

## Discrimination in the Equilibrium Search Model with Wage-Tenure Contracts\*

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This paper extends Burdett and Coles (2003)'s search model to two types of workers and firms and derives the equilibrium earnings distributions for both types of workers. It is proven that minority workers have a higher unemployment rate than majority workers; discriminating firms make lower profit than non-discriminating firms; offers to minority workers by non-discriminating firms are consistently superior to those by discriminating firms, and at the same wage level, majority workers almost always experience a faster wage increase than the minority workers.

*Key Words:* Discrimination; Search model; Wage-tenure contract.

*JEL Classification Numbers:* J31, J41, J71.

### 1. INTRODUCTION

Race and gender differentials in the labor market are persistent and widespread. The black-white pay gap has remained around 20% since the mid-1970s (Altonji and Blank, 1999). Even after controlling for human capital and other factors, blacks still earn about 10% less than whites on average (Lang and Lehmann, 2010).<sup>1</sup> In addition to wage differentials, blacks have historically higher unemployment rates and longer unemploy-

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<sup>1</sup>Neal and Johnson (1996) find the unexplained wage gap between blacks and whites is significantly narrowed, or even disappeared in some subgroups after controlling for

ment duration (Fairlie and Sundstrom, 1999). Similar stylized facts are also found in the gender literature. A series of papers by Blau and Kahn (2000, 2003, 2006) find that the gender pay gap in the US has stayed roughly constant at 25% since the mid-1990s; they also find that on average, there is a 0.3 log-point differential for 22 countries examined over the 1985-94 period. Gender differences in unemployment are also widely observed. For example, Azmat, Guell and Manning (2006) document a large gender gap in unemployment rates in many OECD countries.<sup>2</sup> Du and Dong (2009) find longer unemployment durations for women in post-restructuring urban China while Ollikainen (2003) observes longer duration for men in Finland. This paper, built on the framework of search model with wage-tenure contracts (see for example, Burdett and Coles 2003), is able to generate the stylized facts and at the same time touches on wage-tenure profiles. Few papers on discrimination theory have attempted to generate predictions in this regard.<sup>3</sup>

In what follows, we will outline a discrimination search model with wage-tenure contracts and describe equilibrium results. To discuss the effect of discrimination on labor market outcomes, we introduce two types of workers and firms: (1) majority workers *A* and minority workers *B*; (2) discriminating firms *D* and non-discriminating firms *N*.<sup>4</sup> Workers are assumed to be identical except for their appearance. Firms who experience a disutility from hiring minority workers recruit them at a slower rate. So, for type *A* workers firms are homogenous while for type *B* workers they are heterogeneous. In this paper, discrimination is associated with 3 parameters: the fraction of *D*-firms, the degree of recruiting discrimination and the disutility taste *D*-firms have when hiring *B*-workers, all of which are assumed to be exogenously determined. Our model belongs to a class of random search models. Firms post tenure-based contracts for both types of workers, recruit workers and pay wages specified in the contracts. Work-

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AFQT. However, other researchers find wage differentials re-emerge when years of schooling are further controlled with AFQT (Rodgers and Spriggs, 1996).

<sup>2</sup>Altonji and Blank (1999), however, find the unemployment among women has been as low or lower than among men since early 1980s. Participation rates, on the other hand, are historically lower among women. Therefore, when it comes to the overall rate of non-employment, it is always higher among women.

<sup>3</sup>The positive effect of tenure on wages has been identified in many studies (Altonji and Shakotko 1987; Topel 1991; Altonji and Williams 2005). However, there is competing empirical evidence on gender disparity in wage returns to tenure/experience. Some find that the overall wage return to tenure/experience is lower for women than men (Light and Ureta 1995; Munasinghe et al. 2008), while others find steeper wage-tenure profiles for women than men (Becker and Lindsay 1994; Hersch and Reagan 1997). The difference in returns to tenure between races is found to be insignificant in Bratsberg and Terrell (1998) and the returns to actual experience lower for blacks.

<sup>4</sup>One can think majority and minority workers are male and female in the context of gender, or white and black in the context of race.

ers, both unemployed and employed search for jobs randomly, accept the offers which arrive at an exogenous rate if and only if the expected lifetime value from the new offer is higher than the current one. Firms cannot fire workers or counter-offer workers' outside offers.

In equilibrium, the optimal contract for  $B$  workers provided by  $N$ -firms is uniformly better than that provided by  $D$ -firms. Though by offering a higher tenure-wages, the  $N$ -firm extracts a lower profit from each  $B$  worker, it can hire more  $B$  workers who are willing to stay for a longer period so that the total profit  $B$  workers have created in the  $N$  firm exceeds that in a  $D$  firm. In addition, since both firms make the same profit from type  $A$  workers, the total profit is also higher for  $N$  firms than  $D$  firms.

The second finding of the discrimination search model with wage-tenure contracts concerns the relationship between the discrimination associated parameters and wage ranges for minority workers. It proves that, the fewer  $D$  firms are in the labor market, the higher the minimum wage and the lower the maximum wage  $B$  workers can expect in  $D$  firms. Similarly, the more severe the recruiting discrimination or distaste  $D$  firms hold, the higher the lower bound and the lower the upper bound for wages in  $D$  firms. The maximum wage in  $N$ -firms, is negatively related to all three parameters.

We also find that the lowest wage  $A$  workers are willing to accept is smaller than a  $B$  worker's lowest acceptable wage and both lowest wages are smaller than the unemployment insurance. This is because  $A$  workers can expect a faster wage increase and a larger probability of receiving a new offer than  $B$  workers and at the same time, both types of employed workers get a wage promotion that the unemployed do not get. The sign of the mean wage gap between type  $A$  and  $B$  workers, however, is uncertain. If  $D$  firms don't hire any  $B$  workers, it is shown that the average  $A$  worker earns more than the average  $B$  worker while in a general case, the fraction of discriminating firms and their distaste towards minority workers have to be large enough to generate the stylized average wage gap.

Subsequently, we show that in a special case of a CRRA utility function with the coefficient approaching zero, the model degenerates to a simplified version of Bowlus and Eckstein (2002) and has certain similar implications. How the average wage is affected by the discrimination-related parameters is next illustrated in the numerical example, where we also simulate the profile of wage dynamics for both types of workers. It is found that, the wage-tenure effect is positive and it is steeper for  $A$  workers than  $B$  worker in most cases.

The contribution of this paper is the development of a discrimination search model with wage-tenure contracts that, among other things, generates race/gender differences in unemployment rates, durations of unemployment, and wage dynamics. In the theoretical literature on labor market

discrimination, the taste-based theory of discrimination (Becker, 1971) and statistical discrimination (Aigner and Cain, 1977) are often subject to criticism on the grounds that discrimination cannot be sustained in the long run.<sup>5</sup> Taste discrimination models within a search framework, on the other hand, are very promising in explaining persistent wage differentials (Altonji and Blank, 1999). An early example is Black (1995) who studied discrimination in an equilibrium search model. In that paper, cost is introduced in job search processes and discriminating firms are assumed to hire only majority workers. He shows in the model that the wage minority workers receive is lower than the wage of their majority counterparts and the wage differential increases with the proportion of minority workers in the labor market. In a similar line of research, Bowlus and Eckstein (2002), by allowing for on-the-job search, construct a discrimination search model that generates wage dispersion among equally productive workers (see also Burdett and Mortensen, 1998). Moreover, they are able to distinguish the skill differences and discrimination in explaining the residual wage differences between races. This paper follows the assumption of on-the-job search, but replaces the constant wage assumption with wage-tenure contracts which was first introduced in Burdett and Coles (2003). It allows for the possibility of predicting differences in wage-tenure profiles.

The next section sets up the model and discusses workers' and firms' optimal decisions. Section 3 characterizes the equilibrium solutions and section 4 shows the equilibrium properties. In section 5, we show in a special case, that the optimal wage-tenure contracts degenerate to a constant wage and our discrimination search model with wage-tenure contracts degenerates to a variant of Bowlus and Eckstein (2002). Further, to facilitate comparisons of average wages and their dynamics, we carry out a numerical exercise in section 6. Finally, section 7 concludes and points out promising future research. All proofs are given in the appendix.

## 2. THE MODEL

### 2.1. The Environment

Consider an economy consisting of two types of workers and firms. The total work force is  $n$ , of which the majority workers (type  $A$ ) are  $(1 - \theta)n$  and the minority workers (type  $B$ ) are  $\theta n$ . Among all the firms in the labor market, a fraction  $\sigma$  has a distaste for minority workers, denoted by  $D$ ; and  $(1 - \sigma)$  are non-discriminating firms denoted by  $N$ . Workers are assumed to be equally productive (productivity level  $P$ ), and have utility function  $u(w)$ , where?  $u' > 0, u'' < 0$ . They are finitely lived, with a death

<sup>5</sup>See Cain (1986) for a good review on the classic theories, Lang and Lehmann (2010) and Charles and Guryan (2011) for a recent review on progresses in both theories and empirics on race discrimination.

rate  $\delta$ . To balance the population, it's assumed that birth rate equals death rate and the newly born people enter the labor force immediately as unemployed. Unemployed workers can obtain an insurance compensation  $b$  per instant. Workers—both employed and unemployed—search for better opportunities to maximize their expected lifetime utility.

On the other hand, a firm posts a wage-tenure contract and hires workers to maximize its profit. The wage-tenure contract is denoted by  $w(t)$ , where  $t$  denotes tenure—the duration a worker stays in the firm. Suppose the offer arrival rate is  $\lambda$  for  $A$ -workers, both employed and unemployed; while for  $B$ -workers, it depends on the type of firm the offer originates from. If it is from  $N$  firms, the arrival rate is still  $\lambda$ ; if it is from  $D$  firms, the offer arrival rate is  $(1 - k)\lambda$ , where  $k \in (0, 1)$  reflects the degree of recruiting discrimination.<sup>6</sup> The larger  $k$  is, the more severe the discrimination.  $D$  firms experience a disutility  $d$  from hiring  $B$  workers, which enters the profit function directly. Therefore, the instantaneous profit from a  $B$  worker who has stayed in the  $D$  firm for a duration  $t$  is:  $P - w_B^D(t) - d$ . In addition, assume firms cannot fire workers but workers can quit for a better job without suffering any punishment from the previous employer. Time preferences of workers and firms are zero and there are no recalls in the process.

