Foreign Aid, Public Spending, Optimal Fiscal and Monetary Policies, and Long-Run Growth

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March, 2008

Abstract

This paper presents a group of models showing the strikingly different implications of foreign aid to the private sector and public sector. In the first model, with decentralized decision-making and without optimal choices of fiscal policies on behalf of the government, foreign aid to the private sector has no effect on the long-run capital accumulation and it raises private consumption one to one; whereas foreign aid to the government leads to more public spending and higher private capital accumulation. In another model with optimal choices of both fiscal and monetary policies, foreign aid to the private sector gives rise to higher inflation and income taxation. Although aid to the private sector raises private money holdings and consumption, it reduces capital accumulation. However, when foreign aid is provided to the public sector, the government cuts both the inflation rate and the income tax rate, raises public spending, and provides more incentives for private capital accumulation and money holdings. In the long run, aid to the public sector leads to more private capital accumulation, consumption, money holdings, and welfare.

Journal of Economic Literature Classification Numbers: E2, F34, F35, O1, O4.
Key Words: Foreign aid; Capital accumulation; Income taxation; Inflation; Growth.

1We thank the editor and two anonymous referees for very constructive suggestions on revising this paper. Of course, all remaining errors are ours.
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1 Introduction

Foreign aid to developing countries can be classified into two categories: aid to the public sector and aid to the private sector. Aid to the public sector takes either the form of budget support or project aid. With budget support, money is given directly to a recipient government from donors as development assistance. With project aid, funds are used by donors to implement a specific project. Both forms are used to enhance the accumulation of public capital. Aid to the private sector typically takes the form of emergency aid and private sector development assistance. Emergency aid (including food aid) is given to countries in urgent need of food supplies, clothing, especially in a natural disaster. In the U.S., private sector development assistance includes establishing enterprise funds and providing technical assistance. They are designed to promote the expansion of the private sector in developing countries. Thus, aid to the private sector either increases consumption or enhances the accumulation of private capital.

In this paper, we ask a natural question: does foreign aid to the public and private sector makes a difference for long-run investment and growth? And second, what is the effects on fiscal and monetary polices and government spending when foreign aid goes to the government and the private sector, respectively? These questions have been hotly debated in policy discussions; however, to the best of our knowledge, we are the first to submit these questions under theoretical scrutiny.

Our results show a few strikingly different implications of foreign aid to the private sector and public sector. 1) In the first model with decentralized decision-making and without optimal choices of fiscal policies on behalf of the government, foreign aid to the private sector has no effect on the long-run capital accumulation and it raises private consumption one to one; but foreign aid to the government leads to more public spending and higher private capital accumulation. 2) In the second model with the optimal choices of fiscal policy on behalf of the government, foreign aid to the private sector reduces private capital accumulation and leads the government to levy a higher income tax and raise public spending. Rising foreign aid to the government, however, results in a lower income tax, a higher level of public spending, and more private capital accumulation and consumption. 3) In the third model with the optimal choices of both fiscal and monetary policy, foreign aid to the private sector gives rise to higher inflation and income taxation. At the same time, this kind of aid raises private money holdings and consumption, but it reduces capital accumulation. When foreign aid is provided to the public sector, the government cuts both the inflation rate and the income tax rate, raises public spending, and provides more incentives for private capital accumulation and money holdings. In the long run, this kind of aid leads to more private capital accumulation, consumption, money holdings, and welfare.

Our paper sends important messages to researchers doing empirical researches on foreign aid and economic growth. First, any serious empirical tests on the effects of foreign aid should explicitly specify an analytical framework and differentiate foreign aid to the private sector from foreign aid to the public sector. As shown in our theoretical exercises, these two kinds of aid may have very different effects on savings, investment,
and economic growth in developing countries. Second, the choices of fiscal and monetary policy may depend on, or even be supported by, foreign aid, and further empirical work in line with Burnside and Dollar (2000), Barro and Lee (2005), Easterly, Levine, and Roodman (2003), and Wane (2005) should deal with the effects of aid on policies and economic growth simultaneously. That is to say, we cannot simply control policy variables when examining the effects of aid on economic growth. World Bank (1998, p.47) puts it nicely: aid can be the midwife of good policies.


The paper is organized as follows. Section 2 sets up an intertemporal growth model with two kinds of foreign aid — aid to the government and aid to the private sector. Given an exogenous income tax rate, it examines how private consumption, private investment, and public spending react to these two kinds of foreign aid. Section 3 studies the optimal responses of fiscal policies (public spending and income taxation) to foreign aid. Section 4 deals with the optimal inflation rate, optimal government spending, and optimal income tax. Section 5 concludes.

2 Aid, Government Spending, and Capital Formation

We consider a traditional Ramsey-Cass-Koopmans model with government expenditure and foreign aid. Given the government’s behavior, the representative agent is postulated to choose his private consumption path, $c$, and capital accumulation path, $k$, to maximize his discounted utility, namely

$$\max \int_0^\infty u(c, g)e^{-\beta t}dt$$

s.t. \[ \frac{dk}{dt} = (1-\tau)f(k, g) - c + a_1, \quad \text{given } k(0) = k_0, \] (1)

where $\beta > 0$ is the time discounted rate, $\tau$ is the flat income tax rate, $f(k, g)$ is the output, $a_1$ is the foreign aid to the representative agent or the private sector, and $u(c, g)$ is the instantaneous utility function, which is defined on private consumption $c$ and government expenditure $g$. These kinds of utility functions and production functions are
introduced by Arrow and Kurz (1970) and used recently by Barro (1990) and Turnovsky (2000), among many others. And we suppose that the agent derives positive, but diminishing marginal utility from consumption and government expenditure, namely

\[ u_c > 0, u_g > 0, u_{cc} < 0, u_{cg} > 0, u_{gg} < 0. \]  (2)

It is further assumed that the production function, \( f(k, g) \), is increasing, concave, and twice differentiable in its two inputs: private capital stock and government spending:

\[ f_k > 0, f_g > 0, f_{kk} < 0, f_{gg} < 0. \]  (3)

For simplicity, there is no capital depreciation in the model.

