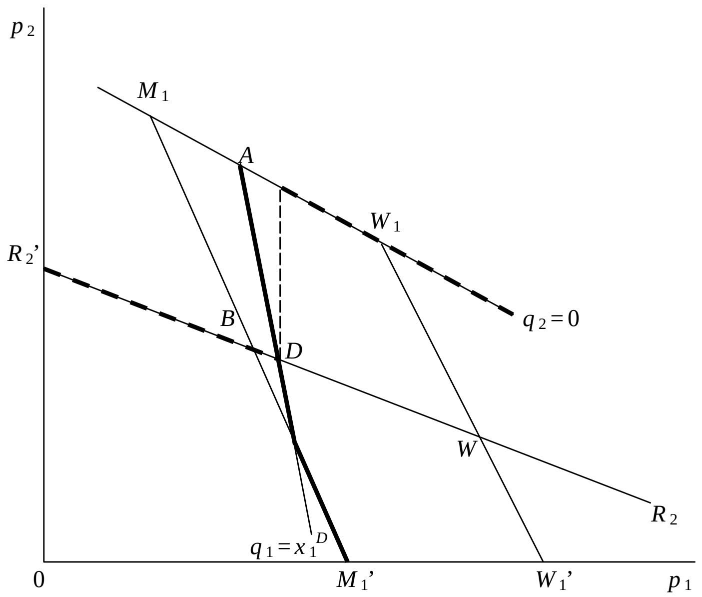

FIG. 2. Case 2: $\partial q_i/\partial p_j < 0$ and $\partial R_i/\partial p_j < 0$

its post-entry profit $\pi_2^D(p_1^D, p_2^D)$ is zero and it does not enter. From (8), firm 2's reaction curve is discontinuous at D, made up of the two segments illustrated by the dotted line in Figure 1 (Figure 2).

Now, we discuss the equilibrium outcomes of Cases 1 and 2 by classifying the possibilities as follows.

(i) $p_1^D \leq p_1^B$

In each of Cases 1 and 2, firm 2 cannot make a positive profit in its post-entry equilibrium even if firm 1 does not adopt WRCP. From Assumption 4, firm 2 does not enter the market. Firm 2's entry is blocked. Hence, the equilibrium occurs at M_1 in Figure 1 (Figure 2) and firm 1 enjoys a pure monopoly.

(ii) $p_1^B < p_1^D$

In each of Cases 1 and 2, firm 1 can deter entry. We discuss the equilibrium outcomes when firm 1 deters entry. In the first stage, firm 1 chooses x_1^D and w_1^D corresponding to D and adopts WRCP. Let w_1 be a variable which can take any value zero and over. From (7), we see that firm 1 can select any price equal to or higher than its Bertrand equilibrium price p_1^B without WRCP. Firm 1's marginal cost exhibits a discontinuity at $q_1 = x_1^D$. Firm 1's reaction curve is kinked at the level equal to $q_1 = x_1^D$. Firm 1's reaction curve is illustrated by the kinked bold line as a result of WRCP.

On the other hand, firm 2's reaction curve is discontinuous at D, made up of the two segments illustrated by the dotted line. Since firm 2's profit is zero at D, it does not enter. Thus, the equilibrium occurs at A in Figure 1 (Figure 2).

From the preceding discussions, we can state the following proposition:

PROPOSITION 1. *In each of Cases 1 and 2, firm 2 never enter the market because firm 1 can adopt WRCP.*

5. QUANTITY-SETTING MODEL AND ENTRY-DETERRING EQUILIBRIUM OUTCOMES

In this section, we extend the price-setting model discussed in Sections 2-4 to a quantity-setting model. The two-stage quantity-setting model is as follows. In the first stage, firm 1 adopts WRCP. At the beginning of the second stage, firm 2 observes firm 1's policy in the first stage and decides whether or not to enter the market. In the second stage, if firm 2 enters, both firms achieve a duopoly equilibrium with quantity-setting. On the other hand, if firm 2 does not enter, firm 1 prevails as a monopoly.

Therefore, firm 1's profit is

$$\pi_(x_1, w_1, q_1, q_2) = \begin{cases} p_1(q_1, q_2)q_1 - \nu_1 q_1 & \text{if } q_1 \leq x_1, \\ p_1(q_1, q_2)q_1 - \nu_1 q_1 - (q_1 - x_1)w_1 & \text{if } q_1 \geq x_1, \end{cases} \tag{9}$$

where $q_i \in [0, \infty)$ is firm i's output and $p_1 : \square_+^2 \to \square_+$ is firm 1's inverse demand function.

