Intertemporal Portfolio Choice under Multiple Types of Event Risks

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This paper examines the effects of major event risk on the optimal intertemporal asset allocation in a continuous time setting. We start by firstly proposing a general framework in which we model three types of event risks: i) the individual jumps of asset prices, ii) the individual jumps of the underlying states; and iii) the joint jumps. Most previous papers in the portfolio choice literature can be included in the framework as special cases. We next illustrate the use of this framework in three examples and find i) hedging demand due to jumps are in general several times larger than that due to diffusions; ii) multiple types of jumps are not only supported by the US stock market data but also play different roles in agents' asset allocation. In particular, jumps in state induce little and negative hedging components under the individual state jumps and the joint jumps, respectively.

Key words: portfolio choice, general framework, event risk, diffusion hedging, jump hedging, jump misspecification

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1 Introduction

Since Merton's orginal work (1971), the problem of portfolio choice in the presence of richer stochastic environment has become a topic of increasing interest. Recent examples include Kim and Omberg (1996), Campbell and Viceira (1999), Xia (2001), Watcher (2002), Liu et al (2003), Chacko and Viceira (2003), Liu (2007) etc. In almost all these papers, the stochastic investment opportunity is only composed of the diffusion of the risky assets as well as the underlying states, and the result is the diffusion hedging component in the aggregate demand for assets. Diffusion processes explain very well, for example, the frequent but local changes of stock prices often within one or two standard deviations from its mean value. In reality, however, stock price jumps suddenly and radically. For example, on October 19, 1987, the Dow index fell by 508 points, or 23% of its total market capitalization within only a few hours. Such major events obviously cannot be explained by diffusion and in practice they matter a lot for people's portfolio choice: since those who hold a lot of stocks will face the risk of running into bankrupty when disasters strike, investors would simply try to avoid aggressive investment strategy. As a result, the possible event risks are likely to effectively decrease people's stock holdings.

Few papers in the portfolio choice literature model event risks[3], and the only one we know so far is Liu et al (2003) who studies the implication of the joint stock price and variance jump on people's stock holdings. Their main finding is that agent facing event risk acts as if some portion of his wealth may become illiquid and thus become less willing to hold leverage positions in stock, which is exactly what the intuition tells us. However, the assumption that stock price and its variance always jump together seems untenable and they don't study the relative importance of the different hedging components in investors' stock demand. More importantly, they only examine the effect of jumps on asset allocation in a very specific setup. A general framework will obviously deepen our understanding about the feature of event risk and the induced jump hedging in a systematic way.

In the literature, Liu (2007) proposes a general framwork about diffusion hedging. Ours setup is built on his framework but with enlarged opportunity set characterized by the event risks as well as the risks due to diffusions. In particular we consider three types of jumps: the individual jump of the risky asset prices, the individual jump of the underlying states and the joint jump of both prices and states. This modeling includes all the possible type of jumps in reality and enables us to compare their different implications on agent's asset allocation. Another extension of our framework is to relax Liu (2007)'s assumption about standard Brownians so that the more recent portfolio choice models with asymmetric information and/or unobservable parameters can also be included asspecial cases.

In our framework, as in Liu (2007), the original portfolio choice problem is transformed mathematically into the study of PDE (partial differential equation) with respect to f, a function of time as well as the underlying states. We firstly summarize the general results about the closed-from solutions to the original portfolio problem from the perspective of solving PDE, and particularly we point out why the literature alway need to assume away intermediate consumption and/or market incompleteness for the existence of

[3] In contrast, modelling events risks is common in the option pricing literature for explaining the well known "volatility smirks". See, for example, Duffie et al (2000), Pan (2002), Liu et al (2005), etc.

closed-form solutions. At the presence of event risks, however, no closed-form solutions can be expected but similar assumptions (together with some technique conditions) will enable us to decompose the original PDE into a series of ODEs which can be readily solved numerically. Trying to solve the original PDE directly requires plausible boundary conditions often missing from the economic models and in addition, the robustness of the numerical solution to different assumptions about bounday conditions is also unknown.

We next use three examples to illustrate the use of this general framework with the focus on the effect of event risks on the portfolio choice. We naturally extend Liu et al (2003) to the scenario where the stock price and its variance both jump individually as well as jointly. This way of modeling is confirmd by our empirical calibration using the US stock market data. We next study the relative importance of different hedging components in the aggregate stock demand and find that the jump hedging is much more important than the diffusion hedging, and different types of jump hedging play different roles in the agent's stock holdings. In another model we follow Kim and Omberg (1996) and Watcher (2002) and use the equity premium instead of the variance as the state but extend it to include possible price jumps. The jump hedging component is found to be even more important in this setup, and in addition we generate different horizon effect, diffusion hedging, etc. The comparison between the specific models show the importance of the choice of states, and the identification of suitable states which constitute the stochastic investment opportunity for different asset allocation problems should be an important direction for future research within our framework.

The remainder of the paper is organized as follows. Section 2 presents the general framework. Section 3 relates the framework to the literature by demonstrating its generality and usefulness; Section 4 presents three specific examples and finally section 5 concludes with a summary of possible directions for future research.

2 The general framework

Assume there are one riskless asset and m risky assets in the economy whose prices are $P_{0t,}$ and $P_t = (P_{1t}, ...P_{mt})'$ respectively:

$$\frac{dP_0}{P_0} = r(X)dt \tag{1}$$

$$\frac{dP}{P} = \mu(X)dt + \Sigma(X)dB + U^p dN^p + UdN \tag{2}$$

where $X_t = (X_{1t}, ...X_{nt})'$ are the n underlying states which follow:

$$dX = \mu^x dt + \Sigma^x dB^x + V^x dN^x + VdN \tag{3}$$

where B and B^x are M by 1 and N by 1 Brownians and $\Sigma(X)$ and Σ^x are m by M and n by N variance-covariance matrics respectively. Assume $\rho' dt = cov(dB, dB^x)$; $\Phi dt = V(dB^x)$; $\Gamma dt = V(dB)$, whose dimensions are M by N, N by N and M by M, respectively. We further assume that $\Sigma\Gamma\Sigma'$ is invertible.

In addition to the diffusion terms, we introduce event risks to the otherwise standard model. As in the literature, major events are modeled as jumps that follow Poisson processes and we consider the most general case in which three types of jumps are involved: the individual jump of asset prices with jump size U^p and arrival intensity $\lambda^p(X)$; the individual jump of the states with jump size V^x and the arrival intensity $\lambda^x(X)$; and the joint jump of both prices and states with arrival intensity $\lambda(X)$ and jump sizes U and V for prices and states respectively. Following Duffie et al(2000) all the jump sizes can be random and have the "jump transform" of θ^p, θ^x, θ^u, θ^v respectively.[4] We assume the jumps are independent of diffusion, and independent of each other; and the arrival time and the random jump sizes are also independent.

We consider the standard additive CRRA utility[5] and the representative agent maximizes:

$$\max_{\{\phi_t, C_t\}_{t=0}^T} E_0[\int_0^T \alpha e^{-\beta t}\frac{C_t^{1-\gamma}}{1-\gamma}dt + (1-\alpha)e^{-\beta T}\frac{W_T^{1-\gamma}}{1-\gamma} \tag{4}$$

where the ϕ is an m by 1 variable denoting the portfolio weight of the risky assets; C_t is the consumptio rate; W_T is the agent's terminal wealth. Following Liu (2007), we introduce α to control the relative importance of the intermediate consumption and the terminal wealth to the agent.

At the presence of potential jumps, the agent's wealth process follows

$$dW = [W(\phi'(\mu - r) + r) - C]dt + W\phi'\Sigma dB + W\phi' U^p dN^p + W\phi' U dN \tag{5}$$

where the jump component for the wealth process is derived as follows:

$$\begin{aligned}\frac{W_t - W_{t-}}{W_t} &= \frac{\sum_{i=1}^m P_{it} - \sum_{i=1}^m P_{it-}}{P_0 + \sum_{i=1}^m P_{it}} = \sum_{i=1}^m \frac{P_{it}}{P_0 + \sum_{j=1}^m P_{jt}}\frac{P_{it} - P_{it-}}{P_{it}} \\ &= \sum_{i=1}^m \phi_{it}\frac{P_{it} - P_{it-}}{P_{it}} = \sum_{i=1}^m \phi_{it}(U^{pi}dN^p + U^i dN) = \phi' U^p dN^p + \phi' U dN\end{aligned}$$

We use the stochastic control approach to attack the problem[6]. Letting $J(t, W, X)$ be the indirect utility, we arrive at:

[4] the "jump transform" is defined as $\theta(c) = \int_{R^n} \exp(c \cdot z)dv(z)$, where $v(z)$ is the c.d.f. for the n by 1 random jump size Z.

[5] A more general preference is called "stochastic differential untility" firstly proposed by Duffie and Epstein (1992) as the continous version counterpart of the traditional Epstein-Zin preference.