Define the Hamiltonian associated with the representative agent’s optimization problem as

\[ H = u(c, g) + \lambda[(1 - \tau)f(k, g) - c + a_1], \]

where \( \lambda \) is the costate variable associated with the budget constraint (1), and it represents the marginal utility of private capital accumulation.

The first-order conditions for optimization are:

\[ u_c = \lambda, \]  \hspace{1cm} (4)

\[ \frac{d\lambda}{dt} = \beta\lambda - \lambda(1 - \tau)\frac{\partial f(k, g)}{\partial k}, \]  \hspace{1cm} (5)

and the dynamic equation of capital accumulation in (1) plus the transversality condition

\[ \lim_{t \to \infty} \lambda ke^{-\beta t} = 0. \]  (6)

Equation (4) is the familiar condition, which means that for the consumer to be in equilibrium, the marginal utility of consumption must equal the marginal utility of private wealth. Equation (5) is the familiar Euler equation describing the motion of the marginal utility of private wealth.

From equation (4), we can express short-run consumption \( c \) as a function of marginal utility of wealth \( \lambda \) and government expenditure \( g \), i.e., \( c(\lambda, g) \). And we have

\[ \frac{dc}{d\lambda} = \frac{1}{u_{cc}} < 0, \frac{dc}{dg} = -\frac{u_{cg}}{u_{cc}} > 0. \]  (7)

If the utility function is separate in private consumption and government expenditure, then, we have

\[ \frac{dc}{d\lambda} = \frac{1}{u_{cc}} < 0, \frac{dc}{dg} = 0. \]  (8)

Substituting private consumption function into the utility function, we get the discounted private welfare as \( \int_0^\infty u(c(\lambda, g), g)e^{-\beta t}dt \).
The government collects the income tax \( \tau f(k, g) \) from the private agent and receives an amount of foreign aid \( a_2 \). Assuming a balanced budget at any time \( t \), its budget constraint can be written as
\[
g = \tau f(k, g) + a_2. \tag{9}
\]

First, we discuss the full general equilibrium with a fixed rate of income taxation. The macroeconomic equilibrium is described by equations (1), (4), (5), (9), and (6), from which, we can determine the equilibrium private consumption, capital accumulation, government expenditure and the marginal utility of wealth. Given the rate of income taxation and foreign aid to the private sector and the government, the dynamic system is saddle-point stable in terms of the variables of private consumption and capital stock. Hence there exists a unique perfect-foresight equilibrium.

In the steady state, we have the following equations determining the equilibrium values of private consumption, capital accumulation, and government spending:
\[
\begin{align*}
\beta - (1 - \tau) \frac{\partial f(k, g)}{\partial k} &= 0, \quad (10) \\
g - \tau f(k, g) - a_2 &= 0, \quad (11) \\
(1 - \tau)f(k, g) - c + a_1 &= 0. \quad (12)
\end{align*}
\]

The following proposition can be derived from equations (10)-(12):

**Proposition 1** In a decentralized economy without the optimal choices of government fiscal policies (including government spending and the income tax rate), foreign aid to the private sector has no effects on long-run capital accumulation and government spending, it increases private consumption one to one, whereas foreign aid to the government increases government spending, private capital stock, and private consumption.

Proof: Taking total differentiation of equations (10)-(12), we have
\[
\begin{pmatrix}
-(1 - \tau) \frac{\partial^2 f(k, g)}{\partial k^2} & 0 & -(1 - \tau) \frac{\partial^2 f(k, g)}{\partial k \partial g} \\
-\tau \frac{\partial f(k, g)}{\partial k} & 0 & 1 - \tau \frac{\partial f(k, g)}{\partial g} \\
\beta & -1 & (1 - \tau) \frac{\partial f(k, g)}{\partial g}
\end{pmatrix}
\begin{pmatrix}
dk \\
dc \\
dg
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
-1
\end{pmatrix}
da_1 + \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
da_2.
\]

Therefore,
\[
\begin{align*}
\frac{dk}{da_1} &= \frac{\Delta_k}{\Delta} a_1 = 0, \quad \frac{dc}{da_1} = \frac{\Delta_c}{\Delta} a_1 = 1, \quad \frac{dg}{da_1} = \frac{\Delta_g}{\Delta} a_1 = 0, \\
\frac{dk}{da_2} &= \frac{\Delta_k}{\Delta} a_2 > 0, \quad \frac{dc}{da_2} = \frac{\Delta_c}{\Delta} a_2 > 0, \quad \frac{dg}{da_2} = \frac{\Delta_g}{\Delta} a_2 > 0,
\end{align*}
\]

where
\[
\Delta = -(1 - \tau)\left[ \frac{\partial^2 f(k, g)}{\partial k^2} (1 - \tau) \frac{\partial f(k, g)}{\partial g} + \frac{\partial^2 f(k, g)}{\partial k \partial g} \tau \frac{\partial f(k, g)}{\partial k} \right],
\]
\[
\Delta_{a_1} = 0, \Delta_{a_1} = \Delta, \Delta_{a_1} = 0,
\]

\[
\Delta_{a_2} = (1 - \tau) \frac{\partial^2 f(k, g)}{\partial k \partial g} \frac{\partial f(k, g)}{\partial k} > 0,
\]

\[
\Delta_{c_2} = -(1 - \tau) \left[ \frac{\partial^2 f(k, g)}{\partial k^2} (1 - \tau) \frac{\partial f(k, g)}{\partial g} - \frac{\partial^2 f(k, g)}{\partial k \partial g} \beta \right] > 0,
\]

\[
\Delta_{g_2} = -(1 - \tau) \frac{\partial^2 f(k, g)}{\partial k^2} > 0.
\]

\textit{Q.E.D.}

We provide the economic intuition as follows. For the decentralized economy in this section, foreign aid to the private sector increases private income and stimulates private consumption and short-run investment as found by Obstfeld (1999). But in the long run, the optimal private capital stock is determined by the modified golden rule of equation (10), which is not related to foreign aid to the private sector. Hence, foreign aid to the private sector increases private consumption one to one in the long run. Similar results have been obtained by Obstfeld (1999) and Gong and Zou (2000, 2001) in different theoretical settings with only one type of foreign aid: aid to the private sector. As for foreign aid to the government, given the income tax rate, it results in more government spending, which in turn increases the marginal productivity of private capital. That leads to more private investment and capital accumulation in the long run. With more private capital stock and government spending, there will be more private output and consumption in the long run as well.