**2.2. Workers' Optimal Decision**

Let  $V_i(t|\widetilde{w}_i^j)$  be the expected lifetime utility of a type  $i$  ( $i = A, B$ ) worker who has tenure  $t$  under the wage-tenure contract  $\widetilde{w}_i^j$  and uses an optimal quit strategy in the future. The term  $\widetilde{w}_i^j$  denotes the wage-tenure contract a type  $i$  worker has signed with firm  $j$  ( $j = D, N$ ).  $F_A(V_0)$ ,  $F_B^N(V_0)$  and  $F_B^D(V_0)$  are the offer distributions for  $A$  and  $B$  where superscripts  $N, D$  denote non-discriminating and discriminating firms and  $V_0$  is the starting expected lifetime value of the offer. Thus, the offer distribution measures the proportion of firms who provide workers a starting offer value no greater than  $V_0$ . Since all firms treat  $A$  the same, there is no difference in the offer distributions for  $A$  provided by  $N$  or  $D$  firms. Let  $\underline{V}_A(\overline{V}_A)$  denote the infimum (supremum) of the support of  $F_A$  and  $\underline{V}_B^j(\overline{V}_B^j)$  the infimum (supremum) of the support of  $F_B^j$  where  $j = N, D$ .

<sup>6</sup>Parameter  $k$  can also be interpreted as indicating the difference in search intensity. Therefore, it only reflects the degree of recruiting discrimination when we assume both types of workers exert the same level of effort in looking for jobs. Indeed, the existence of recruiting discrimination against minority workers such as blacks and women are widely documented (see, for example, Goldin and Rouse 2000; Bertrand and Mullainathan 2004; and Pager et al. 2009).

First consider the situation of employed workers. The standard Bellman equations for employed type  $A$  and type  $B$  workers are:

$$0 = u(w_A(t)) - \delta V_A(t|\widetilde{w}_A) + \lambda \int_{V_A(t|\widetilde{w}_A)}^{\overline{V}_A} [V_0 - V_A(t|\widetilde{w}_A)] dF_A(V_0) + \frac{dV_A(t|\widetilde{w}_A)}{dt}; \quad (1a)$$

$$0 = u(w_B(t)) - \delta V_B(t|\widetilde{w}_B^j) + (1 - \sigma)\lambda \int_{V_B(t|\widetilde{w}_B^j)}^{\overline{V}_B^N} \max\{0, [V_0 - V_B(t|\widetilde{w}_B^j)]\} dF_B^N(V_0) \\ + \sigma(1 - k)\lambda \int_{V_B(t|\widetilde{w}_B^j)}^{\overline{V}_B^D} \max\{0, [V_0 - V_B(t|\widetilde{w}_B^j)]\} dF_B^D(V_0) + \frac{dV_B(t|\widetilde{w}_B^j)}{dt}. \quad (1b)$$

Note that, an  $A$  worker receives an offer at rate  $\lambda$ , whereas a  $B$  worker has a probability of  $(1 - \sigma)\lambda$  receiving an offer from  $N$  firms and a probability of  $\sigma(1 - k)\lambda$  receiving an offer from  $D$  firms. The optimal quit strategy implies that they will quit and accept the new offer if and only if its starting value is greater than the current value.<sup>7</sup> The last term in both equations calculates the instantaneous change in the expected lifetime value.

Similarly, we can get the Bellman equations for unemployed workers of both types:

$$0 = u(b) - \delta V_{AU} + \lambda \int_{V_{AU}}^{\overline{V}_A} [V_0 - V_{AU}] dF_A(V_0); \quad (2a)$$

$$0 = u(b) - \delta V_{BU} + (1 - \sigma)\lambda \int_{V_{BU}}^{\overline{V}_B^N} [V_0 - V_{BU}] dF_B^N(V_0) \\ + \sigma(1 - k)\lambda \int_{V_{BU}}^{\overline{V}_B^D} [V_0 - V_{BU}] dF_B^D(V_0). \quad (2b)$$

The expected lifetime value of an offer from firms should be no less than the unemployed lifetime value  $V_U$ ; otherwise, no worker would be hired. Therefore,  $\underline{V}_A \geq V_{AU}$  and  $\underline{V}_B^j \geq V_{BU}$  ( $j = D, N$ ).

### 2.3. Firms' Optimal Decision

The optimization problem faced by a firm is to choose two wage-tenure contracts, one for  $A$  workers and the other for  $B$  workers, to maximize the total expected profit at the steady state. To begin with, we need to derive the expressions of total expected profit for each firm.

<sup>7</sup>Since the relationship between the current expected lifetime value and the supremum of offers from  $N(D)$ -firm is not clear yet, the maximum of zero and instantaneous change that occurs when the worker accepts the offer ensures the non-negativity and economic meaning. Intuitively, the current value should always be smaller than  $\overline{V}_B^N$ , which means the first max is trivial; however, it may or may not be smaller than  $\overline{V}_B^D$  which makes the second max indispensable.

Since the quit rate of a type  $A$  worker who has stayed  $t$  periods under the wage-tenure contract  $w_A(t)$  is  $\lambda(1 - F_A(V_A(t|\widetilde{w}_A)))$ , the survival probability of such a worker is:

$$\psi_A(t|\widetilde{w}_A) \triangleq \exp \left\{ - \int_0^t [\delta + \lambda(1 - F_A(V_A(s|\widetilde{w}_A)))] ds \right\}. \quad (3a)$$

Similarly, the survival probability of worker  $B$  is:

$$\begin{aligned} & \psi_B(t|\widetilde{w}_B^j) \quad (3b) \\ \triangleq & \exp \left\{ - \int_0^t \{ \delta + (1 - \sigma)\lambda[1 - F_B^N(V_B(s|\widetilde{w}_B^j))] + \sigma(1 - k)\lambda[1 - F_B^D(V_B(s|\widetilde{w}_B^j))] \} ds \right\}. \end{aligned}$$

Let  $G_A(V)$  denote the steady state proportion of  $A$  workers who have an expected lifetime utility less than or equal to  $V$  (including the unemployed); and correspondingly,  $G_B(V)$  for worker  $B$ . Thus, at the steady state, a firm posting an offer  $V$  can recruit  $[\lambda G_A(V)(1 - \theta)n]$   $A$  workers and  $\lambda G_B(V)\theta n$  (if  $N$ -firm) or  $\lambda(1 - k)G_B(V)\theta n$  (if  $D$  firm)  $B$  workers. The steady state profits of  $N$  and  $D$  firms are then functions of the wage-tenure contracts:

$$\begin{aligned} \Omega^N(V_0^A, V_0^B) = & \lambda G_A(V_0^A)(1 - \theta)n \int_0^\infty \psi_A(t|\widetilde{w}_A)[P - w_A(t)] dt \quad (4a) \\ & + \lambda G_B(V_0^B)\theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^N)[P - w_B^N(t)] dt; \end{aligned}$$

$$\begin{aligned} \Omega^D(V_0^A, V_0^B) = & \lambda G_A(V_0^A)(1 - \theta)n \int_0^\infty \psi_A(t|\widetilde{w}_A)[P - w_A(t)] dt \quad (4b) \\ & + \lambda(1 - k)G_B(V_0^B)\theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^D)[P - w_B^D(t) - d] dt. \end{aligned}$$

In each equation, the first part is the profit from  $A$  and the second part is the profit from  $B$ . The integration calculates the expected profit that each worker brings to the firm; the part before the integration measures the steady state number of workers hired at given offers. So, the multiplication reflects the firms' expected profit from each type of worker. As both firms treat  $A$  equally, profit earned from  $A$  is the same between firms in equilibrium.

To derive the optimal decisions of firms, we need to solve the profit maximization problems. Due to additivity, we can solve separately for  $A$ ;  $B$  in  $N$  firms and  $B$  in  $D$  firms. Each sub-problem can be solved in two steps:

(i) Conditional on the offer chosen, the optimal wage-tenure contract solves:

$$\begin{aligned} \max_{\widetilde{w}_i^j(t)} \int_0^\infty \psi_i(t|\widetilde{w}_i^j) [P - w_i^j(t)] dt \\ \text{s.t. } \psi_i(t|\widetilde{w}_i^j) \text{ satisfies (3)} \\ V_i(t|\widetilde{w}_i^j) \text{ satisfies (1)} \\ \text{and, } \psi_i(0|\widetilde{w}_i^j) = 1; V_i(0|\widetilde{w}_i^j) = V_0^{ij}. \end{aligned}$$

(ii) The optimal offer solves:

$$\begin{aligned} \max_{V_0^{ij}} G_i(V_0^{ij}) \int_0^\infty \psi_i(t|\widetilde{w}_i^j) [P - w_i^j(t)] dt \\ \text{s.t. } w_i^j(t) \text{ solves (i),} \end{aligned}$$

where  $i = A, B; j = N, D$ .

When it comes to type  $B$  workers in  $D$  firms, the disutility taste  $d$  should be further subtracted from  $P - w_i(t)$ .

### 3. EQUILIBRIUM

Since worker  $A$  faces homogenous firms in the labor market, the market equilibrium outcomes for this sub-problem are exactly the same as specified in Burdett and Coles (2003). To solve for the steady state equilibrium for worker  $B$ , we first show in proposition 3.2 that the optimal offer for  $B$  provided by  $D$  firms is uniformly smaller than that provided by  $N$ -firms.