[6] Another common way for attacking portfolio choice problem is called the martingale approach firstly proposed by Cox and Huang (1989).

$$
\begin{aligned}
&\max_{\phi,C}\{\alpha e^{-\beta t}\frac{C^{1-\gamma}}{1-\gamma}+\frac{\partial J}{\partial t}+\frac{1}{2}W^2\phi'\Sigma\Gamma\Sigma'\phi J_{ww}+W(\phi'(\mu-r)+r)J_w \\
&-cJ_w+W\phi'\Sigma\rho'\Sigma^{x\prime}J_{wx}+\frac{1}{2}tr(\Sigma^x\Phi\Sigma^x J_{xx})+\mu^{x\prime}J_x \\
&+\lambda^p[EJ(t,W(1+\phi'U^p),X)-J]+\lambda^x[EJ(t,W,X+V^x)-J] \\
&+\lambda[EJ(t,W(1+\phi'U),X+V)-J]\}=0
\end{aligned} \tag{6}
$$

with the boundary condition:

$$
J(t,W,X)=(1-\alpha)e^{-\beta T}\frac{W_T^{1-\gamma}}{1-\gamma} \tag{7}
$$

For the CRRA preferences, J is conjectured to have the following form:

$$
J(t,W,X)=e^{-\beta t}\frac{W^{1-\gamma}}{1-\gamma}(f(t,X))^{\gamma} \tag{8}
$$

FOCs yield the optimal policies as:

$$
C=\alpha^{\frac{1}{\gamma}}\frac{W}{f} \tag{9}
$$

$$
\begin{aligned}
\phi \quad = \quad & \frac{1}{\gamma}(\Sigma\Gamma\Sigma')^{-1}(\mu-r)+(\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime})\frac{\partial\ln f}{\partial X} \\
&+\frac{\lambda^p}{\gamma}(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U^p)^{-\gamma}U^p] && (10) \\
&+\frac{\lambda}{\gamma}(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U)^{-\gamma}U(\frac{f(t,X+V)}{f(t,X)})^{\gamma}] && (11)
\end{aligned}
$$

In the expression for ϕ, the first term is the usual myopic demand whereas the remaining terms are the hedging components. In particular, the second term is the hedging to state diffusion while the third and fourth are the hedging to individual price jumps and the joint jumps, respectively. The expections are with respect to the random jump sizes. Note that there is no separate hedging component to the individual state jumps.

Substitute the opitimal policies into (6), and tedious algebra yields the following HJB (Hamilton-Jacobian-Bellman equation) in terms of PDE for f:

$$
\begin{aligned}
0 = \alpha^{\frac{1}{\gamma}} + \frac{\partial f}{\partial t} + \frac{1}{2}tr(\Sigma^x\Phi\Sigma^{x'}f_{xx'}) + [\mu^x + \frac{1-\gamma}{\gamma}(\Sigma\rho'\Sigma^{x\prime})'(\Sigma\Gamma\Sigma')^{-1}(\mu - r)]'f_x \\
+\frac{\gamma-1}{2f}f_x'[\Sigma^x\Phi\Sigma^{x\prime} - (\Sigma\rho'\Sigma^{x\prime})'(\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime})]f_x + \{\frac{1-\gamma}{2\gamma^2}(\mu - r)'(\Sigma\Gamma\Sigma')^{-1}(\mu - r) \\
+\frac{1-\gamma}{\gamma}r - \frac{\beta}{\gamma} - \frac{\lambda^{p2}(1-\gamma)}{2\gamma^2}E[(1+\phi'U^p)^{-\gamma}U^p]'(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U^p)^{-\gamma}U^p] \\
-\frac{\lambda^2(1-\gamma)}{2\gamma^2}E[(1+\phi'U)^{-\gamma}U(\frac{f(t,X+V)}{f(t,X)})^{\gamma}]'(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U)^{-\gamma}U(\frac{f(t,X+V)}{f(t,X)})^{\gamma}] \\
-\frac{\lambda\lambda^p(1-\gamma)}{\gamma^2}E[(1+\phi'U)^{-\gamma}U(\frac{f(t,X+V)}{f(t,X)})^{\gamma}]'(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U^p)^{-\gamma}U^p] \\
+\frac{\lambda^p}{\gamma}[E(1+\phi'U^p)^{1-\gamma} - 1] + \frac{\lambda^x}{\gamma}[E(\frac{f(t,X+V^x)}{f(t,X)})^{\gamma} - 1] + \frac{\lambda}{\gamma}[E[(1+\phi'U)^{1-\gamma}(\frac{f(t,X+V)}{f(t,X)})^{\gamma}] - 1]\}f
\end{aligned} \tag{12}
$$

with the boundary condition:

$$f(T, X) = (1-\alpha)^{\frac{1}{\gamma}} \tag{13}$$

Note that the first term $\alpha^{\frac{1}{\gamma}}$ is due to the intermediate consumption. It is gone when consumption is assumed away.

Our general framework is based on the seminal contribution by Liu (2007) but contains quite a few important extensions. For example, in Liu (2007), both Brownians, B and B^x, are assumed to be standard and both variance matrics, Σ and Σ^x, are assumed to be square. All these restrictions have been relaxed by introducing Φ and Γ and by allowing different number of risky assets and states from their respective Brownians. The results about diffusion are now explicitly expressed as functions of the second moments of the two processes' diffusion terms: $\Sigma\Gamma\Sigma'$, $\Sigma^x\Phi\Sigma^{x\prime}$ and $\Sigma\rho'\Sigma^{x\prime}$, and particularly the diffusion hedging component $(\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime})$ is just the regression coefficient from the diffusion component of dX to that of $\frac{dP}{P}$[7] and hence measures the correlation between the risky asset and the state. When this coefficient is zero, risky asset can no longer be used to hedge against the diffusion of the states, hence the vanish of the diffusion hedging. The second term $\frac{\partial \ln f}{\partial X}$ summarizes the agent's attititude towards changes of the stochastic investment opportunity characterized by X. Note sign$(\frac{\partial \ln f}{\partial X}) = sign(\frac{\partial V}{\partial X})$ for risk averse ($\gamma > 0$) agent.

More importantly, the jump components are missing in Liu (2007) whereas in our model there are two additional jump hedging components in ϕ: $\frac{\lambda^p}{\gamma}(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U^p)^{-\gamma}U^p]$ and $\frac{\lambda}{\gamma}(\Sigma\Gamma\Sigma')^{-1}E[(1+\phi'U)^{-\gamma}U(\frac{f(t,X+V)}{f(t,X)})^{\gamma}]$. Note the second component, the hedging to the joint jumps is composed of two parts: $\frac{\lambda}{\gamma}(\Sigma\Gamma\Sigma')^{-1}(1+\phi'U)^{-\gamma}U$ which is the hedging to the price jump, and $(\frac{f(t,X+V)}{f(t,X)})^{\gamma}$ which is due to the state jumps and can be either "deflator" or "inflator" of the price hedging depending on how X jumps and whether f decreases or increases in the states. We have a lot more additional terms in the PDE for f starting from the ninth terms in the PDE. The first three quadratic terms in λs are from $\frac{1}{2}W^2\phi'\Sigma\Gamma\Sigma'\phi J_{ww}$ while the last three terms measure the change of the value function when jumps occur. They all vanish

[7] In Liu (2005), this coefficient is given by $\Sigma'^{-1}\rho'\Sigma_x'$ which is less clear for interpretation.

when there is no jump, i.e. when either the λs or the jump sizes are set to zero.

3 Relation with the literature

In the literature Liu (2007) is a good generalization of the standard portfolio choice models where i) Brownians are standard and market completeness is usually assumed; ii)no event risk exists. These models range from the original Heston (1993) to the more recent stock-bond mix problem where the risky bond processes are taken from the DTSM (dynamic term structure models). In the following, I give two "nonstandard" portfolio choice models in the literature that can be included as special cases of our general framework but not that of Liu (2007)'s:

The first example is Yihong Xia (2001) in which she studies the effects of learning on the asset allocation. Though she starts from modelling uncertain parameters, a feature missing in our framework, after a change of the filtration and the corresponding Browninas her setup can be transformed into a three-state-one-risky setup with the states $X = (b, v, s)'$ and stock price P. In addition, $\mu = \bar{\mu} + b(s - \bar{s})$, $\Sigma = \sigma_P$, $\Sigma^x = \begin{pmatrix} v_1 & v_2 \\ 0 & 0 \\ 0 & \sigma_s \end{pmatrix}$, $dB = d\hat{z}_P$, $dB^x = \begin{pmatrix} d\hat{z}_P \\ d\hat{z}_s \end{pmatrix}$, $\rho' = \begin{pmatrix} 1 \\ \rho_{sP} \end{pmatrix}$, $\Phi = \begin{pmatrix} 1 & \rho_{sP} \\ \rho_{sP} & 1 \end{pmatrix}$, $\Gamma = 1$, and $f = \phi^{\frac{1}{\gamma}}$ in our notation which means: $\frac{\partial \ln f}{\partial X} = \frac{1}{\gamma} \frac{\partial \ln \phi}{\partial (b,v,s)'} = \frac{1}{\gamma}[\phi_b, \phi_v, \phi_s]'/\phi$. Note the transformed Brownians are nonstandard and cannot be dealt with in Liu (2007)'s framework. Using our general results, the derivation of the optimal policies becomes obvious. For example, the diffusion hedging is simply given by

$$\begin{aligned} (\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime})\frac{\partial \ln f}{\partial X} &= \frac{1}{\sigma_P^2}\sigma_P[v_1 + \rho_{sP}v_2, 0, \rho_{sP}\sigma_s]\frac{1}{\gamma}[\phi_b, \phi_v, \phi_s]'/\phi \\ &= \frac{\sigma_P}{\sigma_P^2}(v_1 + \rho_{sP}v_2)\frac{\phi_b}{\phi} + \frac{\phi_s}{\gamma\sigma_P\phi}\sigma_s\rho_{sP} \end{aligned}$$

Next we just use (18) in her paper to get her main analytical results in (22).

We use Liu et al (2003) as our second example which is also by far the only paper I know that studies portfolio choice at the presence of even risk. Since they only consider joint jump, $\lambda^p = \lambda^x = 0$. In addition they write $\Sigma\Gamma\Sigma'$ as V^2 and assume the (joint) arrival intensity is given by λV, hence the jump hedging: $\frac{\lambda}{\gamma}E[(1+\phi X)^{-\gamma} X e^{BY}]$, where $e^{BY} = (\frac{f(t,X+V)}{f(t,X)})^{\gamma}$ under their notation and the conjectured formula for the indirect utility. As they remark, the jump hedging component is similar to a buy-and-hold demand for an investor who maximizes his expected terminal wealth given by $E_0 \frac{[(1+\phi X)W_0]^{1-\gamma}}{1-\gamma}$.