Proposition 1 suggests that, ceteris paribus, aid to the government is more effective in raising long-run private capital accumulation than the aid to the private sector. This theoretical result seems counter to what the World Bank, the International Monetary Fund, and many other development agencies around the world are doing — namely, targeting aid to the private sector.

Our theoretical model indicates that, when examining the effects of foreign aid on developing economies, it is important to differentiate foreign aid to the public sector from the one to the private sector in empirical studies. We all know that the effects of foreign aid on investment and growth has been a controversial topic since the 1960s. In a series of papers by Hollis Chenery and his associates, they have found that, on the basis of the Harrod-Domar model and realistic parameters on different developing countries, foreign aid and foreign capital inflows can accelerate investment and speed up the transition to a targeted self-sustained growth path.\(^3\) Since the 1970s, the critics of foreign aid have argued that external resource inflows may mainly increase consumption, depress domestic savings, and slow down investment and output growth.\(^4\) The controversy seems


to continue mainly on the empirical side.\footnote{For conflicting empirical findings on the impact of external finance on savings, investment and output growth, see Rahman (1968), Papanek (1972, 1973), Fry (1978, 1980), Levy (1987, 1988a,b), and Giovannini (1983, 1985), among many others.} But more recently, Boone (1996) finds that foreign aid has hardly any effect on investment; in particular, foreign aid mainly serves to augment the consumption of those who are relatively well-off in developing countries. Barro and Lee (2005) have found that a higher IMF loan-participation rate reduces economic growth and IMF lending also lowers investment. Even if foreign aid is tied to specific sectors and purposes, Feyzioglu, Swaroop, and Zhu (1998) and van de Walle and Cratty (2005) have found that most of foreign aid appears to be fungible, and many developing countries have diverted foreign aid to public consumption.\footnote{see also Pack and Pack (1990, 1993), Svensson (2003), and Wane (2005).} These recent empirical findings naturally suggest that foreign aid has very little positive impact on capital formation and output growth in developing countries. Whereas our theoretical prediction regarding the effect of foreign aid to the private sector supports these recent findings, our theoretical prediction regarding the effect of foreign aid to the public sector indicates just the opposite. It is quite possible that these two effects offset each other when we take foreign aid as an aggregate variable (including aid to both the public and private sectors) in various empirical studies.

3 Aid and Optimal Fiscal Policy

As the empirical work by Burnside and Dollar (2000) has shown, aid has a positive impact on growth in developing countries when accompanied by good fiscal, monetary, and trade policies but has little effect in the presence of poor policies. In their regression analysis, they obtain a significant positive effect of aid on economic growth when controlling budget surplus, inflation, and openness. This empirical finding illustrates the usefulness of our model setup with both aid to the government and aid to the private sector, because sound economic policies adopted by the government are closely associated with, and supported (even conditioned) by, the aid to the government. Thus, it is natural to ask: How do economic policies react to foreign aid? We take up optimal fiscal policy in this section and the optimal inflation rate in the context of public finance in section 4. The analytical tool is a rather standard extension of the intertemporal second-best approach to government policies exemplified in the work by Turnovsky and Brock (1980), Brock and Turnovsky (1981), Chamley (1985a,b, 1986), and Turnovsky (2000), among many others.

3.1 The Analytical Model

Following the intertemporal or dynamic second-best approach, the government is the leader in the Stackleberg game and the private agent is the follower. Therefore, the government chooses private consumption (which is a function of government spending and the marginal utility of private wealth in equation (7)), private capital stock,
public spending, and income taxation subject to the first-order conditions (in particular, the Euler equation) for private agent’s intertemporal optimization, private budget constraint, and government budget constraint. Technically, given the decisions of the private agent and government’s budget constraint (9), the government will choose all endogenous variables in the system (especially public expenditure and the income tax rate) to maximize social welfare

$$\max \int_0^\infty [u(c(\lambda, g), g) e^{-\beta t}] dt$$

s.t.  
$$\frac{dk}{dt} = (1 - \tau) f(k, g) - c(\lambda, g) + a_1,$$  
$$\frac{d\lambda}{dt} = \beta \lambda - \lambda [(1 - \tau) \frac{\partial f(k, g)}{\partial k}],$$

with the budget constraint (9) and the initial private capital stock given by $k(0) = k_0$ and the two kinds of foreign aid given by $a_1$ and $a_2$, respectively.

Associated with the government’s optimization problem, we define the Hamiltonian

$$H = u(c(\lambda, g), g) + \xi [(1 - \tau) f(k, g) - c(\lambda, g) + a_1] + \eta(\beta \lambda - \lambda [(1 - \tau) \frac{\partial f(k, g)}{\partial k}]) + \mu(\tau f(k, g) + a_2 - g),$$

where $\xi$ is the costate variable associated with the constraint (13) and represents the social marginal utility of private capital stock (or private wealth), $\eta$ is the costate variable with the constraint (14) and represents the social marginal utility of the marginal utility of private wealth, and $\mu$ is the Lagrangian multiplier associated with the government’s budget constraint.