If firm 1's marginal cost for output is constantly equal to ν_1, then its reaction function is defined by

$$R_1^{\nu}(q_2) = \arg \max_{\{q_1 \geq 0\}} (p_1(q_1, q_2)q_1 - \nu_1 q_1). \tag{10}$$

On the other hand, if firm 1's marginal cost for output is constantly equal to $\nu_1 + w_1$, then its reaction function is defined by

$$R_1^{\nu+w}(q_2) = \arg \max_{\{q_1 \geq 0\}} (p_1(q_1, q_2)q_1 - (\nu_1 + w_1)q_1). \tag{11}$$

Then firm 1's reaction function is defined by

$$R_1(q_2) = \begin{cases} R_1^{\nu}(q_2) & \text{if } q_1 < x_1, \\ x_1 & \text{if } q_1 = x_1, \\ R_1^{\nu+w}(q_2) & \text{if } q_1 > x_1. \end{cases} \tag{12}$$

Firm 1's reaction function is kinked at the level equal to $q_1 = x_1$.

On the other hand, firm 2's profit is

$$\pi_2(q_1, q_2) = p_2(q_1, q_2)q_2 - \nu_2 q_2 - f_2, \tag{13}$$

where $p_2 : \square_+^2 \to \square_+$ is firm 2's inverse demand function. In this section, we also suppose Assumption 4. Therefore, firm 2's reaction function is defined by

$$R_2(q_1) = \begin{cases} \arg\max_{\{q_2 \geq 0\}}(p_2(q_1, q_2)q_2 - \nu_2 q_2) & \text{if } \pi_2(q_1, q_2) > 0, \\ 0 & \text{if } \pi_2(q_1, q_2) \leq 0. \end{cases} \tag{14}$$

The Cournot Nash equilibrium is defined as a pair (q_1^C, q_2^C) of output levels, where $q_1^C \in R_1(q_2^C)$ and $q_2^C \in R_2(q_1^C)$.

In this model, we make the following assumptions.

Assumption 5. p_i is twice continuously differentiable with $\partial p_i/\partial q_i < 0$ (downward-sloping demand) and $\partial p_i/\partial q_j > 0$ (complementary goods).

Assumption 6. $\partial^2 \pi_i/\partial q_i^2 < 0$ (convexity).

Assumption 7. If $(R_i(q_j), q_j) \in \square_{++}^2$, then $0 \leq |R_i'(q_j)| < 1$ (stability condition).

In this section, we consider the following two cases.

Case 3: $dR_i/dq_j > 0$.

Case 4: $dR_i/dq_j < 0$.

We discuss the equilibrium outcomes resulting from WRCP in Cases 3 and 4. Case 3 (resp. Case 4) is the case of strategic complements (resp. substitutes) in which goods are complements. In Case 3 (resp. Case 4), firm i's reaction curve is upward-sloping (resp. downward-sloping) because of strategic complements (resp. substitutes). Firm i's profit increases (resp. decreases) with the rise (resp. fall) of firm j's output on firm i's reaction curve.

Cases 3 and 4 are illustrated in Figures 3 and 4, respectively. M_1M_1' is firm 1's reaction curve when the marginal cost is constantly equal to ν_1, W_1W_1' is firm 1's reaction curve when the marginal cost is constantly equal to $\nu_1 + w_1$, and R_2R_2' is firm 2's reaction curve when the marginal cost is constantly equal to ν_2. We suppose that R_2R_2' meets M_1M_1' at $C = (q_1^C, q_2^C)$ and W_1W_1' at $W = (q_1^W, q_2^W)$ as shown in Figure 3 (Figure 4). Furthermore, we suppose $D = (q_1^D, q_2^D)$ as a point on firm 2's reaction curve, where its post-entry profit $\pi_2^D(q_1^D, q_2^D)$ is zero and it does not enter. From (14), firm 2's reaction curve is discontinuous at D, made up of the two segments illustrated by the dotted line in Figure 3 (Figure 4).

Now, we discuss the equilibrium outcomes of Cases 3 and 4 by classifying the possibilities as follows.