**PROPOSITION 1.** *Let  $V_{0N}^B$  denote the optimal offer for  $B$  given by  $N$ -firms and  $V_{0D}^B$  the optimal offer provided by  $D$  firms; then we have  $V_{0N}^B \geq V_{0D}^B$ .*

Proposition 1 simplifies the subsequent analysis substantially.<sup>8</sup> As  $V_{0N}^B \geq V_{0D}^B$ , equations (1b) and (3b) can be rewritten for  $B$  in  $N$  and  $D$  firms separately. Specifically, the Bellman equation for  $B$  workers working in  $N$

<sup>8</sup>Burdett and Coles (2010) prove that offer values can be ranked according to the productivity level of firms. Consider the market of  $B$  workers only, if we think the marginal productivity of  $D$  firms as  $P - d$  and  $N$  firms as  $P$ , proposition 3.2 here is implied by their result.

firms is reduced to:

$$0 = u(w_B^N(t)) - \delta V_B^N(t|\widetilde{w}_B^N) + (1 - \sigma)\lambda \int_{V_B^N(t|\widetilde{w}_B^N)}^{\overline{V}_B^N} [V_0 - V_B^N(t|\widetilde{w}_B^N)] dF_B^N(V_0) + \frac{dV_B^N(t|\widetilde{w}_B^N)}{dt}; \tag{5}$$

for those working in  $D$  firms the Bellman equation becomes:

$$0 = u(w_B^D(t)) - \delta V_B^D(t|\widetilde{w}_B^D) + (1 - \sigma)\lambda [EV_B^N - V_B^D(t|\widetilde{w}_B^D)] + \sigma(1 - k)\lambda \int_{V_B^D(t|\widetilde{w}_B^D)}^{\overline{V}_B^D} [V_0 - V_B^D(t|\widetilde{w}_B^D)] dF_B^D(V_0) + \frac{dV_B^D(t|\widetilde{w}_B^D)}{dt}. \tag{6}$$

Similarly, survival probabilities of  $B$  workers who are employed by  $N$  firms and  $D$  firms change from (3b) to:

$$\psi_B(t|\widetilde{w}_B^N) = \exp \left\{ - \int_0^t [\delta + (1 - \sigma)\lambda(1 - F_B^N(V_B(s|\widetilde{w}_B^N)))] ds \right\}; \tag{7}$$

$$\psi_B(t|\widetilde{w}_B^D) = \exp \left\{ - \int_0^t [\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda(1 - F_B^D(V_B(s|\widetilde{w}_B^D)))] ds \right\}. \tag{8}$$

This makes disentanglement of the sub-problems for  $B$  workers in  $N$  and  $D$  firms possible. The following proposition describes the equilibrium outcomes in the labor market. The crucial step in the proof is to define  $G_B^D(V_0)$  and  $G_B^N(V_0)$  to replace  $G_B(V_0)$ . Let  $G_B^D(V_0)$ ,  $V_0 \in [V_B^D, \overline{V}_B^D]$  be the proportion of  $B$  workers who have an expected lifetime value no greater than  $V_0$  in all  $B$  workers excluding those working in  $N$  firms and  $G_B^N(V_0)$ ,  $V_0 \in [V_B^N, \overline{V}_B^N]$  be the proportion of  $B$  workers with expected lifetime value no greater than  $V_0$  in all  $B$ -workers. Then, the proof of the equilibrium outcomes could fit nicely in that of Burdett and Coles (2003). Moreover, through constructing the overall  $G_B(V_0)$  from  $G_B^D(V_0)$  and  $G_B^N(V_0)$ , we show that the lower bound of the starting wage in  $N$ -firms is the upper limit of starting wages offered by  $D$  firms. The assumption of differentiable  $F_B^j(\cdot)$  is necessary to derive the equilibrium. Otherwise, a mass point exists in  $F_B^j(\cdot)$  at the extreme offer value and wages in  $N$ -firms can be smaller than wages in  $D$ -firms when the rank of offer values remains (Burdett and Coles 2010). Detailed proof refers to the appendix.

PROPOSITION 2. *Given  $w_A, \overline{w}_B > 0$  and  $F_A(V)$ ,  $F_B^N(V)$ ,  $F_B^D(V)$  are increasing and continuously differentiable, there exists a unique market equilibrium. At the steady state equilibrium, the baseline salary scale for*

worker  $A$  satisfies:

$$\frac{P - \overline{w}_A}{P - \underline{w}_A} = \left( \frac{\delta}{\delta + \lambda} \right)^2; \quad (9)$$

$$u(\underline{w}_A) = u(b) - \frac{\sqrt{P - \underline{w}_A}}{2} \int_{\underline{w}_A}^{\overline{w}_A} \frac{u'(x)}{\sqrt{P - x}} dx. \quad (10)$$

The optimal wage-tenure contract for worker  $A$  follows the dynamic path:

$$\frac{dw_A}{dt} = \frac{\delta(P - w_A)}{u'(w_A)} \int_{w_A}^{\overline{w}_A} \frac{u(x)}{\sqrt{(P - \overline{w}_A)(P - x)}} dx. \quad (11)$$

For worker  $B$ , the baseline salary scale satisfies:

$$\frac{P - \overline{w}_B^D - d}{P - \underline{w}_B^D - d} = \left( \frac{\delta + (1 - \sigma)\lambda}{\delta + (1 - \sigma k)\lambda} \right)^2; \quad (12)$$

$$u(\underline{w}_B^D) = u(b) - \frac{\sqrt{P - \underline{w}_B^D - d}}{2} \int_{\underline{w}_B^D}^{\overline{w}_B^D} \frac{u'(x)}{\sqrt{P - x - d}} dx; \quad (13)$$

$$\underline{w}_B^N = \overline{w}_B^D; \quad (14)$$

$$\frac{P - \overline{w}_B^N}{P - \underline{w}_B^N} = \left( \frac{\delta}{\delta + (1 - \sigma)\lambda} \right)^2. \quad (15)$$

And the dynamics of baseline salaries are:

$$\frac{d\overline{w}_B^D}{dt} = \frac{(\delta + (1 - \sigma)\lambda)(P - \overline{w}_B^D - d)}{u'(\overline{w}_B^D)} \int_{\overline{w}_B^D}^{\overline{w}_B^D} \frac{u'(x)}{\sqrt{(P - \overline{w}_B^D - d)(P - x - d)}} dx; \quad (16)$$

$$\frac{d\underline{w}_B^N}{dt} = \frac{\delta(P - \underline{w}_B^N)}{u'(\underline{w}_B^N)} \int_{\underline{w}_B^N}^{\overline{w}_B^N} \frac{u'(x) dx}{(P - \overline{w}_B^N)(P - x)}. \quad (17)$$

Baseline salary scale is a succinct way to describe all the equilibrium solutions. For any starting value  $V_0$  from the support of offer distribution  $F_A$ , there exists a point  $t_0$  such that  $V_0 = V_s(t_0)$  where the subscript  $s$  denotes baseline. So the wage-tenure contract with a starting value  $V_0$  can be expressed as  $w(t|V_0) = w_s(t + t_0)$ ; that is, any equilibrium wage-tenure contract can be found on the baseline salary scale starting with a specific point  $t_0$ . In this paper, we suppress the  $s$ -subscript for simplicity of presentation. The optimal decision implied in the proposition 2 is: for worker  $A$ , a firm can set any wage between  $[\underline{w}_A, \overline{w}_A]$  as the starting wage

offer and backload it as described in the optimal wage-tenure dynamic (11); the total profit from  $A$  will be the same across firms no matter which wage-tenure contract they choose. Since  $\frac{dw_A}{dt}$  is positive, the optimal wage increases with tenure and the upper limit of the increment is  $\overline{w_A}$ . Obviously, the wage support for type  $A$  workers can be solved by combining (9) and (10).

Similarly, for worker  $B$ ,  $D$  firms can set any starting wage between  $[w_B^D, \overline{w_B^D})$  and then backload the wage using the rule described in (16). Profit from type  $B$  workers is the same across the discriminating firms.  $N$ -firms can determine any starting wage between  $[w_B^N, \overline{w_B^N})$ , increase the wage with tenure as described in (17) and make the same profit as any other  $N$  firms. One point to note is that although  $w_B^N = \overline{w_B^D}$ ,  $V_B^N \neq \overline{V_B^D}$ . Rather, employees hired in  $N$  firms with a payment  $w_B^N$  have a higher expected lifetime value than the high-earners in  $D$  firms, i.e.,  $V_B^N > \overline{V_B^D}$ ; because workers with  $w_B^N$  can expect an immediate increase in the payment while those approaching  $\overline{w_B^D}$  cannot.

It can be derived that at equilibrium, the earnings distributions are given by:

$$K_w^A(w) = \frac{\delta}{\lambda} \left[ \sqrt{\frac{P - w_A}{P - w}} - 1 \right]; \tag{18}$$

$$K_w^B(w) = \begin{cases} \frac{\delta}{(1-\sigma k)\lambda} \left[ \sqrt{\frac{P - w_B^D - d}{P - w - d}} - 1 \right], & \text{if } w \in [w_B^D, \overline{w_B^D}] \\ \frac{\delta + (1-\sigma k)\lambda}{(1-\sigma k)\lambda} \sqrt{\frac{P - w_B^N}{P - w} - \frac{\delta}{(1-\sigma k)\lambda}}, & \text{if } w \in [w_B^N, \overline{w_B^N}] \end{cases}; \tag{19}$$

and the unemployment rates of each type of workers are:

$$u_A = \frac{\delta}{\lambda + \delta}; \tag{20}$$

$$u_B = \frac{\delta}{\delta + (1 - \sigma k)\lambda}. \tag{21}$$

It is found that disutility  $d$  has no effect on  $u_B$ ; and  $B$ 's unemployment rate is always higher than  $A$ 's unemployment rate as long as there is discrimination in the labor market ( $\sigma k \neq 0$ ).