The first-order conditions for government optimization are:

$$u_g + u_c c_g + \xi((1 - \tau) f_g(k, g) - c_g) - \eta \lambda (1 - \tau) \frac{\partial f(k, g)}{\partial k} = \mu,$$  
$$\eta \lambda \frac{\partial f(k, g)}{\partial k} + \mu f(k, g) = 0,$$

$$\frac{d\xi}{dt} = \beta \xi - \xi (1 - \tau) \frac{\partial f(k, g)}{\partial k} + \eta(1 - \tau) \frac{\partial^2 f(k, g)}{\partial k \partial g} - \mu \frac{\partial f(k, g)}{\partial k},$$

$$\frac{d\eta}{dt} = \beta \eta - u_c c_\lambda + \xi c_\lambda - \eta (\beta - (1 - \tau) \frac{\partial f(k, g)}{\partial k}),$$

$$\frac{dk}{dt} = f(k, g) - g - c(\lambda, g) + a_1 + a_2,$$

$$\frac{d\lambda}{dt} = \beta \lambda - \lambda [(1 - \tau) \frac{\partial f(k, g)}{\partial k}].$$
\[ g = \tau f(k, g) + a_2, \] (9)

and the transversality conditions
\[ \lim_{t \to \infty} \xi ke^{-\beta t} = 0, \quad \lim_{t \to \infty} \eta \lambda e^{-\beta t} = 0. \] (19)

From equations (9), (15), and (16), we can express government tax rate, \( \tau \), the multiplier, \( \mu \), and government expenditure, \( g \), as functions of \( k, \xi, \eta, \lambda, \) and \( a_2 \), namely,
\[ \tau = \tau(k, \xi, \eta, \lambda, a_2), \] (20a)
\[ \mu = \mu(k, \xi, \eta, \lambda, a_2), \] (20b)
\[ g = g(k, \xi, \eta, \lambda, a_2). \] (20c)

Substituting equations (20) into equations (17), (18), (13), and (14), we get the full dynamic system of the economy,
\[
\frac{d\xi}{dt} = \beta \xi - \xi(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial f(k, g)}{\partial k} + \eta \lambda(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial^2 f(k, g)}{\partial k^2} - \mu(k, \xi, \eta, \lambda, a_2) \tau(k, \xi, \eta, \lambda, a_2) \frac{\partial f(k, g)}{\partial k},
\] (17')
\[
\frac{d\eta}{dt} = \beta \eta - u_c c + \xi c - \eta(\beta - [(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial f(k, g)}{\partial k}]),
\] (18')
\[
\frac{dk}{dt} = (1 - \tau(k, \xi, \eta, \lambda, a_2)) f(k) - c(\lambda) + a_1,
\] (13')
\[
\frac{d\lambda}{dt} = \beta \lambda - \lambda[(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial f(k, g)}{\partial k}].
\] (14')

Equations (13'), (14'), (17'), and (18') give the full dynamics of the economy, from which we can determine the dynamic properties of the capital stock and the three multipliers. And from equation (20), we can determine the dynamic properties of the income tax rate, government expenditure, and private consumption level.

The steady state \((k, \xi, \eta, \lambda)\) reaches, when \( \frac{d\xi}{dt} = \frac{d\eta}{dt} = \frac{dk}{dt} = \frac{d\lambda}{dt} = 0 \). That is to say,
\[
\eta \lambda(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial^2 f(k, g)}{\partial k^2} - \mu \tau(k, \xi, \eta, \lambda, a_2) \frac{\partial f(k, g)}{\partial k} = 0,
\] (21)
\[
\beta \eta - uc \lambda + \xi c - \eta(\beta - [(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial f(k, g)}{\partial k}] = 0,
\] (22)
\[
(1 - \tau(k, \xi, \eta, \lambda, a_2)) f(k) - c(\lambda) + a_1 = 0,
\] (23)
\[
(1 - \tau(k, \xi, \eta, \lambda, a_2)) \frac{\partial f(k, g)}{\partial k} = \beta.
\] (24)

From these equations, we can determine the steady-state \(k, \lambda, \xi, \) and \( \eta\). Then, from the short-run equilibrium (20), we can determine the steady-state \( c, g, \) and \( \tau\). In this
paper, we will not discuss the existence and stability of the steady state in its general form. What we are interested in is the effects of foreign aid on the optimal fiscal policies and other endogenous variables, so we focus on the comparative-dynamics solutions. It goes without saying that we have assumed the existence of at least one equilibrium, which is also locally unique.

In principle, if we take total differentiation of equations (21)-(24), we can derive the effects of foreign aid on the steady-state capital stock and the three multipliers; and from equation (20), we can derive the effects of foreign aid on the steady-state consumption level, tax rate, and government expenditure. The general results are rather complicated and we will instead deal with two explicit examples and explore the implications of the two kinds of foreign aid for policy choices and endogenous variables.

3.2 Comparative Dynamics for Two Special Cases

Example 1  Suppose that the production function and the utility function are
\[
u(c, g) = \ln c + \omega_2 \ln g, \quad f(k) = A k^\phi,
\]
where \(\omega_2\), \(A\), and \(\phi\) are positive constants. In this case, public spending only generates private utility, and it has no effect on private production.

From equations (20) and (21)-(24), the steady-state capital stock, government expenditure, private consumption, and the tax rate can be derived from the following conditions
\[
k\phi\left(A\phi k^{\phi - 1} - \beta\right) + a_2 - \omega_2 \frac{A\phi k^{\phi - 1} - \beta}{\beta(\phi - 1)} a_1 \phi - \omega_2 \left(\frac{\beta}{\phi} k + a_1\right) = 0, \quad (25)
\]
\[
k\phi\left(A\phi k^{\phi - 1} - \beta\right) + a_2 = g, \quad (26)
\]
\[-\omega_2 \frac{A\phi k^{\phi - 1} - \beta}{(\beta k + a_1)\phi(\phi - 1)} \frac{k}{\phi} (A\phi k^{\phi - 1} - \beta) + a_2 = \eta, \quad (27)
\]
\[
\frac{\beta}{\phi} k - c + a_1 = 0, \quad (28)
\]
\[
1 - \frac{\beta}{A\phi k^{\phi - 1}} = \tau. \quad (29)
\]

From equation (25), we can determine the equilibrium capital stock, and from equations (26), (28), and (29), we can determine the equilibrium government expenditure, private consumption level, and the income tax rate. Then we have