Next we want to talk about the "closed-form-solution" problem in the portfolio choice literature. In our framework, this is equivalent to whether the PDE for f can be solved in closed form or not since the value function and the optimal policies are all expressed in terms of f. We firstly shut down all the jumps to get a simplified PDE with $\alpha^{\frac{1}{\gamma}}$ being the only nonhomogenous term. We now understand why the literature always assumes away intermediate consumption: this is just to generate a homogeneous PDE about f and its derivatives so that under certain technique conditions, the PDE can be decomposed into a series of ODEs that can be readily solved in closed forms. The most common technical conditions are "affine returns" and "quadratic returns". See Liu (2007) for a detailed discussion.

Recently, some authors obtain closed-form solutions at the presence of consumption[8]. As pointed out by Liu (2007), the point is to transform the original PDE into a new homogenous PDE of $\hat{f}$ by a change of variable from $f(t,X)$ to $\hat{f}(t,X)$ where $f(t,X) = \alpha^{\frac{1}{\gamma}} \int_t^T \hat{f}(u,X)du + (1-\alpha)^{\frac{1}{\gamma}} \hat{f}(t,X)$. The key step for this approach is the validity of changing order between integration and differential operator, something like whether we have $\frac{\partial}{\partial x}\int f(u,x)du = \int \frac{\partial}{\partial x} f(u,x)du$ or not. This imposes an important restriction that the original PDE must be linear and in our case the only exception is the term $\frac{\gamma-1}{2f} f_x'[\Sigma^x \Phi \Sigma^{x\prime} - (\Sigma\rho'\Sigma^{x\prime})'(\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime})]f_x$[9] which is due to the market incompleteness. If the market is complete, this term is gone[10] and we have $f(t,X) = \alpha^{\frac{1}{\gamma}} \int_t^T e^{A_u + B_u' X} du + (1-\alpha)^{\frac{1}{\gamma}} e^{A_t + B_t' X}$, where the integration term also found in Watcher (2002) is due to the change of variable.

What if we have both intermediate consumption and an incomplete market? Watcher (2002) considers this problem from the perspective of extended martingale method proposed by He and Pearson (1991)[11]. She concludes that the possible help from this extension is largely illusory and the problem with intermediate consumption under incomplete markets "is much harder to solve". From the perspective of dynamic programming, this problem is equivalent to whether we give closed-form solutions to PDEs with both nonhomogenous and nonlinear terms, and we leave it for furture research.

At the presence of jumps, the PDE becomes highly nonlinear and in addition it must be solved together with ϕ, which is mathematically called the differential-algebraic system. Despite of this complexity, the original PDE can still be decomposed into ODEs under the same technique conditions specified by Liu (2007). As a quick example, consider the scalar case when $m = M = n = N = 1$, assume away consumption ($\alpha = 0$), market incompleteness ($\Sigma^x \Phi \Sigma^{x\prime} - (\Sigma\rho'\Sigma^{x\prime})'(\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime}) = 0$) and individual jumps ($\lambda^p = \lambda^x = 0$), and impose: i) $r = a_0 + a_1 x$; ii)$\mu^x = b_0 + b_1 x$; iii)$\mu - r = kx$; iv)$\Sigma^x = \sigma_x \sqrt{x}$; v)$\Sigma = \sigma\sqrt{x}$; vi)$\lambda = lx$, where a_0, a_1, b_0, b_1, k, l, σ_x, σ are all constants.[12] If we conjecture $f(t,X) = \exp(A_t + B_t X)$ and note $\frac{\partial f}{\partial t} = (\dot{A}_t + \dot{B}_t X)f$, $f_x = B_t f$, $f_{xx} = B_t^2 f$, $(\frac{f(t,X+V)}{f(t,X)})^\gamma = e^{\gamma B_t V}$, we derive from the HJB:

$$
\begin{aligned}
-(\dot{A}_t + \dot{B}_t X) \quad = \quad & \frac{1}{2}\sigma_x^2 X B_t^2 + (b_0 + b_1 X + \frac{1-\gamma}{\gamma}\frac{\rho k \sigma_x}{\sigma} X)B_t + \frac{1-\gamma}{2\gamma^2}\frac{k^2}{\sigma^2} X \\
& + \frac{1-\gamma}{\gamma}(a_0 + a_1 X) - \frac{\beta}{\gamma} - \frac{1-\gamma}{2\gamma^2}\frac{l^2}{\sigma^2} X E^2[(1+\phi U)^{-\gamma} U e^{\gamma B_t V}] \\
& + \frac{l}{\gamma} X E[(1+\phi U)^{1-\gamma} U e^{\gamma B_t V} - 1]
\end{aligned}
$$

with the boundary:

[8] See, for example, Watcher (2002)

[9] Note although this term is nonlinear, it is still homogeneous in f and its derivatives.

[10] This can be seen clearly when the Brownians are standard and Σ is square and invertible. In this case, $\Sigma^x \Phi \Sigma^{x\prime} - (\Sigma\rho'\Sigma^{x\prime})'(\Sigma\Gamma\Sigma')^{-1}(\Sigma\rho'\Sigma^{x\prime}) = \Sigma^x (I - \rho\rho')\Sigma^{x\prime}$, and market completeness implies $\rho = I$, hence the vanish of this term.

[11] The original martingale method proposed by Cox and Huang (1989) can only be applied under complete markets.

[12] The conditions we impose is the so-called "affine returns" commonly found DTSM and in option pricing literature.

$$e^{A_T+B_TX} = 1$$

Comparing the constant and linear terms of X on both sides leads to

$$\begin{aligned}\dot{A}_t &= -b_0B_t - \tfrac{1-\gamma}{\gamma}a_0 + \tfrac{\beta}{\gamma};\ A_T = 0\\ \dot{B}_t &= -\tfrac{1}{2}\sigma_x^2B_t^2 - (b_1 + \tfrac{1-\gamma}{\gamma}\tfrac{\rho k\sigma_x}{\sigma})B_t - \tfrac{1-\gamma}{2\gamma^2}\tfrac{k^2}{\sigma^2} - \tfrac{1-\gamma}{\gamma}a_1\\ &+\tfrac{1-\gamma}{2\gamma^2}\tfrac{l^2}{\sigma^2}E^2[(1+\phi U)^{-\gamma}Ue^{\gamma B_tV}] - \tfrac{l}{\gamma}E[(1+\phi U)^{1-\gamma}Ue^{\gamma B_tV}-1];\ B_T = 0\end{aligned}$$

with

$$\phi = \frac{k}{\gamma\sigma^2} + \frac{\rho\sigma_x}{\sigma}B_t + \frac{l}{\gamma\sigma^2}E[(1+\phi U)^{-\gamma}Ue^{\gamma B_tV}]$$

Note without the jump terms, the ODE about B_t is just the Riccati equation which readily allows closed-form solutions. Because of its last two highly nonlinear terms introduced by joint jump, the ODE about B_t and hence the whole system must be solved numerically and together with ϕ. Still the decompostion makes things a lot easier because we have avoided solving the original PDE directly.

4 Examples

4.1 The Merton model with jumps

In this section we apply our general framework to three specific models with the focus on the effect of event risks on the portfolio choices. We start from the simplest one by considering the original Merton (1971) model with possible stock price jumps as follows:

$$\frac{dS}{S} = \mu dt + \sigma dB + UdN$$

where the jump size U and the arrival intensity λ are both assumed to be constant. The optimal stock share is given by

$$\phi = \frac{\mu - r}{\gamma\sigma^2} + \frac{\lambda}{\gamma\sigma^2}(1+\phi U)^{-\gamma}U$$

where r and γ are the risk free rate and coefficient of relative risk aversion, respectively. Note there is no diffusion hedging in this model.

Table 1 shows the quick calibration of the model. Note at the presence of event risk,s the equity premium and the stock volatility are changed to $\mu + \lambda U - r$ and $\sqrt{\sigma^2 + \lambda U^2}$ respectively. In Panel A, a 1% risk free, 8% equity premium and 20% volatility roughly match the historical data for US market (see Cochrane (2001)), while risk aversion coefficient of 3 is roughly the median value considered plausible by Mehra and Prescott (1985). We assume that in every 20 years the stock market drops by an average of 20%. This assumption is moderate since the last major stock crisis (1987) just happened less than twenty years ago with nearly one quarter of the market value gone.

Panel B gives the derived parameter values and the stock demands. We obtain a sizable jump hedging of -13.5% which implies that jumps are important for investors' stock choice. If we assume away the jump, the stock weight increases from 0.6557 to $\frac{8\%}{3\times(20\%)^2}$ =0.6667 which shows the effect of event risk on the decrease of investors' stockholdings. This effect is more pronounced for smaller γ. For example when $\gamma = 2$, the portfolio weight decreases from 1.1842 to 0.9806 when the jump is shut down.[13].

4.2 Stochastic volatility with multiple jumps

4.2.1 the setup

The above example gives us some general idea of how large the jump hedging could be. However, it is static because of the time invariant investment opportunity due to the lack of underlying states. Since Heston (1993), the literature increasingly assumes stochastic instantaneous stock variance and use it as the state. Following this tradition we assume:

$$\frac{dS}{S} = (r + \eta V)dt + \sqrt{V}dB^s + X^s dN^s + XdN$$

$$dV = (\kappa - \xi V)dt + \sigma\sqrt{V}dB^v + Y^v dN^v + YdN$$

Our model is an extension of Liu et al (2003) in that three types of jumps are assumed: the individual jump of the stock price with arrival intensity $\lambda^s V$ and jump size X^s; the individual jump of the instantaneous variance (of the diffusive return) with intensity $\lambda^v V$ and size Y^v; and the joint jump with the common arrival intensity λV and jump sizes X and Y for price and variance jumps, respectively, where λ^s, λ^v λ and the jump sizes are all assumed to be constant.[14] In addition, the variance is assumed to follow a mean reverting process with the mean $\bar{V} = \frac{\kappa}{\xi - Y^v\lambda^v - Y\lambda}$ because of the jumps.