The maximized total profits earned by a  $D$  firm and a  $N$  firm are:

$$\Omega^D = \lambda(1-\theta)n\frac{P-\bar{w}_A}{\delta} + \lambda(1-k)\theta n\delta\frac{P-\bar{w}_B^D-d}{[\delta+(1-\sigma)\lambda]^2}; \quad (22)$$

$$\Omega^N = \lambda(1-\theta)n\frac{P-\bar{w}_A}{\delta} + \lambda\theta n\frac{P-\bar{w}_B^N}{\delta}. \quad (23)$$

So the difference in profits in  $N$  and  $D$  firms is:

$$\Omega^N - \Omega^D = \lambda\theta n\frac{\delta[k(P-\bar{w}_B^D) + (1-k)d]}{[\delta+(1-\sigma)\lambda]^2} > 0. \quad (24)$$

This is a general finding in the discrimination literature. Though  $D$  firms earn higher profit from a single  $B$  worker by paying a lower wage, the total profit is less than that in  $N$  firms; because the negative effect of lower employment and higher quit rate in a  $D$  firm outweighs the positive effect of a lower wage. Besides, the disutility taste  $D$  firms have towards  $B$  workers widens the profit gap further. The larger  $\theta$ ,  $n$ ,  $k$  and  $d$  is, the larger the gap.<sup>9</sup> This indicates that having more minority workers in the labor market places the discriminating firms in a worse situation; and, the more prejudiced the discriminating firms are, the higher loss they will bear.

#### 4. EQUILIBRIUM PROPERTIES

To facilitate the comparisons of average wages between two types of workers, we calculate the mean wages from (9), (12), (14), (15), (18) and (19), which gives

$$Ew_A = \int_{\underline{w}_A}^{\bar{w}_A} wdK_w^A(w) = \bar{w}_A + \frac{\delta}{\lambda}(\bar{w}_A - \underline{w}_A) - \frac{2\delta}{\delta + \lambda}(P - \underline{w}_A); \quad (25)$$

$$\begin{aligned} Ew_B &= \int_{\underline{w}_B^D}^{\bar{w}_B^N} wdK_w^B(w) \quad (26) \\ &= \bar{w}_B^N + \frac{\delta}{(1-\sigma k)\lambda}(\bar{w}_B^N - \underline{w}_B^D) \\ &\quad - \frac{2}{1-\sigma k} \left[ \frac{\delta\sigma(1-k)}{\delta+(1-\sigma k)\lambda}(P - \underline{w}_B^D - d) + \frac{(1-\sigma)[\delta+(1-\sigma k)\lambda]}{\delta}(P - \bar{w}_B^N) \right]. \end{aligned}$$

<sup>9</sup>Though values of  $k$  and  $d$  also influence  $\bar{w}_B^D$  in the expression of profit difference, the negative correlation between  $k$ ,  $d$  and  $\bar{w}_B^D$  (which to be shown in section 4) will enhance the positive relationship between  $k$ ,  $d$  and the profit gap.

Note that the unemployed workers are not included in the calculation. Under some general conditions, we discuss the equilibrium properties in the following proposition:

PROPOSITION 3. *If  $b \leq \frac{3}{4}(P - d)$ ,  $\frac{u'(w_B^D)}{u'(w_B^N)} > \frac{\eta}{\eta+2}$  and  $\frac{\partial w_B^D}{\partial \sigma} < \frac{2\lambda(P - w_B^D)}{\delta + (1 - \sigma)\lambda}$  where  $\eta = \frac{\delta + (1 - \sigma)k\lambda}{\delta + (1 - \sigma)\lambda}$  is the relative hazard rate, then the equilibrium has the following properties:*

- (1)  $\frac{\partial w_B^D}{\partial \sigma} < 0$     $\frac{\partial w_B^D}{\partial \sigma} > 0$     $\frac{\partial w_B^N}{\partial \sigma} < 0$
- (2)  $\frac{\partial w_B^D}{\partial k} > 0$     $\frac{\partial w_B^D}{\partial k} < 0$     $\frac{\partial w_B^N}{\partial k} < 0$
- (3)  $\frac{\partial w_B^D}{\partial d} > 0$     $\frac{\partial w_B^D}{\partial d} < 0$     $\frac{\partial w_B^N}{\partial d} < 0$
- (4)  $w_A < w_B^D \leq b < w_B^N < \bar{w}_A$
- (5)  $Ew_A > Ew_B$  when discriminating firms only hire A workers (i.e.,  $k = 1$  and  $\sigma \neq 0$ )

The discriminating wage bounds solved from equations (12) and (13) and non-discriminating wage bounds solved from equations (14) and (15) are functions of productivity  $P$ , unemployment insurance  $b$ , birth-death rate  $\delta$ , normal offer arrival rate  $\lambda$  and three discrimination indicators ( $\sigma, k, d$ ). Under conditions specified in proposition 3, the comparative statics of wage bounds with respect to the three discrimination associated parameters, described in properties (1)-(3), can be easily obtained.

Property (1) shows that the higher the proportion of  $D$ -firms in the market, the wider the range of discriminating wages will be; and the range extends in both directions. On the contrary, the degree of recruiting discrimination has an opposite effect: severe discrimination in the hiring process will lead to a narrowing of the discriminating wage range which converges to the unemployment insurance (which is implied by property (4)). Disutility has the same effect on discriminating wage bounds. Finally, the highest non-discriminating wage decreases as any of the three parameters increases.

The next two properties compare the equilibrium wages between two types of workers. Several points are noteworthy. First, the lowest acceptable wage is lower than the unemployment insurance, which is a unique result within the search model with wage-tenure contracts. In Burdett and Mortensen (1998), firms set a constant wage rather than a wage-tenure contract, hence the lowest acceptable wage is the unemployment insurance  $b$  (when the offer arrival rate is the same for both the employed and the unemployed). Under the wage-tenure framework, however, workers are willing to work at a wage lower than the unemployment insurance only because they can expect an immediate increase in the payment. In fact,

the expected lifetime value at the lowest wage is virtually equal to that at the status of unemployment.

Second,  $A$ 's lowest acceptable starting wage is less than the lowest starting wage for  $B$ . This is because on the one hand, worker  $A$ 's wage increases with tenure more quickly than  $B$ 's; on the other hand, compared to  $B$ ,  $A$  is more likely to get a new and better job offer in the labor market.

Third, that the upper bound of  $A$ 's wages being higher than their counterpart's is within expectation, since discriminating firms are unlikely to set too high a wage due to their disutility tastes.

In a special case where discriminating firms hire only type  $A$ , property (5) shows that "minority workers receive lower wages than workers not facing discrimination" (Black, 1995). However, this finding cannot be generalized. In the numerical example, we will show that if  $D$ -firms can hire  $B$  ( $0 \leq k < 1$ ), the average worker  $B$  might be able to earn a slightly higher wage than worker  $A$ .

## 5. SPECIAL CASE

In this section, a special case of the CRRA utility function:  $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$  ( $\gamma \rightarrow \infty$ ) is considered. Tractable equilibrium solutions that are derived from proposition 2 can shed more light on the labor market with discrimination. Proposition 4 below summarizes the equilibrium results in this special case.

**PROPOSITION 4.** *Given that both types of workers have the same CRRA utility function:  $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$  with  $\gamma \rightarrow \infty$ , the following statements hold:*

(1) *The optimal strategy of a firm is to set fixed wages instead of the wage-tenure contracts.*

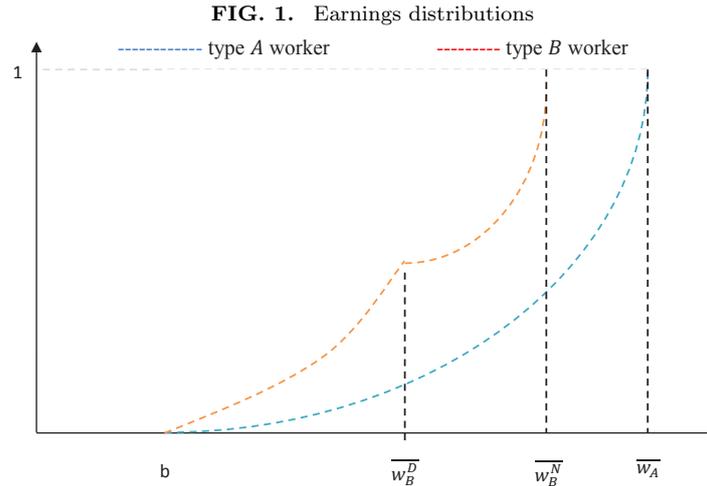
(2) *The wage bounds are:  $\underline{w}_A = b$ ;  $\overline{w}_A = P - (\frac{\delta}{\lambda+\delta})^2(P - b)$ ;  $\overline{w}_B^D = b$ ;  $\overline{w}_B^D = \overline{w}_B^N = P - d - (\frac{\delta+(1-\sigma)\lambda}{\delta+(1-\sigma k)\lambda})^2(P - b - d)$ ;  $\overline{w}_B^N = P - (\frac{\delta}{\delta+(1-\sigma)\lambda})^2 d - (\frac{\delta}{\delta+(1-\sigma k)\lambda})^2(P - b - d)$ ; and,  $\underline{w}_A = \underline{w}_B^D = b < \overline{w}_B^D < \overline{w}_B^N < \overline{w}_A$ .*

(3)  *$A$ 's earnings distribution first order dominates  $B$ 's earnings distribution, i.e.,  $K_w^A < K_w^B$  for all  $w$ .*

(4)  *$Ew_A > Ew_B$  and the mean wage gap increases with  $(\sigma, k, d)$ .*

As  $\gamma \rightarrow \infty$ , workers are infinitely risk averse; thus the optimal wage contract is constant wages. The equilibrium search model with wage-tenure contracts then degenerates to Burdett and Mortensen (1998) and the discriminating wage-tenure equilibrium search model degenerates to a sim-

plified version of Bowlus and Eckstein (2002).<sup>10</sup> Figure 1 describes the earnings distributions for both types of workers and apparently  $A$ 's cumulative earnings distribution first order dominates  $B$ 's distribution. From first order dominance, property (4) is directly obtained. In addition, the same reservation wages between  $A$  and  $B$  is resulted from the assumption that the offer arrival rate is invariant between the employed and unemployed workers. The upper wage limit of  $B$  is less than that of  $A$  because of the existence of the three non-zero discrimination parameters  $(\sigma, k, d)$ .