Proposition 2 In this special example, foreign aid to the private sector reduces long-run capital accumulation, whereas it raises consumption, the income tax rate, and government spending. At the same time, foreign aid to the government reduces the tax rate, raises private capital stock, private consumption, and government spending.
More explicitly, the comparative dynamics are given as follows:

\[
\frac{dk}{da_2} = \frac{\beta \phi}{-(1 + \omega_2)\beta^2 + A\beta \phi k^{\phi-1} - Aa_1 k^{\phi-2}}; \quad (30a)
\]

\[
\frac{dk}{da_1} = \frac{\phi(\beta - k^{\phi-1})}{(\phi - 1)(1 + \omega_2)\beta^2 - A\beta \phi k^{\phi-1} + Aa_1 k^{\phi-2}}; \quad (30b)
\]

\[
\frac{d\tau}{da_2} = -\frac{\beta}{\phi A} (1 - \phi) k^{-\phi} \frac{dk}{da_2}, \quad (30c)
\]

\[
\frac{d\tau}{da_1} = -\frac{\beta}{\phi A} (1 - \phi) k^{-\phi} \frac{dk}{da_1}, \quad (30d)
\]

\[
\frac{dc}{da_2} = \frac{\beta}{\phi A} \frac{dk}{da_2}, \quad \frac{dc}{da_1} = \frac{\beta}{\phi A} \frac{dk}{da_1} + 1; \quad (30e)
\]

\[
\frac{dg}{da_2} = \phi A k^{\phi-1} \frac{dk}{da_2} + 1, \quad \frac{dg}{da_1} = \phi A k^{\phi-1} \frac{dk}{da_1}. \quad (30f)
\]

To gain some intuitive sense, we let \(\phi = 0.5, \beta = 0.05,\) and \(a_1 = 1,\) and we let the value of foreign aid to the government sector, \(a_2,\) vary from 0 to 10. We get the effects of foreign aid to the public sector on private capital stock, private consumption, government expenditure, and the income tax as shown in figure 1.

Figure 1 shows that with the increase in the foreign aid to the government, the steady-state capital stock, consumption, government expenditure, and output increase, and the income tax rate decreases. The economic intuition is obvious. With the intertemporal optimal choice, the government chooses to lower the income taxation rate after receiving more foreign aid. A lower income tax raises the incentive for private savings and investment. Therefore, more output is produced. In the long run, the private agent saves more, produces more, and consumes more.

Next, we take \(\phi = 0.5, \beta = 0.05,\) and fix foreign aid to the government at \(a_2 = 1.\)

If we let the value of foreign aid to the private sector vary from 0 to 10, we obtain the effects of foreign aid to the private sector on the capital stock, consumption, government expenditure, and income taxation as shown in figure 2.

Figure 2 shows that with the increase in the foreign aid to the private sector, the steady-state capital stock and output decrease, but private consumption, government expenditure, and the income tax rate increase. The economic intuition for the results in figure 2 is more complicated than the one in section 2 where the government does not make optimal intertemporal choices regarding public spending and income taxation. In this case, the government will raise its output tax rate and collect more revenues from the private agent when the latter receives more foreign aid. A higher tax rate is justified because government spending enters the private agent’s utility function. It is in the private agent’s interest to spend more on public goods as a result of the rise in private income (more foreign aid) and a corresponding rise in private consumption, and the only way for the government to implement this optimal choice is a higher tax on output in the model. The rise in output taxation reduces the incentive to save and invest by the private agent, and, therefore, the long-run capital stock and output are reduced.
The obvious shortcoming of example 1 is the absence of government spending in the production function. But its strength lies in the significant difference in the effects of foreign aid to the public and private sector on capital accumulation and growth even from a viewpoint of pure consumption of private goods and public goods. When government spending also enters the production function in a more general setting, the results in example 1 are reinforced as illustrated in the following example.

**Example 2** Suppose the utility function and the production function are

\[
\begin{align*}
    u(c, g) &= \ln c + \ln g, \\
    f(k, g) &= A k^\phi g^{1-\phi},
\end{align*}
\]

where \( \phi \in (0, 1) \) is a positive constant. The production function here allows the role of public spending in enhancing the productivity of private capital.

Now, the steady-state capital stock, consumption, taxation, and government expenditure satisfy the following four simultaneous equations

\[
\begin{align*}
    -\beta(\phi - 1)(1 - A(1 - \phi)(\frac{k}{g})^\phi) + (\frac{\beta k}{\phi} + (1 - A(\frac{k}{g})^\phi + \frac{\beta k}{\phi} g \frac{a_1}{a_2})\beta(\phi - 1) \\
    + (1 - A(\frac{k}{g})^\phi + \frac{\beta k}{\phi} g \frac{a_1}{a_2})\phi A\phi(\frac{k}{g})^\phi - (1 - A(\frac{k}{g})^\phi + \frac{\beta k}{\phi} g \frac{a_1}{a_2})\phi = 0,
\end{align*}
\]

where \( a_2 = 1 - A(\frac{k}{g})^\phi + \frac{\beta k}{\phi} g \frac{a_1}{a_2}, \)

\[
\begin{align*}
    c = \frac{\beta k}{\phi} + a_1, \\
    \tau = 1 - \frac{\beta}{A\phi k^{\phi - 1}g^{1-\phi}}.
\end{align*}
\]

For a few selected values of parameters, we have the effects of foreign aid to the private and public sector, \( a_1 \) and \( a_2 \), on the steady-state capital stock, consumption, tax rate, and public spending as shown in figures 3 and 4, respectively.

In figure 3, as foreign aid to the government directly increases government revenues, government expenditure rises accordingly. This will lead to a rising marginal productivity of private capital, and hence, more private output and consumption for a given output tax. At the same time, with more aid to the government, the social-welfare-maximizing government lowers its tax rate on private production, and creates further incentive for private savings and investment.

On the other hand, figure 4 indicates that, with more foreign aid to the private sector, the government will put more tax on private production and raise public spending. Even though more public spending improves the marginal productivity of private capital, the
private agent can afford to save less and consume more with more foreign aid. Of course, a higher tax also reduces the incentive for the private agent to invest. Therefore, in the long run, the economy ends up with less private capital accumulation and less output production as a result of a higher level of foreign aid to the private sector.