Assume $\rho dt = cov(dB^s, dB^v)$. From our general results:

$$\phi = \frac{\eta}{\gamma} + \rho\sigma\frac{\partial \ln f}{\partial V} + \frac{\lambda}{\gamma}(1+\phi X)^{-\gamma}X(\frac{f(t,V+Y)}{f(t,V)})^{\gamma} + \frac{\lambda^s}{\gamma}(1+\phi X^s)^{-\gamma}X^s$$

[13] Intuitively, a less risk averse agent would prefer to hold more leveraged positions but cannot do so because of the possible illiquidy situation they will face when jump occurs. Thus, the effect of event risk is larger on investors with low γ who would otherwise act more aggressively.

[14] We assume that the λs are constant to make the system affine. We assume constant jump sizes for simpliciy. Another reason is that it is difficult to calibrate the stochastic jump sizes from the data since jumps are rare events.

$$
\begin{aligned}
0 \;=\;& \alpha^{\frac{1}{\gamma}} + \frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 V f_{vv} + (\kappa - \xi V + \frac{1-\gamma}{\gamma}\rho\sigma\eta V) f_v + \frac{\gamma-1}{2}\sigma^2 V(1-\rho^2)\frac{f_v^2}{f} \\
& + \{\frac{1-\gamma}{2\gamma^2}\eta^2 V + \frac{1-\gamma}{\gamma} r - \frac{\beta}{\gamma} - \frac{\lambda^{s2} V(1-\gamma)}{2\gamma^2}[(1+\phi X^s)^{-\gamma} X^s]^2 - \frac{\lambda^2 V(1-\gamma)}{2\gamma^2} \cdot \\
& [(1+\phi X)^{-\gamma} X (\frac{f(t,V+Y)}{f(t,V)})^{\gamma}]^2 - \frac{\lambda^s \lambda V(1-\gamma)}{\gamma^2}(1+\phi X)^{-\gamma} X (\frac{f(t,V+Y)}{f(t,V)})^{\gamma}(1+\phi X^s)^{-\gamma} X^s \\
& + \frac{\lambda^s V}{\gamma}[(1+\phi X^s)^{1-\gamma} - 1] + \frac{\lambda^v V}{\gamma}[(\frac{f(t,V+Y^v)}{f(t,V)})^{\gamma} - 1] + \frac{\lambda V}{\gamma}[(1+\phi X)^{1-\gamma}(\frac{f(t,V+Y)}{f(t,V)})^{\gamma} - 1]\} f
\end{aligned}
$$

Since our focus is the optimal portfolio choice, we assume away consumption or set $\alpha = 0$. Hence the boundary condition:

$$f(T, V_T) = 1$$

Conjecture the following form of solution

$$f(\tau, V) = \exp(A_\tau + B_\tau V) \text{ with } \tau = T - t$$

and we obtain the following ODE-algebraic system

$$
\begin{gathered}
\dot{A}_\tau = \kappa B_\tau + \tfrac{1-\gamma}{\gamma} r - \tfrac{\beta}{\gamma};\; A_0 = 0 \\
\dot{B}_\tau = \tfrac{1}{2}\sigma^2 B_\tau^2 + (\tfrac{1-\gamma}{\gamma}\rho\sigma\eta - \xi) B_\tau + \tfrac{\gamma-1}{2}\sigma^2(1-\rho^2) B_\tau^2 + \tfrac{1-\gamma}{2\gamma^2}\eta^2 \\
-M_1 - M_2 - M_3 + M_4 + M_5 + M_6;\quad B_0 = 0 \\
\phi = \tfrac{\eta}{\gamma} + \rho\sigma B_\tau + \tfrac{\lambda}{\gamma}(1+\phi X)^{-\gamma} X e^{\gamma B_\tau Y} + \tfrac{\lambda^s}{\gamma}(1+\phi X^s)^{-\gamma} X^s
\end{gathered}
$$

where

$$
\begin{gathered}
M_1 = \tfrac{\lambda^{s2}(1-\gamma)}{2\gamma^2}[(1+\phi X^s)^{-\gamma} X^s]^2 \\
M_2 = \tfrac{\lambda^2(1-\gamma)}{2\gamma^2}[(1+\phi X)^{-\gamma} X e^{\gamma B_\tau Y}]^2 \\
M_3 = \tfrac{\lambda\lambda^s(1-\gamma)}{\gamma^2}(1+\phi X^s)^{-\gamma} X^s (1+\phi X)^{-\gamma} X e^{\gamma B_\tau Y} \\
M_4 = \tfrac{\lambda^s}{\gamma}[(1+\phi X^s)^{1-\gamma} - 1] \\
M_5 = \tfrac{\lambda^v}{\gamma}[e^{\gamma B_\tau Y^v} - 1] \\
M_6 = \tfrac{\lambda}{\gamma}[(1+\phi X)^{1-\gamma} e^{\gamma B_\tau Y} - 1]
\end{gathered}
$$

4.2.2 Relative importance of the different jump hedging components

Before calibrating the model to the real data, we want to compare the relative importance of the three types of jumps on agents' portfolio choice. To this end, we do some temporary calibration first with the results given in Table 2.

Panel A calibrates parameters not related to the jumps. Columns 1 to 4 are directly from the last example for the equity premium, the stock volatility, the risk free rate, and the coefficient of relative risk aversion, respectively. The discrete macroeconomics literature usually sets the time discount factor, β, to

.98 per year, or .02 if time is continous. Hence, column 5. Columns 6 to 7 are taken from Pan (2002)'s estimations of her SV0 model with some adjustment. In particular, the convergence speed of the mean reverting stock variance in her paper is just ξ due to the lack of variance jump. We write it as $\xi - Y^v\lambda^v - Y\lambda$ in our model.

Panel B calibrates the jumps. we assume the same jump sizes for the two price jumps and two variance jumps, respectively, and we assume the identical arrival intensity for all the three types. We choose the same average arrival intensity as in the last example[15], but we increase the price jump size to -50% so as to dramatize the results. The variance jump size is taken from Liu et al (2003), which means that when jump occurs, the volatility will on average increase up to 50%. The derived parameters are given in Panel C which will be used as the benchmark values for our temporary calibration.

To compare the relative importance among different types of jumps on the agent's asset allocation, we plot in Figure 1 the aggregate demand for stocks under the following six cases: i) the benchmark; ii) the individual price jump is shut down; iii) the individual variance jump is shut down; iv) both individual jumps are shut down; v) the joint jump is shut down; vi) all three types of jumps are shut down. In each of the cases, the original values of the equity premium, the stock volatility are both kept constant for the purpose of comparison. For example, the stock volatility which is still set to 20% is given by $\sqrt{(1+\lambda X^2)\bar{V}}$ in ii) with the individual stock price jump shut down. Hence the implied λ and $\bar{V}$ are different from their benchmark values and need to be recomputed. Similarly all the other related parameters like η, κ, etc also need to be recomputed. We follow this principle in all the following calibrations and do the recomputation whenever necessary.

We now have the intertemporal asset allocation as stock weight changes over time. This is because B_τ shows up in both of the diffusion hedging and the hedging to the the joint jump. The stock weight ϕ monotonically increases in the investment horizon[16] and converges to a constant when $\tau \to \infty$. Within this paper we generally consider a horizon up to 2 years since there is little time dependency beyond it[17].

By comparison with Liu et al (2003), we have identifed two new features about the jump hedging components: 1) the individual variance jump has little effect on the portfolio choice. Note the bechmark case and the case when V-individual is shut down almost coincide with each other, and so do the cases when we shut down the S-individual and when we shut down both individual jumps. Theoretically, individual V-jump still matters for ϕ through B_τ[18], but this indirect effect is very small. More importantly, there is no separate hedging component to individual variance jump for two reasons: i) the individual V-jump is independent of the other two price jumps, hence the investor cannot use stock to hedge against it; ii) the individual V-jump doesn't show up in the wealth process and hence has no effect on the agent's terminal wealth. Because of this, in the following calibration with real data we simply shut down the individual V-jump.

2) The agent has a stronger hedging demand to the individual price jump than that to the joint jump. This can be seen by comparing the increased amount of stockholding when we only shut down the individual

[15] Since the three jumps are assumed to be independent, the average arrival intensity is just $\lambda^s\bar{V} + \lambda^v\bar{V} + \lambda\bar{V} = .05$.
[16] This is not always so. Actually ϕ decreases in the τ when $\gamma < 1$.
[17] In some cases below, we consider investment horizon up to 5 years.
[18] Note both λ^v and Y^v enters the term M_5 in the ODE for B_τ.

P-jump and when we shut down both individuals. Put it in another way, individual price jump is more effective in decreasing investors' stockholding than the joint one which may seem counterintuitive at first glance. To explain it mathematically, note $|\frac{\lambda^s}{\gamma}(1+\phi X^s)^{-\gamma}X^s| > |\frac{\lambda}{\gamma}(1+\phi X)^{-\gamma}Xe^{\gamma B_\tau Y}|$ or equivalently, $e^{\gamma B_\tau Y} < 1$ with all the other being equal. This is because the agent's value function and hence the f function decreases in V[19], hence $(\frac{f(t,V+Y)}{f(t,V)})^\gamma = e^{\gamma B_\tau Y} < 1$ for positive variance jumps[20]. As mentioned before, the hedging componet to the joint jump is composed of two parts: the first part, $\frac{\lambda}{\gamma}(1+\phi X)^{-\gamma}$, is the hedging to the price jump within the joint jump just like that to the individual P-jump. However, the second term due to the variance jump, $(\frac{f(t,V+Y)}{f(t,V)})^\gamma$ works as the deflator in this model which decreases the investor's incentive to hedge against the price jump risk. Intuitively, a simultaneous variance jump means a higher probability for the dropped stock price to be back to normal, hence less incentives for the investors to decrease their stockholding beforehand to prepare for the future illiquidity caused by price drops.

We plot the deflator over the investment horizon for different Y, variance jump sizes in the joint jump, in Figure 2. In our temporary calibration, the deflator is also the ratio between the joint jump hedging and the individual P-jump hedging. This ratio is shown to increase in both τ and Y. For example, at $\tau = 2$, the agent's hedging to the joint jump is about 98% of that to the individual P-jump at $Y = 0.1$, but it decreases to less than 85% when the joined variance jump size increases to 0.7.