Moreover, the larger  $(\delta, k, d)$  is, the smaller  $B$ 's average wage is. Since  $(\sigma, k, d)$  does not enter type  $A$  worker's wage, the average wage gap increases as  $(\sigma, k, d)$  increases. This conclusion is in line with the empirical findings. For example, Charles and Guryan (2008) plot the black-white wage gap against prejudicial attitude and find a wider gap at regions where many people will not vote for the black candidate for presidency or are against interracial marriages.

### 6. A NUMERICAL EXAMPLE

As mentioned in section 4, it is interesting to examine the effect of the three discrimination-relevant parameters on the difference in the mean wages between type  $A$  and  $B$  workers. We assume in the section that

<sup>10</sup>Bowlus and Eckstein (2002) extend Burdett and Mortensen (1998)'s model to discuss the contributions of discrimination and skill differences to the wage gaps. In their paper, the offer arrival rate is assumed to be different between the employed and the unemployed and therefore unlike what we get in this special case, the reservation wage is larger than the unemployment compensation.

all workers have the same CRRA utility function. Let  $P = 300$ ,  $b = 100$ ,  $\lambda = 0.03$  and  $\delta = 0.003$ . If the coefficients of relative risk aversion are 0.9, 1.4 and 1.9, equation (25) gives that  $A$ 's average wages are 273.3307, 275.3025 and 276.8115 respectively. It seems that the more risk averse workers are, the higher the average wage they would earn.

For worker  $B$ , we vary the values of  $(\sigma, k, d)$  to see how the mean wage changes accordingly. Results are presented in Table 1 in which the first panel fixes  $d$  and  $k$ , and changes the measure of discriminating firms  $\sigma$ ; the second panel changes the recruiting discrimination  $k$  and keeps the other two measures unchanged; and the third one modifies disutility taste  $d$  given certain values of  $\sigma$  and  $k$ . The findings are as follows: First, the mean wage of type  $B$  worker decreases in  $\sigma$  and  $d$ , but increases in  $\gamma$  while the relationship with  $k$  is uncertain. Second, the fraction of  $D$ -firms plays a key role in the average wage; the other three parameters, though matter to some extent, have only limited influence on the wage outcomes. Third, if only  $D$ -firms exist in the labor market (see the case  $\sigma = 1$  in Panel 1), the wage gap is very large; however, the gap will drop dramatically when  $N$ -firms begin to appear. In addition, Panel (2) indicates that the wage gap does not change much even when  $D$ -firms are forbidden to discriminate in hiring (see  $k = 0$ ); on the other hand, what appears to be against expectation is that severe discrimination in recruitment leads to higher average wage for  $B$  and hence smaller wage gap (see  $k = 0.9$ ). However, one should realize that this does not mean type  $B$  workers are better off because only a few will be hired in this situation and the overall welfare of type  $B$  workers is in fact jeopardized. Finally, compared to  $A$ 's average wage, the numbers in Table 1 are almost consistently smaller, which accords with the common sense that discriminated workers have lower average wage.<sup>11</sup>

Next, we illustrate the difference in wage dynamics between the two types of workers in Figure 2. To be representative, we choose a most realistic case where  $\gamma = 0.9$ ,  $k = 0.8$ ,  $\sigma = 0.8$  and  $d = 80$  and an extreme case in which  $B$ 's mean wage exceeds that of type  $A$  worker.<sup>12</sup>

There are several points worth noting. First, the slope of the wage-tenure contract is positive, meaning that the wage always increases with

<sup>11</sup>One exception is when  $\sigma = 0.2$  in Panel 1,  $B$ 's average wage is slightly larger than  $A$ 's. These rare cases seem to imply that the fraction of discriminating firms has to be large enough to generate the result of minority workers earning less than majority workers on average. Becker (1971) gives the exact condition  $\sigma$  should satisfy to derive the wage differential in the framework of competitive labor market. Aigner and Cain (1977) find a similar result in a group of low skilled workers, that discriminated-against workers have a higher average wage than their counterparts under the assumption of same mean productivity and different variances.

<sup>12</sup>Given those values, the simulated average wages for  $A$  and  $B$  are 273.3307 and 229.4995 respectively, very close to 273.9 and 230.96 derived from real data (Bowlus and Eckstein, 2002).

**TABLE 1.**The mean wage of type *B* workers(1)  $d = 80$   $k = 0.2$ 

	$\gamma = 0.9$	$\gamma = 1.4$	$\gamma = 1.9$
$\sigma = 0.2$	276.6618	276.7661	276.8648
$\sigma = 0.4$	269.3880	269.9290	270.3657
$\sigma = 0.6$	258.4842	259.7430	260.6239
$\sigma = 0.8$	238.7852	240.6891	241.9693
$\sigma = 1.0$	196.9306	199.2000	200.7968

(2)  $d = 80$   $\sigma = 0.5$ 

	$\gamma = 0.9$	$\gamma = 1.4$	$\gamma = 1.9$
$k = 0.0$	264.9373	265.9374	266.6517
$k = 0.3$	264.4216	265.2213	265.8328
$k = 0.6$	264.4312	264.8563	265.2231
$k = 0.9$	265.7274	265.7662	265.8044

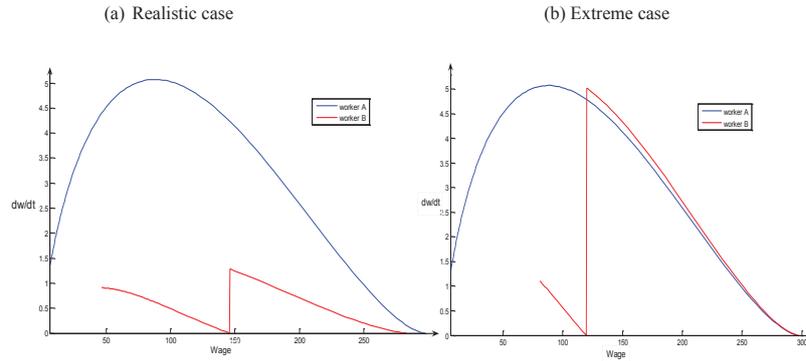
(3)  $\sigma = 0.5$   $k = 0.5$ 

	$\gamma = 0.9$	$\gamma = 1.4$	$\gamma = 1.9$
$d = 10$	269.3757	270.4330	271.2300
$d = 80$	264.3334	264.9009	265.3683
$d = 150$	259.9760	260.1040	260.2230

tenure. Second, for type *A* workers, the increase accelerates at the beginning, and slows down gradually; on the other hand, for type *B* workers the increasing rate drops from the very beginning. Besides, the slope of *A*'s wage-tenure contract is, in general, larger than *B*'s, especially in *D*-firms. *N*-firms, though owning no prejudice towards worker *B*, have less incentives to backload their wages as quickly as they do to worker *A* because there are fewer outside opportunities to worker *B*. If, however, only a small number of firms discriminate against worker *B* so that they can still seek many job offers from non-discriminating firms, then the slope of wage-tenure contracts designed for *B* workers by *N*-firms can be very close to, or even exceed the wage increase rate of worker *A* (see Figure 2(b)).

## 7. CONCLUSIONS

This paper develops a discrimination search model with wage-tenure contracts and predicts: 1) minority workers have a higher unemployment rate and a longer duration of unemployment; 2) non-discriminating firms make higher profits than discriminating firms; 3) the lowest acceptable wage for a minority worker is greater than that for a majority worker while the highest expectable wage of a minority worker is lower; 4) generally, mi-

**FIG. 2.** Wage Dynamics

nority workers earn less than majority workers on average, and their wage increases more slowly than their counterpart. Moreover, we also show how the fraction of discriminating firms, distaste and recruiting discrimination affect the wage ranges and mean wages for both types of workers.

There are some limitations in the discrimination search model with wage-tenure contracts. First, it does not consider the status of nonparticipation and other characteristics of jobs but wages in the labor market. This is crucial in comparing gender differences in labor market outcomes. Bowlus (1997) shows women have a greater tendency to exit jobs to nonparticipation due to family, pregnancy or health issues. Flabbi and Moro (2010) measure women's preference for work flexibility and find an impact on wage distributions. The second limitation of the model exists in the empirical application. We follow the identification strategy in Bowlus and Eckstein (2002) but it would be better if we can generate an econometric approach from the model and do some robustness check. Finally, we suggest some future researches on this line. One can apply the model to data and compare the discrimination estimate with that of Bowlus and Eckstein (2002) and Flabbi (2010). Besides, we can study taste discrimination in the directed search model with wage-tenure contract (Shi, 2009) and see what different predictions can be obtained. Or, it may be modified to some extent to explain glass ceiling/sticky floor effects found in empirical work.