(i) $q_1^C \leq q_1^D$

In each of Cases 3 and 4, firm 2 cannot make a positive profit in its post-entry equilibrium even if firm 1 does not adopt WRCP. From Assumption

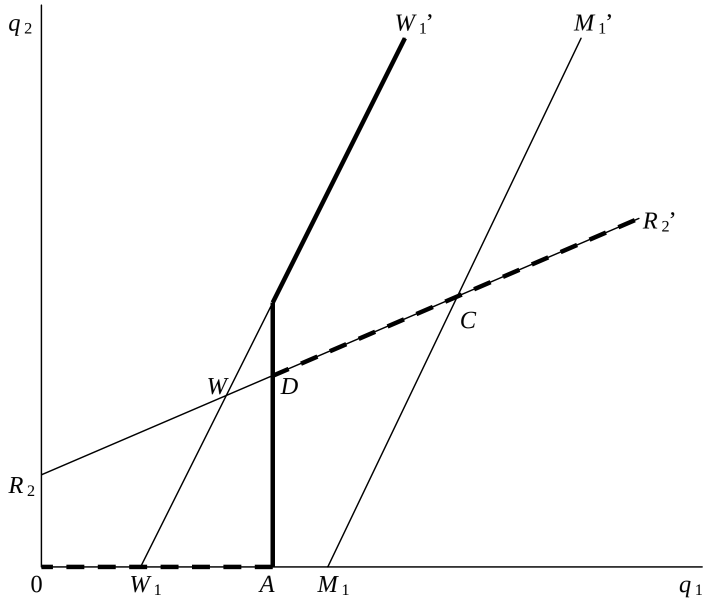

FIG. 3. Case 3: $\partial q_i/\partial q_j > 0$ and $\partial R_i/\partial q_j > 0$

4, firm 2 does not enter the market. That is, firm 2's entry is blocked. Thus, the equilibrium occurs at M_1 in Figure 3 (Figure 4) and firm 1 enjoys a pure monopoly.

(ii) $q_1^D < q_1^C$

In each of Cases 3 and 4, firm 1 can deter entry. We discuss the equilibrium outcomes when firm 1 deters entry. In the first stage, firm 1 chooses x_1^D and t_1^D corresponding to D and adopts WRCP. Let w_1 be a variable which can take any value zero and over. From (12), we see that firm 1 can select any output equal to or smaller than its Cournot equilibrium output q_1^C without WRCP. Firm 1's marginal cost exhibits a discontinuity at $q_1 = x_1^D$. Firm 1's reaction curve is kinked at the level equal to $q_1 = x_1^D$. Firm 1's reaction curve is illustrated by the kinked bold line as a result of WRCP. On the other hand, firm 2's reaction curve is discontinuous at D, made up of the two segments illustrated by the dotted line. Since firm 2's profit is zero at D, it does not enter. Thus, the equilibrium occurs at A in Figure 3 (Figure 4).

In Case 3, if firm 1's output q_1^D corresponding to D is larger than firm 1's pure monopoly output q_1^M corresponding to M_1 in Figure 3, then the equilibrium occurs at M_1 and firm 1 enjoys a pure monopoly.

From the preceding discussions, we can state the following proposition:

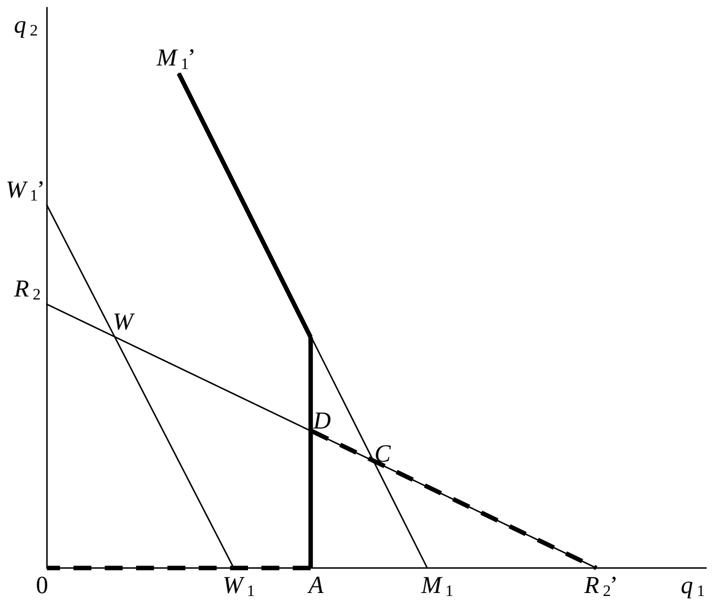

FIG. 4. Case 3: $\partial q_i/\partial q_j > 0$ and $\partial R_i/\partial q_j < 0$

PROPOSITION 2. *In each of Cases 3 and 4, firm 2 never enter the market because firm 1 can adopt WRCP.*

6. CONCLUSION

We have examined a two-stage model of an incumbent firm and a potential entrant. We have considered WRCP as a strategic commitment that increases the marginal cost and analysed entry deterrence in four cases with complementary goods. Consequently, we have found that in each of four cases, the potential entrant never enters the market because the incumbent firm can adopt WRCP. There are many pioneering works dealing with strategic commitments that generate kinks in the firms' reaction curves. We will pursue further research on these works in the future.

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