In summary, the three types of jumps play different roles in the agent's asset allocation. In particular, the individual V-jump alone almost has no effect on investors' hedging demand; when joined with the price jump, it decreases instead of increasing the agent's hedging incentive against the event risk caused by the price jump within the joint jump[21]. As a result, the individual price jump is the most effective among the three types for decreasing investors' stockholding.

Why is it important to set to identical both the arrival intensities and the jump sizes when comparing their relative importance ? Because arrival intensity and jump size have an asymmetric impact on people's portfolio choice as documented in the literature.[22] To illustrate it clearly in our setup, we shut down both individual jumps and consider three senarios for the joint jump in which the product of the arrival intensity and jump size, $\lambda\bar{V}X$, is kept constant: i) $\lambda\bar{V} = .01$, $X = -50\%$; ii) $\lambda\bar{V} = .02$, $X = -25\%$; iii) $\lambda\bar{V} = .01 \times \frac{5}{9}$, $X = -90\%$. The results are shown in Figure 3.

From Figure 3, the jump size is much more effective in decreasing the investor's stockholding than the arrival intensity. In other words, investors are more afraid of a rare but large stock market crash than a more frequent but less severe market downside movement. Intuitively, rare but large jumps is more likely to get the investors into illiquid situation while small jumps strike the investors as closer to the local diffusion changes which they deal with every day. Actually, diffusion can be regarded as the limit of jumps in which the jump size approaches zeros while the arrival intensity approaches infinity[23]. Hence,

[19] That investor's utility increases in the expected stock return and decreases in its variance is the starting point for the classicle static portfolio choice models. See Sharpe (1965).

[20] Of course, we also need the agent to be risk averse, i.e. $\gamma > 0$, which assumed throughout the paper.

[21] This is of course not always so theoretically $e^{\gamma B_\tau Y}$ may also work as inflator

[22] See, for example, Liu et al (2003) or Barro (2006).

[23] See Merton (1990), for example.

we deduce that a jump hedging should be a lot larger than a diffusion hedging.

4.2.3 Model calibrations

To calibrate the model, we use the monthly US financial data for the sample period from January1926 to December 2004 from CRSP. We use the value-weighted index returns (cum-dividend) for the stock return and the 90 day T-bill for the risk free rate, both of which are made real using the CRSP inflation data and then annualized. Panel A of Table 3 shows that the calibrated parameters are not related to jumps.

The first three columns reported the equity premium, the stock return volatility and the risk free rate from our sample period. For the remaining columns we take the results from the temporary calibration in last subsection. In particular, we still use Pan (2002)'s estimations for ρ, σ and the convergence speed. We will later do comparative analysis for these three values to test the robustness of the results.

The calibration of the jump-related parameters takes more time. Firstly from what we've discussed in the last subsection, we shut down the individual V-jump since our focus is the portfolio choice problem. We then focus on identifying all the price jumps during our sample period. It is important to set the criteria so as to identify the rare major events from the common local diffusion. In Liu et al (2003), they use a threshold value of -25% for the monthly stock returns which is almost 6 standard deviations below the mean. This is too extreme in our opinion, since there is only 0.62% probability that stock return will drop 2.5 standard deviations below its mean if approximated by a normal distribution, and the probability is less than 10^{-9} for values 6 standard deviations below the mean! Even after considering the "fat tail" property of stock return, their criteria still seems too extreme. In the following we set 2.5 standard deviations as our criteria. In our sample period, the mean and standard deviation for the monthly US stocks are 0.7% and 5.4%, respectively. Hence a jump occurs whenever the monthly return drops below 0.7%-2.5*5.4%=-12.8%, and we find a total of 13 jumps from our sample[24]. To compute the jump size we use $X - 0.7\%$ where X is the return of the month during which jump occurs.

Next we need to find out whether these price jumps are individual or joint with the variance jumps. To do this, we compute the stock volatility for each of the price jump months as follows: for jumps that take place after 1962, the volatility is computed as the annulized standardard deviation of the daily stock returns from CRSP daily stock file for each of the event months[25]. Before 1962, the daily stock data are not available, and hence, we follow Liu et al (2003) to approximate the volatilty by the annualized standard deviation of the returns for the five-month window centered at the even month. Finally we use the threshhold value of $(40\%)^2$ to determine whether the variance jumps or not.[26] Once a variance jump is identified, we use $X^2 - (18.73\%)^2$ to compute its jump size, where X is the computed volatility. The results are summarized in Table 4.

[24] Liu et al (2003) find just eight jumps for their larger sample period from 1802 to 2000 due to their much stricter criteria. Though their estimated jump size is also larger, their results are still likely to underestimate the impact of jump on investors' portfolio choice.

[25] We use the mutilpler $\sqrt{252}$ for annualizing where 252 is the number of trading days each year.

[26] From Pan (2002)'s estimation, the standard deviation of stock variance (volatility of volatility) is $\sqrt{.0035} = 5.91\%$. $(40\%)^2$ is more than two standard deviations above $(18.73\%)^2$, the stock variance of our sample period which is taken as the mean of the variance. Hence it is not unreasonable to regard variance above this threshhold as due to jump.

From Table 4, in a period of 79 years, there are a total of 13 jumps, 4 of which are the joint jumps with an average jump size of -24.1362% for the price jump, and 0.3875 for the variance jump respectively. The remaining 9 jumps are individual price jumps with average jump size -16.4706%. Intuitively, larger price jumps are more likely to be joined with variance jumps but are also rarer. Interestingly, 8 of 13 jumps are clustered in the 1930s showing the possible correlation among different jumps. For the last 60 years or so, however, this correlation feature is largely gone. Hence the calibration of parameters related to jump in Panel B of Table 3. Finally in Panel C we list the derived parameters.

4.2.4 Implications of the model

We show the model implications by plotting and comparing the aggregate demand, the hedging demand to state diffusion and the hedging to the two types of jumps under a wide range of parameter values. In particular, we do the comparative analysis for ρ, σ, and ξ, since they are all directly taken from Pan (2002)'s estimation where no variance jump is assumed and thus might not be appropriate for our model. The results are shown in Figure 4 to Figure 6. For all these cases, the myopic demand is kept unchanged at $\frac{\eta}{\gamma} = 1.0293$.

Firstly let's take a look at Figure 4 for different ρs. We notice right at the beginning that in general the hedging component to jumps are several times larger than that to diffusion. For example, when $\rho = -.57$ and at $\tau = 1$ the absolute value of the hedging demand due to joint jumps and individual P-jumps are 17.7% and 25.7%, respectively whereas the diffusion hedging is less than 4%. Another difference is that the jump hedgings are always negative, meaning the uniform "decreasing effect" of even risk on the agent's stockholding. In contrast, the diffusion hedging can be either negative or positive depending on the sign of ρ. In particular it vanishes when $\rho = 0$.

Since a nonzero ρ is the underlying reason for the existence of diffusion hedging in that the agent can use certain position of the stock to hedge against the variance diffusion, ρ has the direct and dominant effect on diffusion hedging. Hence, the aggregate demand changes with ρ in exactly the same way as the diffusion hedging does. However ρ doesn't change the diffusion linearly since B_τ is also a function of ρ. Actually ρ matters for jump hedging through B_τ and then onto the jump hedging components of ϕ since B_τ and ϕ must be solved together. In particular, both jump hedging components decrease[27] monotonically in ρ.

From the calibration P-jump has smaller jump size though it happens more frequently, but we know larger jump size matters more for jump hedging than the arrival intensity. However, the figure shows that individual P-jump gives the investor stronger incentive to decrease his stockholding. Another difference between the two jump hedging component is the their different horizon effects. In particular, as the investment horizon shortens, the agent has stronger desire to hedge against the joint jump but weaker desire against the individual P-jump[28]. Both are explained by the variance jump term, $e^{\gamma B_\tau Y}$, within the joint jump hedging which serves as a powerful deflator to the P-jump hedging due to a large calibrated

[27] The increase/ decrease of jump hedging refers to the absolute value.

[28] We focus on the cases when $\rho < 0$ because the literature normally documents a negative correlation between the stock price innovation and variance innovation.

variance jump size, but gradually loses its powers as τ decreases.

Next we consider the model implications for different $\sigma^2 s$ in Figure 5. Since $\bar{V}$ is kept constant at 0.0340 σ^2 can be equivalized as the variance of the variance[29]. Notice the similarity between ρ and σ : both affect the diffusion hedging directly; both enters the expression for diffusion hedging linearly but the effect is not linear because B_τ is a function of both ρ and σ; and both matters for the jump hedging through B_τ. Unlike ρ, however, larger σ induces stronge jump hedging (absolute value) for both jump types. Intuitively a more volatile stock variance gives investors the feeling that the whole stock market becomes more unstable, and hence, a jump seems more likely to be expected.

Finally we consider ξs in Figure 6. Note we also plot on the bottom the two components of the hedging demand to the joint jump: $\frac{\lambda}{\gamma}(1+\phi X)^{-\gamma}X$ or the the hedging to the price jump within the joint jump, and $e^{\gamma B_\tau Y}$, the "deflator" due to the variance jump. Again ξ can be equivalized with the convergence speed for the variance since both the $Y s$ and the λs are kept constant[30]. In our model a higher ξ has two effects: it makes the variance converge faster to its average, and it also makes the stock less volatile on average[31]. In a word, a higher ξ means less stock variance in a double sense. As a result, it has a very strong effect on the diffusion hedging. For example, at $\tau = 1$ the diffusion hedging is over 10% when $\xi = 7$, but it rapidly decreases to just about 2.5% when $\xi = 2$.