## APPENDIX A

## A.1. PROOF OF PROPOSITION 1

Since  $V_{0N}^B$  and  $V_{0D}^B$  are offers chosen by  $N$ - and  $D$ -firms to maximize their respective profit flow at the steady state, it implies

$$\begin{aligned} & \lambda G_B(V_{0N}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^N) [P - w_B^N(t)] dt \\ & \geq \lambda G_B(V_{0D}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^D) [P - w_B^D(t)] dt; \end{aligned}$$

and

$$\begin{aligned} & (1-k) \lambda G_B(V_{0D}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^D) [P - w_B^D(t) - d] dt \\ & \geq (1-k) \lambda G_B(V_{0N}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^N) [P - w_B^N(t) - d] dt. \end{aligned}$$

Note that  $\widetilde{w}_B^j$  ( $j = N, D$ ) is the wage-tenure contract designed to deliver the offer, so it's a function of  $V_{0j}^B$ . The two inequalities then imply:

$$\begin{aligned} & G_B(V_{0N}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^N) [P - w_B^N(t)] dt \\ & - G_B(V_{0N}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^N) [P - w_B^N(t) - d] dt \\ & \geq G_B(V_{0D}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^D) [P - w_B^D(t)] dt \\ & - G_B(V_{0D}^B) \theta n \int_0^\infty \psi_B(t|\widetilde{w}_B^D) [P - w_B^D(t) - d] dt. \end{aligned}$$

If we define:

$$\Psi(V_0^B) \triangleq d G_B(V_0^B) \int_0^\infty \psi_B(t|\widetilde{w}_B) dt,$$

then the above inequality is:

$$\Psi(V_{0N}^B) \geq \Psi(V_{0D}^B),$$

because,

$$\Psi'(V_0^B) = d \frac{\partial G_B(V_0^B)}{\partial V_0^B} \int_0^\infty \psi_B(t|\widetilde{w}_B) dt + d G_B(V_0^B) \int_0^\infty \frac{\partial(\psi_B(t|\widetilde{w}_B))}{\partial V_0^B} dt > 0,$$

due to the increasing property of  $G_B(V_0^B)$  and  $\psi_B(t|\widetilde{w}_B)$  with respect to  $V_0^B$ , we have  $V_{0N}^B \geq V_{0D}^B$ .

## A.2. PROOF OF PROPOSITION 2

For the derivation of equilibrium results for worker  $A$ , refer to Burdett and Coles (2003). Below is a similar derivation of equilibrium results for worker  $B$ .

(1) First consider the optimal wage-tenure contract designed for  $B$ -workers by discriminating firms.

Given the starting offer  $V_0$ , the wage-tenure function solves:

$$\max_{w(t)>0} \int_0^\infty \psi_B(t|\widetilde{w}_B^D)[P - w_B^D(t) - d]dt,$$

where

$$\dot{\psi}_B = -[\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda(1 - F_B^D(V_B^D))] \psi_B; \quad (\text{A.1})$$

$$\begin{aligned} \dot{V}_B^D &= \delta V_B^D - u(w_B^D(t)) - (1 - \sigma)\lambda[EV_B^N - V_B^D] \\ &\quad - \sigma(1 - k)\lambda \int_{V_B^D}^{\overline{V}_B^D} [x - V_B^D] dF_B^D(x); \end{aligned} \quad (\text{A.2})$$

with starting values  $\psi_B(0) = 1$ ;  $V_B^D(0) = V_0$ .

To solve the dynamic optimization problem, define the Hamiltonian:

$$\begin{aligned} H &= \psi_B[P - w_B^D(t) - d] - \Lambda_\psi[\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda(1 - F_B^D(V_B^D))] \psi_B \\ &\quad + \Lambda_V[\delta V_B^D - u(w_B^D(t)) - (1 - \sigma)\lambda[EV_B^N - V_B^D] \\ &\quad - \sigma(1 - k)\lambda \int_{V_B^D}^{\overline{V}_B^D} [x - V_B^D] dF_B^D(x)] \end{aligned}$$

where  $\Lambda_\psi$ ,  $\Lambda_V$  are costate variables with respect to  $\psi_B$  and  $V_B^D$ .

The necessary conditions are:

$$H_w = -\psi_B - \Lambda_V u'(w_B^D(t)) = 0; \quad (\text{A.3})$$

$$\begin{aligned} \dot{\Lambda}_\psi &= -H_\psi = -[P - w_B^D(t) - d] \\ &\quad + \Lambda_\psi[\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda(1 - F_B^D(V_B^D))]; \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \dot{\Lambda}_V &= -H_V = -\Lambda_V[\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda[1 - F(V_B^D)]] \\ &\quad - \Lambda_\psi \sigma(1 - k)\lambda F_B^{D'}(V_B^D) \psi_B. \end{aligned} \quad (\text{A.5})$$

And the two differential equations  $\psi_B$  and  $V_B^D$  should satisfy (A1), (A2).

Integrate (A4) with the integrating factor  $\psi_B$  yields:

$$\Lambda_\psi \psi_B = \int_t^\infty \psi_B(s|\widetilde{w}_B^D)[P - w_B^D(s) - d]ds + C_1.$$

Define the expected future profit flow from tenure period  $t$  onwards as:

$$\Pi_B^D(t|\widetilde{w}_B^D) = \int_t^\infty \frac{\psi_B(s|\widetilde{w}_B^D)}{\psi_B(t|\widetilde{w}_B^D)} [P - w_B^D(s) - d]ds.$$

Then,

$$\Lambda_\psi = \Pi_B^D(t|\widetilde{w}_B^D) + \frac{C_1}{\psi_B(t|\widetilde{w}_B^D)}$$

Since it's an autonomous control problem, the optimized Hamiltonian is zero, i.e.,  $H = 0$ . Substituting  $\Lambda_\psi, \Lambda_V$  in  $H$  out yields:

$$\begin{aligned} 0 = & [P - w_B^D(t) - d] \\ & - \left\{ \Pi_B^D(t|\widetilde{w}_B^D) + \frac{C_1}{\psi_B(t|\widetilde{w}_B^D)} \right\} [\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda(1 - F_B^D(V_B^D))] \\ & - \frac{1}{u'(w_B^D(t))} [\delta V_B^D - u(w_B^D(t)) - (1 - \sigma)\lambda(EV_B^N - V_B^D)] \\ & - \sigma(1 - k)\lambda \int_{V_B^D}^{\overline{V}_B^D} [x - V_B^D]dF_B^D(x) \end{aligned}$$

Therefore,  $C_1$  has to be zero to make  $\Pi_B^D$  bounded. Thus  $\Lambda_\psi = \Pi_B^D(t|\widetilde{w}_B^D)$  and (A4) turns to be:

$$\frac{d\Pi_B^D(t|\widetilde{w}_B^D)}{dt} = -[P - w_B^D(t) - d] + \Pi_B^D(t|\widetilde{w}_B^D)[\delta + (1 - \sigma)\lambda + \sigma(1 - k)\lambda(1 - F_B^D(V_B^D))]. \tag{A.6}$$

And (A2), (A6) and  $H = 0$  give:

$$\frac{dV_B^D(t|\widetilde{w}_B^D)}{dt} = -u'(w_B^D(t)) \frac{d\Pi_B^D(t|\widetilde{w}_B^D)}{dt}. \tag{A.7}$$

Integrating (A5) with the integrating factor  $\frac{1}{\psi_B}$  and substituting  $\Lambda_\psi$  with  $\Pi_B^D$  yields:

$$\frac{\Lambda_V}{\psi_B} = - \int_0^t \Pi_B^D \sigma(1 - k)\lambda F_B^{D'}(V_B^D)ds + C_2.$$

To Substitute  $\Lambda_V$  in (A3) using the above expression and differentiate with respect to  $t$ , we get:

$$-\frac{u''(w_B^D)}{u'(w_B^D)^2} \frac{dw_B^D(t)}{dt} = \sigma(1-k)\lambda F_B^{D'}(V_B^D)\Pi_B^D \tag{A.8}$$

In addition, the transversality condition implies  $\lim_{t \rightarrow \infty} V_B^D(t|\widetilde{w_B^D}) = \overline{V_B^D}$ .

(2) Next, we present the equilibrium results in terms of baseline wage.

If the solution to the above optimization problem with  $V_0 = \overline{V_B^D}$  is taken as the baseline, then for any starting offer  $V_0 \in [\underline{V_B^D}, \overline{V_B^D})$ , there exists  $t_0$  such that  $V_s^{BD}(t_0) = V_0$ . So, the optimal wage contract of any firm and all the equilibrium solutions could be expressed in terms of the baseline. For example,  $w_B^D(t|V_0) = w_s^{BD}(t_0 + t)$ ,  $V_B^D(t|\widetilde{w_B^D}) = V_s^{BD}(t_0 + t)$  and  $\Pi_B^D(t|\widetilde{w_B^D}) = \Pi_s^{BD}(t_0 + t)$ . Then, it's easy to derive  $w_s^{BD} \uparrow \overline{w_B^D}$  and  $\Pi_s^{BD} \uparrow \overline{\Pi_B^D}$ . Further, from (A2) we can obtain  $\overline{V_B^D} = \frac{u(\overline{w_B^D}) + (1-\sigma)\lambda EV_B^N}{\delta + (1-\sigma)\lambda}$ ; and from (A6), we get  $\overline{\Pi_B^D} = \frac{P - \overline{w_B^D} - d}{\delta + (1-\sigma)\lambda}$ .

Let  $u_B$  denote the unemployment rate,  $d_B$  denote the share of  $B$  workers employed in  $D$ -firms and  $n_B$  the share employed in  $N$ -firms. The flow conditions imply

$$\begin{aligned} \delta &= u_B(\delta + (1-\sigma)\lambda + \sigma(1-k)\lambda); \\ u_B\sigma(1-k)\lambda &= d_B(\delta + (1-\sigma)\lambda); \\ (1-n_B)(1-\sigma)\lambda &= n_B\delta \end{aligned}$$

So, the unemployment rate is  $u_B = \frac{\delta}{\delta + (1-\sigma k)\lambda}$ .