4 Aid and Optimal Monetary Policy with Inflation Finance

In this section, we turn our attention to monetary policy in the familiar framework of Brock and Turnovsky (1981) and Chamley (1985b). This model defines the private agent’s utility function on consumption, real balances, and government spending. Government spending is financed by the output tax, the inflation tax, and foreign aid. First, we deal with the private agent’s optimization in the new setting. Given the government’s behavior (public spending, output taxation, the rate of monetary growth), the private agent chooses the consumption path and capital-accumulation path to maximize his discounted utility, namely

$$\max \int_0^{\infty} u(c, m, g) e^{-\beta t} dt$$

subject to

$$\frac{dk}{dt} + \frac{dm}{dt} = (1 - \tau)f(k, g) - c + a_1 - \pi m,$$

where $m$ is the real balances, $\pi$ is the expected inflation rate, and $a_1$ is still the foreign aid to the private agent. The initial capital stock is given by $k(0) = k_0$. It is further assumed that

$$u_c > 0, u_m > 0, u_g > 0, u_{cc} < 0, u_{mm} < 0, u_{gg} < 0.$$  \hspace{1cm} (33)

We define the Hamiltonian associated with the private agent’s optimization problem

$$H = u(c, m, g) + \lambda[(1 - \tau)f(k, g) - c + a_1 - \pi m] + \mu(A - k - m),$$

where $\lambda$ is the costate variable and $\mu$ is the multiplier associated with the wealth constraint:

$$A = k + m,$$

where $A$ is the sum of total private wealth in terms of the capital stock and real balances.

Now, we have the first-order conditions for the private agent’s optimization:

$$u_c = \lambda,$$

$$\mu = \lambda(1 - \tau) \frac{\partial f(k, g)}{\partial k},$$

where $A$ is the sum of total private wealth in terms of the capital stock and real balances.
\[ u_m = \lambda \pi + \mu, \quad (37) \]
\[ \frac{d\lambda}{dt} = \beta \lambda - \mu, \quad (38) \]
and the transversality condition
\[ \lim_{t \to \infty} \lambda A e^{-\beta t} = 0. \quad (39) \]

Equation (35) states that marginal utility of consumption equals the marginal value of wealth; From equations (36) and (37), we have
\[ u_m = u_c(\pi + (1 - \tau) \frac{\partial f(k, g)}{\partial k}), \quad (40) \]
which says that the marginal utility of money holdings equals the marginal cost of money holdings \((\pi + (1 - \tau) \frac{\partial f(k, g)}{\partial k})\) weighted by the marginal utility of consumption. For a separable utility function, from equations (35), (36), and (38), we have
\[ dc = \frac{c}{\theta}((1 - \tau) \frac{\partial f(k, g)}{\partial k} - \beta), \quad (41) \]
where \(\theta\) is the intertemporal elasticity of elasticity in consumption.

From equation (40), we can express \(m\) as a function of \(c, \tau, k, \pi, g\), i.e.,
\[ m = m(c, \tau, k, \pi, g), \quad (42) \]
and the comparative-statics analysis yields:
\[ \frac{dm}{dc} = \frac{u_{cc}(\pi + (1 - \tau) \frac{\partial f(k, g)}{\partial k})}{u_{mm}} > 0, \quad (43a) \]
\[ \frac{dm}{d\pi} = \frac{u_c}{u_{mm}} < 0, \quad (43b) \]
\[ \frac{dm}{d\tau} = -\frac{u_c f'(k)}{u_{mm}} > 0, \quad (43c) \]
\[ \frac{dm}{dk} = \frac{u_c(1 - \tau) \frac{\partial^2 f(k, g)}{\partial k^2}}{u_{mm}} > 0, \quad (43d) \]
\[ \frac{dm}{dg} = \frac{u_c(1 - \tau) \frac{\partial^2 f(k, g)}{\partial k \partial g}}{u_{mm}} < 0, \quad (43e) \]
which are the short-run effects of \(c, \tau, k, \) and \(\pi\) on money holdings.

To fully spell out the dynamics, we need to specify the government sector. Government revenues come from money creation, foreign aid, \(a_2\), and output taxation. The government spends on public goods, \(g\). With a balanced budget at any point of time, the government has the following budget constraint:
\[ g = \frac{dM}{dt}/p + \tau f(k, g) + a_2, \quad (44) \]

or
\[ g = (\frac{dM}{dt}/M)(M/p) + \tau f(k, g) + a_2. \quad (45) \]

Let the money growth rate be a positive constant \( \theta \):
\[ \frac{dM}{dt}/M = \theta. \]

By definition, \( m = M/p \), we have
\[ \frac{dm}{dt} = (\theta - \pi)m. \]

Then, we can write equation (45) as
\[ g = \frac{dm}{dt} + \pi m + \tau f(k, g) + a_2. \quad (46) \]

Also, by definition, \( m = M/p = m(c, \tau, k, \pi) \), we have
\[ \frac{dm}{dt} = m_c \frac{dc}{dt} + m_{\tau} \frac{d\tau}{dt} + m_k \frac{dk}{dt} + m_{\pi} \frac{d\pi}{dt} + m_g \frac{dg}{dt}. \quad (47) \]

After substituting the private agent’s consumption function into his utility function, we get the social welfare function
\[ \int_0^\infty u(c, m(c, \tau, k, \pi), g) e^{-\beta t} dt. \]

### 4.1 The Steady-State Second-Best Approach

Following Brock and Turnovsky (1981), we adopt the steady-state second-best approach. Under the private sector’s steady-state conditions and the government’s steady-state budget constraint, the government selects the equilibrium consumption, money holdings, government expenditure, output taxation, and inflation rate to

\[
\begin{align*}
\max & \quad u(c, m, g) \\
\text{s.t.} & \quad f(k, g) - c - g + a_1 + a_2 = 0, \\
& \quad (1 - \tau) \frac{\partial f(k, g)}{\partial k} - \beta = 0, \\
& \quad g = \pi m + \tau f(k, g) + a_2;
\end{align*}
\]

and
\[ u_m = u_c((1 - \tau) \frac{\partial f(k, g)}{\partial k} + \pi), \quad (40) \]

where equations (49), (50), and (51) are the steady-state conditions from \( \frac{dk}{dt} = \frac{dc}{dt} = \frac{dm}{dt} = 0 \), and equation (40) is the private optimal condition.
Define the Lagrangian function associated with the above steady-state second-best problem as follows:

\[
\mathcal{L} = u(c, m, g) + \lambda_1[f(k, g) - c - g + a_1 + a_2] + \lambda_2[(1 - \tau)\frac{\partial f(k, g)}{\partial k} - \beta] + \lambda_3[\pi m + \tau f(k, g) + a_2 - g] + \lambda_4[u_c((1 - \tau)\frac{\partial f(k, g)}{\partial k} + \pi) - u_m],
\]

where \(\lambda_1, \lambda_2, \lambda_3, \lambda_4\) are the Lagrangian multipliers associated with the constraints (48), (49), (50), and (40).