Unlike σ^2, a higher ξ has a different effect on the two jump hedging components. In particular, it increases the hedging to the individual P-jump but decreases that to the joint jump. Actually the hedging to the P-jump, whether it is individual or joined with the variance jump, always decreases with the increase of ξ as we can see from subfigure (3,1). The reason is the same as that for a lower σ^2 : a higher ξ makes the whole market seem more stable to the investors, and hence, a less desire to hedge against their future illiquidity. The different behavior between the two components is again due to the deflator. In particular, as shown in subfigure (3,2), higher ξ can effectively increase the "deflator", or in other words, effectively decrease its ability to lower the investors' hedging incentive against the P-jump. For example, when $\tau = 1$, the agent holds less than 70% of the P-jump hedging as the hedging to the joint jump when $\xi = 2$. This proportion increases to over 90% when $\xi = 7$. As a result, the hedging to the joint jump behaves just the opposite to the P-jump hedging within it.

In summary, the model has three main implications that survive the above comparative analysis: i) the hedging demand to both types of jumps are important and are usually several times larger than that due to diffusion hedging; ii) the hedging to the individual P-jump is more important than that to the joint jump; iii) the two jump hedging components can behave very differently (horizon effect, response to parameter changes, etc) due to the deflator in the joint jump hedging.

The above calibration gives a stock weight of over 60% which seems too high for the real world practice.[32] One of the possible reasons, as pointed out by Liu et al (2003) may be the possiblity of survivorship bias for US during the last century. In fact, there are many countries which have experienced huge market declines during relatively short periods of time in the last century, and in some cases, major events such as

[29] The variance of the variance is given by $\sigma^2\bar{V}$.
[30] The convergence speed is given by $\xi - Y^v\lambda^v - Y\lambda$.
[31] This is because $\bar{V} = \frac{\kappa}{\xi - Y^v\lambda^v - Y\lambda}$.
[32] For example, Heaton and Lucas (2000) estimate that the average stock share in financial wealth is just 23%.

wars or polictial crises even led to the stock markets being closed for years[33]. Considering these factors, we recalibrate our model by assuming the stock price on average drops by -50% and the variance jumps up to 70% while all the other parameters remain unchanged. Figure 7 shows the results for aggregate demand.

The aggregate demand effectively rises due to increased hedging against jumps. For example, at $\tau = 1$ and for our benchmark values, the aggregate stockholding is less than 53% instead of 62%. In addition, this alternative calibration can roughly match what's found in the empirical study about people's portfolio choice by generating a stockholding of around 25% for $\gamma = 7$ which is about the largest value for a coefficient of relative risk aversion considered feasible by Mehra and Prescott (1985)[34].

4.2.5 Jump misspecifications

In this subsection, we recalibrate our model to two cases of jump misspecifications normally found in the literature: i) the variance always jumps together with the price; ii) there is no variance jump.[35] To see what the difference could be, Table 5 shows the recalibrations where we only list the parameters that need to be recomputed. Panel A shows the recalibration for i) where -18.8292% is the average jump sizes for the whole 13 price jumps while 0.1717 is the average of $X^2 - (18.73\%)^2$ in the last column of Table 4 which is the "variance jump size" if we believe variance always jump together with the price. In Panel B for ii), we simply shut down variance jumps, i.e., set $\lambda \bar{V} = Y = 0$. Figure 8 shows the difference for the aggregate demand as well as its three components.

The difference is sizable though not very large for our benchmark values: at $\tau = 1$, the aggregate demand is 63% in our model but decreases to 62% if we think price jumps are always joined by variance jump and further to 60% if variance jumps are simply shut down. For diffusion hedging we have the similar amount of underestimation though the relative change is much larger. Of course, what the misspecification matters the most is the two jump hedging components. For example, if variance jump is shut down, then the hedging demand due to individual P-jumps almost doubles from about -25% up to about -50%!

Next in Figure 9, we study the misspecification effect on the aggregate demand only for different γs since γ is normally regarded as the most important parameter for the agent's portfolio choice. Firstly we notice a reversed horizon effect for $\gamma < 1$ in that the older investors should hold more stocks than the younger ones. In addition the aggregate demand is instead overestimated under both misspecificiations for $\gamma < 1$. For the other three cases when $\gamma > 1$, the misspecification effect is larger for larger γ. For example when $\gamma = 1.5$, the ignorance of variance jump only decreases the total demand from 1.2 to 1.16, a 3.3% decrease which increases to 6.2% (from 0.325 to 0.305) when $\gamma = 6$. Finally we notice that for $\gamma > 1$,[36] shutting down variance jump always decreases the total demand. This is because, when transfered from

[33] Some of the nearest examples include the Mexican financial crisis (1994), the South East Asia financial crisis (1997) and Russian stock market crisis (1998).

[34] Some more radical choice of jump sizes, for example, $X = -80\%$ can generate a plausible stock share for just $\gamma = 3$. Many times what matters to the jump hedging is people's psychological response. People may decrease their stockholding for worrying about very large market crash though what happens may be much less severe ones. Hence we justify a very large price jump size as the one that investors are fearing about.

[35] For example, Liu et al (2003) only consider the joint jump, while Pan (2002) only considers the individual price jump.

[36] $\gamma < 1$ is implausible at least in portfolio choice models since it generally implies a too high stockholding. As pointed out by Samuelson (1991), investors who are less risk averse than logarithm can behave very differently from those whose $\gamma > 1$.

joint jump to the individual price jump, investors get stronger desire to hedge, i.e., to decrease their stockholding. It is worth noticing that under our benchmark values, allowing variance to jump together with price decreases ϕ. This is not always the case, as will be seen in Figure 10.

In Figure 10 we show that the jump misspecification effect could be large for certain parameter values. We consider an investment horizon up to 5 years since there is still a lot of time dependency beyond $\tau = 2$. At $\tau = 5$, the percentage underestimation of the aggregate demand when the variance jumps are not considered are 16.3% (from .86 to .7) and 15.6% (from .77 to .65) for $(\xi = 2, \rho = -.9)$ and $(\xi = 2, \sigma = .5)$ respectively. Interestingly, for the case where P-jump and V-jump always occur at the same time, the aggregate demand is firstly overestimated and then it quickly becomes underestimated by up to 20% (from .9 to 1.08) for $(\xi = 2, \rho = -.9)$ at $\tau = 5$! [37]

5 Stochastic equity premium with jump

Though the literature is dominated by using the stock volatility as the state, the equity premium is also important for people's portfolio choice decision. Below we consider a stochastic equity premium model by extending Kim and Omberg (1996) and Watcher (2002) to the event risk case as follows:

$$\frac{dS}{S} = \mu dt + \sigma\sqrt{Z}dB^s + X^s dN^s + XdN$$

$$dZ = (\kappa - \xi Z)dt + \sigma^z\sqrt{Z}dB^z + Y^z dN^z + YdN$$

where

$$Z \equiv \mu - r^{38}$$

The arrival intensities for the three types of jumps and the two diffusion variances are assumed to be $\lambda^s Z$, $\lambda^z Z$, λZ, $\sigma^2 Z$ and $(\sigma^z)^2 Z$, respectively, where λ^s, λ^z λ, σ, and σ^z are all constants. The assumption makes sense because in reality stock market prosperity characterized by high equity premiums is always followed by larger stock price volatility and even market crash, possibly due to excessive speculation.[39]. Like the stochastic variance case, the equity premium is also assumed to follow a mean reverting process, which combines the assumption about λs and variance to make the whole system affine.

Again let $\rho dt = cov(dB^s, dB^z)$ and assume away consumption, the portfolio policy and the HJB are given by:

[37] Note from the above analysis, smaller ξ means a more unstable stock market. Hence Liu et al(2003)'s setup runs the risk of greatly overestimating the stockholding by investors in the newly emerging market power where the stock market is usually more volatile.

[38] Strictly speaking the equity premium is $(1 + \lambda^s X^s + \lambda X)Z$ not Z because of the existence of jumps. We just continue to call our state equity premium for convenience.

[39] The latest of which is, of course, the burst of the high tech bubble.

$$\phi = \frac{1}{\gamma\sigma^2} + \frac{\rho\sigma^z}{\sigma}\frac{\partial \ln f}{\partial Z} + \frac{\lambda}{\gamma\sigma^2}(1+\phi X)^{-\gamma}X(\frac{f(t,Z+Y)}{f(t,Z)})^{\gamma} + \frac{\lambda^s}{\gamma\sigma^2}(1+\phi X^s)^{-\gamma}X^s$$

$$\begin{aligned} 0 \;=\; & \frac{\partial f}{\partial t} + \frac{1}{2}\sigma^{z2}Zf_{zz} + (\kappa - \xi Z + \frac{1-\gamma}{\gamma}\frac{\rho\sigma^z}{\sigma}Z)f_z + \frac{\gamma-1}{2}\sigma^{z2}Z(1-\rho^2)\frac{f_z^2}{f} \\ & +\{\frac{1-\gamma}{2\gamma^2}\frac{Z}{\sigma^2} + \frac{1-\gamma}{\gamma}r - \frac{\beta}{\gamma} - \frac{\lambda^{s2}Z(1-\gamma)}{2\sigma^2\gamma^2}[(1+\phi X^s)^{-\gamma}X^s]^2 \\ & -\frac{\lambda^2 Z(1-\gamma)}{2\sigma^2\gamma^2}[(1+\phi X)^{-\gamma}X(\frac{f(t,Z+Y)}{f(t,Z)})^{\gamma}]^2 - \frac{\lambda^s\lambda Z(1-\gamma)}{\sigma^2\gamma^2}\cdot \\ & (1+\phi X)^{-\gamma}X(\frac{f(t,Z+Y)}{f(t,Z)})^{\gamma}(1+\phi X^s)^{-\gamma}X^s + \frac{\lambda^s Z}{\gamma}[(1+\phi X^s)^{1-\gamma}-1] \\ & +\frac{\lambda^v Z}{\gamma}[(\frac{f(t,Z+Y^z)}{f(t,Z)})^{\gamma}-1] + \frac{\lambda Z}{\gamma}[(1+\phi X)^{1-\gamma}(\frac{f(t,Z+Y)}{f(t,Z)})^{\gamma}-1]\}f \end{aligned}$$