And the employment rate of type  $B$  workers in  $D$ -firms and  $N$ -firms are:

$$d_B = \frac{\delta\lambda\sigma(1-k)}{[\delta + (1-\sigma k)\lambda][\delta + (1-\sigma)\lambda]}; \quad n_B = \frac{(1-\sigma)\lambda}{\delta + (1-\sigma)\lambda}$$

Let  $G_B^D(V_0)$ ,  $V_0 \in [\underline{V_B^D}, \overline{V_B^D})$  be the proportion of  $B$  workers who have an expected lifetime value no greater than  $V_0$  in all the  $B$  workers excluding those working in  $N$ -firms. Then  $G_s^{BD}(t)$  is the corresponding baseline expression which satisfies:

$$G_s^{BD}(0) = \frac{u_B}{u_B + d_B} = \frac{\delta + (1-\sigma)\lambda}{\delta + (1-\sigma k)\lambda} \tag{A.9}$$

and the flow condition for  $B$  workers employed in  $D$  firms with salary point greater than  $t$ :

$$[\delta + (1 - \sigma)\lambda](1 - G_s^{BD}(t)) = \frac{dG_s^{BD}(t)}{dt} + G_s^{BD}(t)\sigma(1 - k)\lambda(1 - F_s^{BD}(t)) \tag{A.10}$$

As every  $D$ -firm makes the same profit from  $B$ -workers at the equilibrium, and  $G_s^{BD} \rightarrow 1$ ,  $\Pi_s^{BD} \rightarrow \bar{\Pi}_B^D$ , from the profit function:

$$\Omega_B^D = \lambda(1 - k)G_s^{BD}(t)\theta n(1 - n_B)\Pi_s^{BD}(t),$$

we can get  $G_s^{BD}(t)\Pi_s^{BD}(t) = \frac{P - \bar{w}_B^D - d}{\delta + (1 - \sigma)\lambda}$ . So,

$$\frac{dG_s^{BD}}{dt}\Pi_s^{BD} + \frac{d\Pi_s^{BD}}{dt}G_s^{BD} = 0$$

Then substituting out  $\frac{dG_s^{BD}}{dt}$  and  $\frac{d\Pi_s^{BD}}{dt}$  using (A6) and (A10) and combining it with (A10) yields:

$$G_w^{BD} = \sqrt{\frac{P - \bar{w}_B^D - d}{P - w_B^D - d}}$$

$$\Pi_w^{BD} = \frac{1}{\delta + (1 - \sigma)\lambda} \sqrt{(P - \bar{w}_B^D - d)(P - w_B^D - d)}$$

Putting the expression of  $G_w^{BD}$  into (A9) thus gets,

$$\frac{P - \bar{w}_B^D - d}{P - w_B^D - d} = \left( \frac{\delta + (1 - \sigma)\lambda}{\delta + (1 - \sigma k)\lambda} \right)^2.$$

The offer distribution could be derived from (A6), (A7), (A8) and the expression of  $\Pi_w^{BD}$ :

$$1 - F_w^{BD} = \frac{\delta + (1 - \sigma)\lambda}{\sigma(1 - k)\lambda} \left[ \sqrt{\frac{P - \bar{w}_B^D - d}{P - w_B^D - d}} - 1 - \frac{1}{2u'(w_B^D)} \int_{w_B^D}^{\bar{w}_B^D} \frac{u'(x)dx}{\sqrt{(P - \bar{w}_B^D - d)(P - x - d)}} \right] \tag{A.11}$$

Further,  $V_s^{BD}(0) = V_{BU}$  at the equilibrium.

Since,

$$\frac{dV_s^{BD}(0)}{dt} = u(b) - u(\bar{w}_B^D),$$

which is derived from the baseline expression of (A2) at  $V_B^D = V_s^{BD}(0)$  and the Bellman equation for unemployed  $B$  workers; and,

$$\frac{dV_s^{BD}(0)}{dt} = \frac{\sqrt{P - w_B^D - d}}{2} \int_{w_B^D}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P - x - d}},$$

which could be derived from substitutions using (A6), (A7), (A11) and the expression of  $\Pi_w^{BD}$ ; we can derive another relationship between the bounds of the support of discriminating wages, i.e.,  $u(w_B^D) = u(b) - \frac{\sqrt{P - w_B^D - d}}{2}$ .

Besides, the dynamics of baseline tenure-wages (equation (16)) could be easily derived from (A8), (A11) and  $\Pi_w^{BD}$  expression.

(3) By the same token, we can get the equilibrium outcomes for  $B$  workers in the non-discriminating firms. Following the same procedures, we can prove that (17) holds. However, the support of the non-discriminating wages is somewhat different in the derivation.

Let  $G_B^N(V_0)$ ,  $V_0 \in [\underline{V_B^N}, \overline{V_B^N})$  be the proportion of  $B$  workers (including the unemployed) who have an expected lifetime value no greater than  $V_0$ .

Then, for the baseline expression, we have  $G_w^{BN} = \sqrt{\frac{P - \overline{w_B^N}}{P - w_B^N}}$ .

So, the overall proportion of type  $B$  workers (including the unemployed) who earn less than or equal to  $w$  at the steady state is:

$$G_w^B(w) = \begin{cases} \frac{\delta}{\delta + (1 - \sigma)\lambda} G_w^{BD}, & \text{if } w \in [w_B^D, \overline{w_B^D}] \\ G_w^{BN}, & \text{if } w \in [\underline{w_B^N}, \overline{w_B^N}] \end{cases}$$

Since  $G_w^B(\overline{w_B^D}) = G_w^B(w_B^N)$  and  $G_w^B(w)$  is monotonically increasing,  $\overline{w_B^D} = \underline{w_B^N}$ . Further, as  $G_s^{BN}(0) = \frac{\delta}{\delta + (1 - \sigma)\lambda}$ , we can get  $\frac{P - \overline{w_B^N}}{P - w_B^N} = \left(\frac{\delta}{\delta + (1 - \sigma)\lambda}\right)^2$ . Thus, (14) (15) are proved.

(4) Finally, we derive the earnings distribution of type  $B$  workers.

Given  $G_w^{BD}$  and  $G_w^{BN}$ , the earning distributions of  $B$  workers in the  $D$ - and  $N$ -firms at the steady state are  $K_w^{BD} = \frac{u_B + d_B}{d_B} [G_w^{BD}(w) - \frac{u_B}{u_B + d_B}]$ , and  $K_w^{BN} = \frac{1}{n_B} [G_w^{BN}(w) - (u_B + d_B)]$ .

So, the overall earning distribution is:

$$K_w^B(w) = \begin{cases} \frac{d_B}{d_B + n_B} K_w^{BD}, & \text{if } w \in [w_B^D, \overline{w_B^D}] \\ \frac{d_B}{d_B + n_B} + \frac{n_B}{d_B + n_B} K_w^{BN} & \text{if } w \in [\underline{w_B^N}, \overline{w_B^N}] \end{cases}$$

Substituting the expressions of  $G_w^{BD}$ ,  $G_w^{BN}$ ,  $u_B$ ,  $d_B$  and  $n_B$  inside, gives equation (19).

**A.3. PROOF OF EQUATIONS (22), (23)**

As shown above:

$$\Omega_B^D = \lambda(1 - k)G_s^{BD}(t)\theta n(1 - n_B)\Pi_s^{BD}(t) = \frac{\lambda(1 - k)\theta n\delta}{[\delta + (1 - \sigma)\lambda]^2}(P - \overline{w_B^D} - d)$$

Similarly,

$$\Omega_B^N = \lambda G_s^{BN}(t)\theta n\Pi_s^{BN}(t) = \lambda\theta n\frac{P - \overline{w_B^N}}{\delta}$$

Profits from A are:

$$\Omega_A = \lambda G_s^A(t)(1 - \theta)n\Pi_s^A(t) = \lambda(1 - \theta)n\frac{P - \overline{w_A}}{\delta}$$

So,  $\Omega^D = \Omega_A + \Omega_B^D$  and  $\Omega^N = \Omega_A + \Omega_B^N$ .

**A.4. PROOF OF PROPOSITION 3**

First, let's consider properties (1)-(3).

Taking partial derivatives of equation (13) with respect to  $(\sigma, k, d)$  yields:

$$A\frac{\partial w_B^D}{\partial \sigma} + B\frac{\partial \overline{w_B^D}}{\partial \sigma} = 0; \quad A\frac{\partial w_B^D}{\partial k} + B\frac{\partial \overline{w_B^D}}{\partial k} = 0; \quad A\frac{\partial w_B^D}{\partial d} + B\frac{\partial \overline{w_B^D}}{\partial d} = \frac{1}{4}C;$$

where

$$A = \frac{u'(w_B^D)}{2} - \frac{1}{4\sqrt{P - \overline{w_B^D}} - d} \int_{\overline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P - x - d}} > 0;$$

$$B = \frac{u'(\overline{w_B^D})}{2} \sqrt{\frac{P - \overline{w_B^D} - d}{P - \overline{w_B^D} - d}} > 0;$$

$$C = \frac{1}{\sqrt{P - \overline{w_B^D}} - d} \int_{\overline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P - x - d}} - \sqrt{P - \overline{w_B^D}} - d \int_{\overline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{(P - x - d)^{3/2}} < 0.$$

Similarly, partial differentiation of equation (12) gives:

$$\begin{aligned} \frac{\partial w_B^D}{\partial \sigma} &= \eta_1^2 \frac{\partial \overline{w_B^D}}{\partial \sigma} - \eta_2(P - \overline{w_B^D} - d); \\ \frac{\partial w_B^D}{\partial k} &= \eta_1^2 \frac{\partial \overline{w_B^D}}{\partial k} + \eta_3(P - \overline{w_B^D} - d); \\ \frac{\partial w_B^D}{\partial d} &= \eta_1^2 \frac{\partial \overline{w_B^D}}{\partial d} + \eta_1^2 - 1; \end{aligned}$$

where:  $\eta_1 = \frac{\delta+(1-\sigma k)\lambda}{\delta+(1-\sigma)\lambda}$ ;  $\eta_2 = \frac{2\lambda(\delta+\lambda)(1-k)}{[\delta+(1-\sigma)\lambda][\delta+(1-\sigma k)\lambda]}$ ;  $\eta_3 = \frac{2\sigma\lambda}{\delta+(1-\sigma k)\lambda}$ .