The first-order conditions for the government’s optimization are:

\[
\frac{\partial \mathcal{L}}{\partial \tau} = -\lambda_2 \frac{\partial f(k, g)}{\partial k} + \lambda_3 f(k, g) - \lambda_4 u_c \frac{\partial f(k, g)}{\partial k} = 0, \quad (52)
\]

\[
\frac{\partial \mathcal{L}}{\partial \pi} = \lambda_3 m + \lambda_4 u_c = 0, \quad (53)
\]

\[
\frac{\partial \mathcal{L}}{\partial c} = u_c - \lambda_1 + \lambda_4[u_c((1 - \tau)\frac{\partial f(k, g)}{\partial k} + \pi) - u_{mc}] = 0, \quad (54)
\]

\[
\frac{\partial \mathcal{L}}{\partial m} = u_m + \lambda_3 \pi + \lambda_4[u_{cm}((1 - \tau)\frac{\partial f(k, g)}{\partial k} + \pi) - u_{mm}] = 0, \quad (55)
\]

\[
\frac{\partial \mathcal{L}}{\partial g} = u_g + \lambda_1[\frac{\partial f(k, g)}{\partial g} - 1] + \lambda_2(1 - \tau)\frac{\partial^2 f(k, g)}{\partial k \partial g} + \lambda_3[\tau \frac{\partial f(k, g)}{\partial g} - 1]
+ \lambda_4[u_c(1 - \tau)\frac{\partial^2 f(k, g)}{\partial k \partial g} + u_{cg}((1 - \tau)\frac{\partial f(k, g)}{\partial k} + \pi) - u_{mg}] = 0, \quad (56)
\]

\[
\frac{\partial \mathcal{L}}{\partial k} = \lambda_1 \frac{\partial f(k, g)}{\partial k} + \lambda_2(1 - \tau)\frac{\partial^2 f(k, g)}{\partial k^2} + \lambda_3 \tau \frac{\partial f(k, g)}{\partial k} + \lambda_4 u_c(1 - \tau)\frac{\partial^2 f(k, g)}{\partial k^2} = 0, \quad (57)
\]

and

\[
f(k, g) - c - g + a_1 + a_2 = 0, \quad (49)
\]

\[
(1 - \tau)\frac{\partial f(k, g)}{\partial k} - \beta = 0, \quad (50)
\]

\[
g = \pi m + \tau f(k, g) + a_2, \quad (51)
\]

\[
u_m = u_c((1 - \tau)\frac{\partial f(k, g)}{\partial k} + \pi). \quad (40)
\]

From equations (49)-(57) and (40), we can derive \(\lambda_1, \lambda_2, \lambda_3, \lambda_4, c, k, \tau, \pi, g, \) and \(m\) as functions of \(a_1\) and \(a_2\).

For the neoclassical production function and strictly concave utility function, there exists at least one equilibrium for the economic system, i.e., there exist \(\lambda_1, \lambda_2, \lambda_3, \lambda_4, c, k, \tau, \pi, g, \) and \(m\) as the functions of \(a_1\) and \(a_2\). Taking total differentiation
of equations (49)-(57), and (40), we can derive the effects of the two kinds of foreign aid on private consumption, the capital stock, the output tax rate, the inflation rate, government expenditure, and money holdings. Obviously these effects are hard to obtain analytically. For our purpose of policy discussions, we rely on an explicit example.

4.2 An Example

Suppose that the utility function and the production function are specified as

\[ u(c, m, g) = \ln c + \ln m + \ln g, \quad f(k) = k^\phi, \]

respectively, and where \( \phi \) is a positive constant \( (0 < \phi < 1) \).

Now, from equations (49)-(57), and (40), we can derive the tax rate, government expenditure, private consumption, the inflation rate, money holdings, and the capital stock as:

\[ \tau = 1 - \beta \frac{\phi k^\phi - 1}{\phi k^\phi - 1}, \quad (58) \]

\[ 0 = \frac{1}{g} \left( 1 - \frac{1}{m} \frac{1}{\phi k^\phi - 1} \right), \quad (59) \]

\[ c = \frac{(\tau k^\phi + a_2)}{\phi^2 k^\phi - 1 + \frac{1}{\phi k^\phi - 1} - \frac{1}{\phi k^\phi - 1} - \frac{2}{\beta}}, \quad (60) \]

\[ \pi = \frac{1}{\phi k^\phi - 1} - \frac{2}{\beta} - \beta, \quad (61) \]

\[ m = \frac{\tau k^\phi + a_2}{\phi^2 k^\phi - 1 + \frac{1}{\phi k^\phi - 1} - \frac{1}{\phi k^\phi - 1} - \frac{2}{\beta}}, \quad (62) \]

\[ 0 = k^\phi - \frac{(\tau k^\phi + a_2)}{\phi^2 k^\phi - 1 + \frac{1}{\phi k^\phi - 1} - \frac{1}{\phi k^\phi - 1} - \frac{2}{\beta}} + \beta \frac{1}{\phi k^\phi - 1} - \frac{2}{\beta} \]

\[ - \frac{\tau k^\phi + a_2}{\phi^2 k^\phi - 1 + \frac{1}{\phi k^\phi - 1} - \frac{1}{\phi k^\phi - 1} - \frac{2}{\beta}} + \beta \frac{1}{\phi k^\phi - 1} + a_1 + a_2. \quad (63) \]

From equation (63), we can determine the equilibrium capital stock. Then from equations (58)-(62) we can determine the income tax rate, government expenditure, private consumption, the inflation rate, and money holdings.