with the boundary condition:

$$f(T, Z_T) = 1.$$

Again conjecture $f(\tau, Z) = \exp(A_\tau + B_\tau Z)$ with $\tau = T - t$, and we obtain

$$\begin{gathered} \dot{A}_\tau = \kappa B_\tau + \tfrac{1-\gamma}{\gamma}r - \tfrac{\beta}{\gamma} \quad A_0 = 0 \\ \dot{B}_\tau = \tfrac{1}{2}\sigma^{z2}B_\tau^2 + (\tfrac{1-\gamma}{\gamma}\tfrac{\rho\sigma^z}{\sigma} - \xi)B_\tau + \tfrac{\gamma-1}{2}\sigma^{z2}(1-\rho^2)B_\tau^2 + \tfrac{1-\gamma}{2\gamma^2\sigma^2} \\ -M_1 - M_2 - M_3 + M_4 + M_5 + M_6 \quad B_0 = 0 \\ \phi = \tfrac{1}{\gamma\sigma^2} + \tfrac{\rho\sigma^z}{\sigma}B_\tau + \tfrac{\lambda}{\gamma\sigma^2}(1+\phi X)^{-\gamma}Xe^{\gamma B_\tau Y} + \tfrac{\lambda^s}{\gamma\sigma^2}(1+\phi X^s)^{-\gamma}X^s \end{gathered}$$

where

$$\begin{gathered} M_1 = \tfrac{\lambda^{s2}(1-\gamma)}{2\sigma^2\gamma^2}[(1+\phi X^s)^{-\gamma}X^s]^2 \\ M_2 = \tfrac{\lambda^2(1-\gamma)}{2\sigma^2\gamma^2}[(1+\phi X)^{-\gamma}Xe^{\gamma B_\tau Y}]^2 \\ M_3 = \tfrac{\lambda\lambda^s(1-\gamma)}{\sigma^2\gamma^2}(1+\phi X^s)^{-\gamma}X^s(1+\phi X)^{-\gamma}Xe^{\gamma B_\tau Y} \\ M_4 = \tfrac{\lambda^s}{\gamma}[(1+\phi X^s)^{1-\gamma}-1] \\ M_5 = \tfrac{\lambda^v}{\gamma}[e^{\gamma B_\tau Y^z}-1] \\ M_6 = \tfrac{\lambda}{\gamma}[(1+\phi X)^{1-\gamma}e^{\gamma B_\tau Y}-1] \end{gathered}$$

We use annual US financial data from 1926 to 2004 to calibrate parameters not related to jumps. The equity premium is computed as the difference between the anual NYSE/AMEX index returns and the 90 day US T-bill returns, and the $\frac{dS}{S}$ sequence is computed as the annual growth rate of the CRSP total market values. The standardard deviation of the equity premium is 20.26% while the correlation coefficient between the premium and $\frac{dS}{S}$ is .977[40]. Finally we discretize the equity premium process as the following:

[40] Note Watcher (2002) simply sets $\rho = 1$. The main reason that ρ is very close to one in our calibration is because the VWRETD file is actually generated from growth rate of total market value file with some adjustment and there is not much

$$Z_{t+1} = \kappa + (1 - \xi + \lambda^z Y^z + \lambda Y) Z_t + \epsilon$$

where ϵ is the error term with a mean of zero. We run the autoregression for the equity premium and use the intercept as the approximation for κ, which will undergo comparative analysis in the following. The results are shown in Column 6 to 9 in Panel A of Table 6, where Columns 1 to 5 are directly taken from the calibration of the previous example.

To calibrate the jumps, we make the simplest assumption that the equity premium doesn't jump[41] and take all the 13 jumps from last example as the individual price jumps. Panel B of Table 6 reports the results. Finally we use $\bar{Z} = \frac{\kappa}{\xi - \lambda^z Y^z - \lambda Y}$ to get the derived parameters in Panel C. The model implications shown in Panel 11 for the aggregate demand and its three hedging components under different ρs and κs whose values are less likely to be accruately estimated. Note the myopic demand is kept constant at $\frac{1}{\gamma \sigma^2}$ =1.1956

Comparing Figure 11 with Figure 4 to 6, the stochastic equity premium model implies a higher aggregate stock demand (66% vs 63%) and higher jump hedging (52.6% vs 17.7%+25.5% =43.2%, absolute value) than that of the stochastic variance model. In addition, the jump hedging is even more important with respect to the diffusion hedging in the current setup. The main differences, however, are i) we now have a negative diffusion hedging instead of postive one. Intuitively the agent wants to short stocks so as to hedge against the negative innovation of the expected stock price changes which also decreases the stock price itself due to their positive correlation. ii) the horizon effect for the aggregate demand is now reversed which means the older investors instead of the younger ones should hold more stock.

Like the stochastic variance case, the changes of parameters not related to jump size or arrival intensities also matter for the jump hedging through B_τ though the effect is small. Both larger ρ and smaller κ tend to strengthen the diffusion hedging since larger ρ makes the holding of a leveraged position of stock a more effective way for hedging premium diffusion, while a smaller κ means smaller average equity premium which makes investors more worried about the premium diffusion. Though the amount of change is small, the relative change is huge: at $\tau = 1$, with the increase of ρ from 0.5 to 0.977, the absolute value of the diffusion hedging doubles from 0.5% to 1%! And the relative change due to κ is even more radical. In addition, the change of diffusion hedging due to parameter changes determines that of the aggregate demand.

The comparison of the above two examples tells us the importance of different modeling of the underlying state which can generate reversed horizon effect, reversed diffusion hedging, etc even for the same stock/ riskfree allocation problem. Of course, it may be even better to study this problem within our general framework by modelling both the variance and the equity premium as the states. The challenge is

variance for US Tbills. In the literature, the instantaneous return, μ, is always calibrated with the realized stock returns (aggregate stock price changes) though it is actually the expected part of the price change. Normally this doesn't matter since the expected part averages to zero. Since we explicitly model the difference between the gross stock price change, $\frac{dP}{P}$, and its expected part μ in our setup to compute their correlation, this standard calibration may be problemic and a better way is to use the professional forecast data about stock returns for μ, like what Fan (2005) does for the expected bond returns.

[41] This is not unreasonable since price jumps usually come unexpected while equity premium represents the expected component of price changes. In figures not reported, we find that the results are largely unchanged if we model jumps for Z.

the mathematical tractability. In particular, it becomes hard to make the systme affine with two states,[42] which means that we have to deal with the original PDE directly. This difficulty, combined with the difficulty at the presence of consumption and market incompleteness, points to a potentially fruitful way for future research, which will be summarized in the conclusion. Finally, we want to point out that one implication that survives different setups is the great importance of jump hedging within the hedging demand components.

6 Conclusion

In this paper, we study the effect of event risk on people's portfolio choice by firstly proposing a general framework and then apply it to three specific examples. Our main findings include: i)hedging demand due to jumps are very important under a wide range of parameter values and different setups, and in general several to dozens of times larger than that to state diffusion; ii) It is important to model multiple types of jumps since they are not only supported from the calibration of the US stock market data, but also plays different roles in agents' asset allocation. In addition, the over/ underestimation of the portfolio weight under jump misspecification could be very large; iii) the different choices of state can generate very different implications even for the same asset allocation problem.

There are several directions for future research both within and beyond our general framework. Within the framework, the most important thing to do is to identify appropriate states for various portfolio choice problems, not only that of stock/riskfree allocation which is the most common in literature, but also stock/ risky bond allocation, allocations between different bond portfolios, stock portfolios, funds, etc. To give an example, some recent papers[43] about stock/ riskfree allocation start with asymmetric information or/and the unobservable parameters of the stock price process. Then, after a change of the information filtration and the corresponding Brownians, the asymmetry or/and unobservability is replaced by a series of states computed as the moments of the unobservables under the new filtration, which exactly fits into our framework.

Another direction is a mathematical one: to study the original portfolio choice problem from the perspective of the corresponding PDE. Roughly speaking, this study can be in two categories: the first is whether we can and how to decompose the original PDE into a series of ODEs at the presence of nonhomogenous (due to consumption) and nonlinear (due to market incompleteness) terms? Without event risk, this is usually equivalent to whether we can find closed-form solutions to the original portfolio choice problem. At the presence of event risk , the decomposition enables us to avoid solving the original PDE directly. The key difference between PDE and ODE in our framework is whether the states are the arguments. If yes as in the PDE case, two pairs of boundary conditions for the states are demanded which are largely missing from the model itself and can only be got via mathematical analysis or even

[42] The problem is with the two terms $(\Sigma\rho'\Sigma^{x\prime})'(\Sigma\Gamma\Sigma')^{-1}(\mu - r)$ and $(\mu - r)'(\Sigma\Gamma\Sigma')^{-1}(\mu - r)$ in the original PDE which becomes $\frac{\Sigma^x Z}{\sqrt{V}}$ and $\frac{Z^2}{V}$ where $Z{=}\mu - r$ in the stock/ riskfree allocation setup. Neither of them can be made affine in Z or V without a deterministic relation between the two states, which means only one state is needed.

[43] See, for example, Xia (2001), Turmuhambetova (2005).

assumptions. This lead to the second category of the study for PDE: a mathematical identification of the boundary conditions if the decomposition is not available; and the robustness of the numerical results under various boundary conditions in the case that these condtions must be assumed[44]. Though mathematical, this research could be very helpful economically since once solved, we can just forget about restrictions such as affine returns which usually cannot be satisfied; and we can study consumption choice as well as portfolio choice under all types of markets, whether they are complete or not.

Beyond our framework, there are also two directions for future work: Firstly and relatively directly, we can extend the CRRA preference to the more general "stochastic differential utility" proposed by Duffie and Epstein (1992), which is the time continous counterpart of the traditional Epstein and Zin preference (1989). Chacko and Viceira (2004) uses this general preference to study stock/riskfree allocation problem in a specific setup wth the reciporical of the stock volatility as the state. It remains an interesting problem to embed this generalized preference into our general framework and to apply it to some other specific setups.