Substituting them into the first group of equations thus proves:

$$\frac{\partial w_B^D}{\partial \sigma} < 0; \quad \frac{\partial \overline{w_B^D}}{\partial \sigma} > 0; \quad \frac{\partial w_B^D}{\partial k} > 0; \quad \frac{\partial \overline{w_B^D}}{\partial k} < 0; \quad \frac{\partial \overline{w_B^D}}{\partial d} < 0.$$

In addition, as  $(A\eta_1^2 + B)\frac{\partial w_B^D}{\partial d} = \frac{\eta_1(\eta_1-1)}{2}[u'(\overline{w_B^D})(\eta_1+2) - u'(w_B^D)\eta_1]$ , when  $\frac{u'(\overline{w_B^D})}{u'(w_B^D)} > \frac{\eta_1}{\eta_1+2}$ , we have  $\frac{\partial w_B^D}{\partial d} > 0$ .

Since  $\underline{w_B^N} = \overline{w_B^D}$ , the partial derivative with respect to  $(\sigma, k, d)$  in (15) yields:

$$\begin{aligned} \frac{\partial \overline{w_B^N}}{\partial k} &= \left(\frac{\delta}{\delta+(1-\sigma)\lambda}\right)^2 \frac{\partial \overline{w_B^D}}{\partial k}; \\ \frac{\partial \overline{w_B^N}}{\partial d} &= \left(\frac{\delta}{\delta+(1-\sigma)\lambda}\right)^2 \frac{\partial \overline{w_B^D}}{\partial d}; \\ \frac{\partial \overline{w_B^N}}{\partial \sigma} &= \left(\frac{\delta}{\delta+(1-\sigma)\lambda}\right)^2 \frac{\partial \overline{w_B^D}}{\partial \sigma} - \frac{2\delta^2\lambda}{[\delta+(1-\sigma)\lambda]^3}(P - \overline{w_B^D}). \end{aligned}$$

So,  $\frac{\partial \overline{w_B^N}}{\partial k}$  and  $\frac{\partial \overline{w_B^N}}{\partial d}$  have the same sign as  $\frac{\partial \overline{w_B^D}}{\partial k}$  and  $\frac{\partial \overline{w_B^D}}{\partial d}$ ; and,  $\frac{\partial \overline{w_B^N}}{\partial \sigma} < 0$  if  $\frac{\partial \overline{w_B^D}}{\partial \sigma} < \frac{2\lambda(P - \overline{w_B^D})}{\delta+(1-\sigma)\lambda}$ .

Next, prove property (4).

From (13) we get:  $u(\underline{w_B^D}) \leq u(b)$ . Thus,  $\underline{w_B^D} \leq b$  because of the increasing property of  $u(w)$ .

To prove the other side, let's assume  $\overline{w_B^D} \leq b$ . The integrated variable hence satisfies  $\underline{w_B^D} \leq x \leq \overline{w_B^D} \leq b$ . So we have:

$$\begin{aligned} \frac{\sqrt{P - \underline{w_B^D} - d}}{2} \int_{\underline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P-x-d}} &\leq \frac{\sqrt{P - \underline{w_B^D} - d}}{2} \int_{\underline{w_B^D}}^b \frac{u'(x)dx}{\sqrt{P-b-d}} \\ &= \frac{\sqrt{P - \underline{w_B^D} - d}}{2\sqrt{P-b-d}} [u(b) - u(\underline{w_B^D})] \end{aligned}$$

If  $b \leq \frac{3}{4}(P-d)$ , then  $\underline{w_B^D} > 4b - 3(p-d)$ . Thus,  $\frac{\sqrt{P - \underline{w_B^D} - d}}{2\sqrt{P-b-d}} < 1$

and  $\frac{\sqrt{P - \underline{w_B^D} - d}}{2} \int_{\underline{w_B^D}}^{\overline{w_B^D}} \frac{u'(x)dx}{\sqrt{P-x-d}} < u(b) - u(\underline{w_B^D})$  which violates equation (13).

Therefore, the assumption is false and we have proved  $\overline{w_B^D} > b$  if  $b \leq \frac{3}{4}(P-d)$ .

Besides, the wage bounds of worker  $A$  can be seen as a special case of worker  $B$ 's where  $d = 0$ ,  $k = 0$  and  $\sigma = 1$ . From properties (1)-(3), property (4) is easily derived, i.e.,  $\underline{w}_A < \underline{w}_B^D$  and  $\overline{w}_A > \overline{w}_B^D$ .

As for property (5), if  $k = 1$ , equation (26) is reduced to

$$Ew_B = \overline{w}_B^N + \frac{\delta}{(1-\sigma)\lambda}(\overline{w}_B^N - \underline{w}_B^N) - \frac{2\delta}{\delta + (1-\sigma k)\lambda}(P - \underline{w}_B^N)$$

where  $\underline{w}_B^N$  and  $\overline{w}_B^N$  satisfy:

$$u(\underline{w}_B^N) = u(b) - \frac{\sqrt{P - \underline{w}_B^N}}{2} \int_{\underline{w}_B^N}^{\overline{w}_B^N} \frac{u'(x)dx}{\sqrt{P-x}} \text{ and } \frac{P - \overline{w}_B^N}{P - \underline{w}_B^N} = \left( \frac{\delta}{\delta + (1-\sigma)\lambda} \right)^2.$$

The only difference in the system of equations compared with those for type  $A$  workers is the offer arrival rate, i.e.,  $\sigma = 0$  for type  $A$  while  $\sigma > 0$  for type  $B$ .

Let  $k \triangleq \frac{\delta}{\delta + (1-\sigma)\lambda}$ , after some algebra the mean wage could be rewritten as:

$$Ew = P - (k^3 + k^2 + k)(P - \underline{w})$$

From the system of equations about  $\underline{w}$  and  $\overline{w}$ , we can get:

$$\frac{\partial \underline{w}}{\partial k} = \frac{u'(\overline{w})(P - \underline{w})}{A + \frac{u'(\overline{w})k}{2}}$$

where  $A = \frac{u'(\underline{w})}{2} - \frac{1}{4\sqrt{P-\underline{w}}} \int_{\underline{w}}^{\overline{w}} \frac{u'(x)dx}{\sqrt{P-x}} > 0$ .

So,

$$\begin{aligned} \frac{\partial Ew}{\partial k} &= -(3k^2 + 2k + 1)(P - \underline{w}) + (k^3 + k^2 + k) \frac{\partial \underline{w}}{\partial k} \\ &= \frac{(P - \underline{w})}{A + \frac{u'(\overline{w})k}{2}} \left[ \frac{u'(\overline{w})(k - k^3)}{2} - A(3k^2 + 2k + 1) \right] < 0 \end{aligned}$$

where the last inequality holds due to:

$$\frac{u'(\overline{w})(k - k^3)}{2} - A(3k^2 + 2k + 1) < \frac{u'(\overline{w})(k - k^3)}{2} - \frac{u'(\underline{w})(3k^3 + 2k^2 + k)}{2} < 0$$

In addition, as  $k$  is increasing in  $\sigma$ , we get  $\frac{\partial Ew}{\partial \sigma} < 0$ . So the proposition is proved.

**A.5. PROOF OF PROPOSITION 4**

(1) and (2) can be directly derived from proposition 1 and proposition 2.  $\overline{w_B^N} < \overline{w_A}$  because

$$\begin{aligned} \overline{w_A} - \overline{w_B^N} = & (P - b) \left[ \left( \frac{\delta}{\delta + (1 - \sigma k)\lambda} \right)^2 - \left( \frac{\delta}{\lambda + \delta} \right)^2 \right] \\ & + d \left[ 1 - \left( \frac{\delta + (1 - \sigma)\lambda}{\delta + (1 - \sigma k)\lambda} \right)^2 \right] \left( \frac{\delta}{\delta + (1 - \sigma)\lambda} \right)^2 > 0 \end{aligned}$$

Next, consider the comparison of earning distributions.

Since

$$\begin{aligned} K_w^A &= \frac{\delta}{\lambda} \left[ \sqrt{\frac{P - b}{P - w}} - 1 \right], \\ K_w^B &= \begin{cases} \frac{\delta}{(1 - \sigma k)\lambda} \left[ \sqrt{\frac{P - b - d}{P - w - d}} - 1 \right], & \text{if } w \in [\underline{w_B^D}, \overline{w_B^D}] \\ \frac{\delta}{(1 - \sigma k)\lambda} \left[ \frac{\delta + (1 - \sigma k)\lambda}{\delta + (1 - \sigma)\lambda} \sqrt{\frac{P - \underline{w_B^N}}{P - w}} - 1 \right], & \text{if } w \in [\underline{w_B^N}, \overline{w_B^N}] \end{cases} \end{aligned}$$

and

$$\begin{aligned} \frac{\delta}{\lambda} &< \frac{\delta}{(1 - \sigma k)\lambda}, \\ \sqrt{\frac{P - b}{P - w}} &< \sqrt{\frac{P - b - d}{P - w - d}}, \\ \sqrt{\frac{P - b}{P - w}} &< \frac{\delta + (1 - \sigma k)\lambda}{\delta + (1 - \sigma)\lambda} \sqrt{\frac{P - \underline{w_B^N}}{P - w}} \end{aligned}$$

we can get  $K_w^A < K_w^B$  for all  $w$ , i.e.,  $A$ 's earnings distribution first-order stochastically dominates  $B$ 's earnings distribution. Therefore,  $Ew_A > Ew_B$  and  $w_A^q > w_B^q$ .

Through tedious calibration, we can get the comparative statics of  $Ew_A - Ew_B$ :

$$\frac{\partial [Ew_A - Ew_B]}{\partial (\sigma, k, d)} > 0.$$

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