With parameter values of \( \beta = 0.05, \phi = 1/3 \), and with foreign aid to the private sector fixed at \( a_1 = 0.5 \), figure 5 illustrates the effects of foreign aid to the government sector, \( a_2 \), on the steady-state capital stock, output, money holdings, government expenditure, output taxation, inflation, and consumption.

Figure 5 shows that with a rise in foreign aid to the government, the steady-state capital stock, output, money holdings, government expenditure, and private consumption all increase, whereas both the income tax rate and the inflation rate decrease. The
reason for this scenario is similar to the one in section 3 without inflation finance for government spending. With a rise in foreign aid for the government and a corresponding rise in government revenues, the government cuts its income tax and inflation tax, which would result in more private investment and money holdings. In the long run the private agent’s capital accumulation, consumption, and welfare all increase.

In the case of foreign aid to the private agent, we have a totally different picture as shown in figure 6.

In figure 6, we set $\beta = 0.05$, $\phi = 1/3$, and foreign aid to the government, $a_2 = 0.5$. Figure 6 illustrates the effects of foreign aid to the private sector, $a_1$, on the steady-state private capital accumulation, output, money holdings, consumption, government expenditure, income taxation, and inflation. As foreign aid to the private agent rises, the steady-state capital stock and output decreases, but private consumption and money holdings still rises because the private agent has more total disposable income even with less output and a higher income tax and a higher inflation tax. As for the government, it raises both the inflation rate and income tax rate and collects more revenues to finance a higher level of public spending.

Hence, we can summarize our findings from this example in the following proposition.

**Proposition 3** With intertemporal second-best choices for the government, foreign aid to the private sector reduces private capital accumulation and increases both the optimal inflation rate and the optimal income tax rate, whereas foreign aid to the government raises capital accumulation and lowers the optimal inflation rate and the optimal income tax rate.

## 5 Conclusion

In this paper, we have presented a group of models showing striking different implications of foreign aid to the private sector and public sector. In the first model, with decentralized decision-making and without optimal choices of fiscal policies on behalf of the government, foreign aid to the private sector has no effect on the long-run capital accumulation; whereas foreign aid to the government leads to more public spending and higher private capital accumulation. When we consider optimal choices of fiscal and monetary policies, foreign aid to the private sector reduces capital accumulation. However, when foreign aid goes to the public sector, the government cuts both the inflation rate and the income tax rate, raises public spending, and provides more incentives for private capital accumulation. In the long run, aid to the public sector leads to more private capital accumulation. These striking differences between aid to the public sector and aid to the private sector should draw attention of any serious researchers who empirically test on the effects of foreign aid.

Our paper also points to the direction of future research. First, in our model, we treat the aid to the public sector as a homogenous flow and we do not distinguish between project aid and general budget assistance. Given that the latter is fungible, the aid might substitute rather than supplement domestic fiscal policy tools. In future
research, we could model public capital as a stock variable and address the fungibility issue. Second, this paper obtains the main results by focusing on several special cases of production functions and utility functions. Although our qualitative results are quite robust, it would be better to consolidate the discussion in a unified model. Again, this is the task of future research.

References


Figure 1: The effects of foreign aid to the government sector $a_2$ on the steady-state capital stock, consumption level, government expenditure, output, and the income tax rate. Where we take the utility function $u(c, g) = \ln c + \ln g$, and the production function $f(k, g) = k^\phi$; The parameters are taken as $\beta = 0.05$, $\phi = 0.5$, and the foreign aid to the private sector is given as $a_1 = 1$. 
Figure 2: The effects of foreign aid to the private sector $a_1$ on the steady-state capital stock, consumption level, government expenditure, output, and the income tax rate. Where we take the utility function $u(c, g) = \ln c + \ln g$, and the production function $f(k, g) = k^{\phi}$; The parameters are taken as $\beta = 0.05$, $\phi = 0.5$, and the foreign aid to the government sector is given as $a_2 = 1$. 
Figure 3: The effects of foreign aid to the government sector $a_2$ on the steady-state capital stock, consumption level, government expenditure, output, and the income tax rate. Where we take the utility function $u(c, g) = \ln c + \ln g$, and the production function $f(k, g) = Ak^\phi g^{1-\phi}$; The parameters are taken as $\beta = 0.05$, $\phi = 0.5$, $A = 0.3$ and the foreign aid to the private sector is given as $a_1 = 1$. 
Figure 4: The effects of foreign aid to the private sector $a_1$ on the steady-state capital stock, consumption level, government expenditure, output, and the income tax rate. Where we take the utility function $u(c, g) = \ln c + \ln g$, and the production function $f(k, g) = Ak^{\phi}g^{1-\phi}$; The parameters are taken as $\beta = 0.05$, $\phi = 0.5$, $A = 0.3$ and the foreign aid to the government sector is given as $a_2 = 1$. 

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Figure 5: The effects of foreign aid to the government sector $a_2$ on the steady-state capital stock, output, money holding, government expenditure, the income tax rate, inflation rate, and the consumption level. Where we take the utility function $u(c, m, g) = \ln c + \ln m + \ln g$, and the production function $f(k) = k^\phi$; The parameters are taken as $\beta = 0.05$, $\phi = 1/3$, and the foreign aid to the private sector is given as $a_1 = 0.5$. 


Figure 6: The effects of foreign aid to the private sector $a_1$ on the steady-state capital stock, output, money holding, government expenditure, the income tax rate, inflation rate, and the consumption level. Where we take the utility function $u(c, m, g) = \ln c + \ln m + \ln g$, and the production function $f(k) = k^\phi$; The parameters are taken as $\beta = 0.05$, $\phi = 1/3$, and the foreign aid to the government sector is given as $a_2 = 0.5$. 