In the real world, asset price determination and asset allocation are actually one problem because they always interwine with each other: price changes generate different demands (allocations) and different allocations drive prices up and down. In the academic world, however, we always assume exogenous price processes with respect to asset allocation[45] when studying portfolio choice problem; On the other hand, asset allocation is always ignored when we study endogeneous asset prices.[46] Hence the second and much more difficult direction is to go beyond our partial equilbrium framework to study the portfolio choice problem in a general equilibrium framework that will gives us a complete understanding of the interaction between price dynamics and investor's portfolio choice.

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[44] Xia (2001) solves her PDE directly by imposing the boudary conditions from either mathematical analysis or assumptions. However, the robustness of her numerical results is not checked.

[45] The stock prices are always directly assumed exogenous; while the bond prices are always assumed driven by exogenous instantaneous bond return.

[46] Veronesi (1999) gives a stock price that is consistent with some equilibrium conditions while studying the consumption/portfolio choice problem at the same time. However, his setup is still not general equilibrium in that the stock price is not computed as the expected discounted value of the dividends explicitly modelled in his paper together with the pricing kernel determined by the optimal consumption policy.

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Table 1: The calibration of Merton (1971) model with price jumps

Panel A					
$\mu + \lambda U - r$	$\sqrt{\sigma^2 + \lambda U^2}$	γ	λ	U	r
8%	20%	3	.05	-20%	1%

Panel B				
μ	σ^2	ϕ	$\frac{\mu - r}{\gamma\sigma^2}$	$\frac{\lambda}{\gamma\sigma^2}(1+\phi U)^{-\gamma} U$
0.0950	0.0375	0.6557	0.7895	-0.1337

Table 2: the temporary calibration of the stochastic variance model with three types of jumps

Panel A							
$(\eta + \lambda^s X^s + \lambda X)\bar{V}$	$\sqrt{(1 + \lambda^s X^{s2} + \lambda X^2)\bar{V}}$	r	γ	β	ρ	σ	$\xi - Y^v \lambda^v - Y\lambda$
8%	20%	.01	3	.02	-.57	.38	5.3

Panel B						
$\lambda^s \bar{V}$	$\lambda^v \bar{V}$	$\lambda \bar{V}$	X^s	X	Y	Y^v
$\frac{.05}{3}$	$\frac{.05}{3}$	$\frac{.05}{3}$	-50%	-50%	$(50\%)^2$-$\bar{V}$	$(50\%)^2$-$\bar{V}$

Panel A								
λ^s	λ^v	λ	$\bar{V}$	Y^v	Y	η	κ	ξ
0.4310	0.4310	0.4310	0.0387	0.2113	0.2113	2.2396	0.2051	5.4821

Table 3: The calibration of stochastic variance model with the US financial data

Panel A							
$(\eta+\lambda^s X^s+\lambda X)\bar{V}$	$\sqrt{(1+\lambda^s X^{s2}+\lambda X^2)\bar{V}}$	r	γ	β	ρ	σ	$\xi - Y^v\lambda^v - Y\lambda$
7.39%	18.73%	.0103	3	.02	-.57	.38	5.3

Panel B						
$\lambda^s\bar{V}$	$\lambda^v\bar{V}$	$\lambda\bar{V}$	X^s	X	Y	Y^v
$\frac{9}{79}$	0	$\frac{4}{79}$	-16.4706%	-24.1362%	0.3875	0

Panel C						
λ^s	λ^v	λ	$\bar{V}$	η	κ	ξ
3.3623	0	1.4944	0.0340	3.0880	0.1802	5.8791

Table 4: The summary of jumps for the US stock market from Jan. 1926 to Dec. 2004

event month	p-jump size %	volatility %	v-jump or not	v-jump size if Y	$X^2-(18.73\%)^2$
19291031	-20.2680	38.70	N	-	0.1147
19300630	-15.9399	37.5349	N	-	0.1058
19300930	-13.6689	25.0049	N	-	0.0274
19310930	-29.2051	47.8378	Y	0.1938	0.1938
19311231	-13.5947	32.5526	N	-	0.0709
19320430	-18.1177	39.4356	N	-	0.1204
19320531	-20.2333	74.4602	Y	0.5194	0.5194
19370930	-14.7938	28.4111	N	-	0.0456
19380331	-24.3750	49.0813	Y	0.2058	0.2058
19400531	-22.7300	39.4673	N	-	0.1207
19800331	-13.6147	24.30	N	-	0.0240
19871030	-22.7168	81.61	Y	0.6309	0.6309
19980831	-15.6081	29.62	N	-	0.0527

Table 5: The recalibration of the two jump misspecification cases.
In Panel A, variance always jump together with the price; In Panel B, there is no variance jump

Panel A						
$\lambda^s\bar{V}$	$\lambda\bar{V}$	X^s	X	Y	λ	$\bar{V}$
0	$\frac{13}{79}$	0	-18.8292%	0.1717	5.4106	0.0342

Panel B						
$\lambda^s\bar{V}$	$\lambda\bar{V}$	X^s	X	Y	λ^s	$\bar{V}$
$\frac{13}{79}$	0	-18.8292%	0	0	5.4106	0.0342

Table 6: the calibration of the model with stochastic equity premium as the state

Panel A							
$(1+\lambda^s X^s+\lambda X)\bar{Z}$	$\sqrt{(\sigma^2+\lambda^s X^{s2}+\lambda X^2)\bar{Z}}$	r	γ	β	ρ	$\sqrt{(\sigma^{z2}+\lambda^z Y^{z2}+\lambda Y^2)\bar{Z}}$	κ
7.39%	18.73%	.0103	3	.02	.977	20.26%	1.6606

Panel B						
$\lambda^s\bar{Z}$	$\lambda^z\bar{Z}$	$\lambda\bar{Z}$	X^s	X	Y	Y^z
0	0	$\frac{13}{79}$	0	-18.8292%	0	0

Panel C						
λ^s	λ^z	λ	$\bar{Z}$	σ^2	σ^{z2}	ξ
0	0	1.5689	0.1049	0.2788	0.3357	15.5349

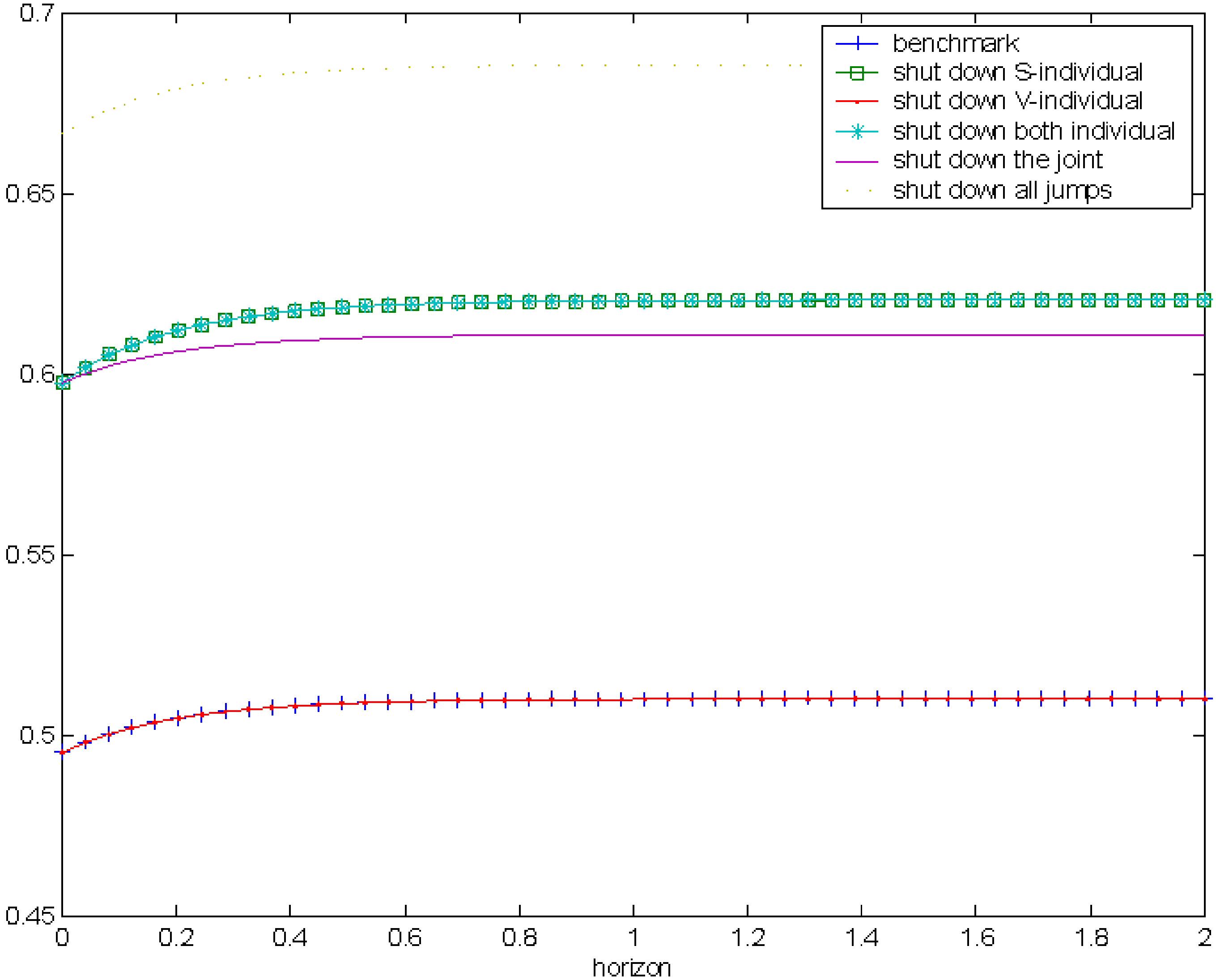

Figure 1: the aggregate demand for stocks under six cases for the temporary calibration of the stochastic variance model.

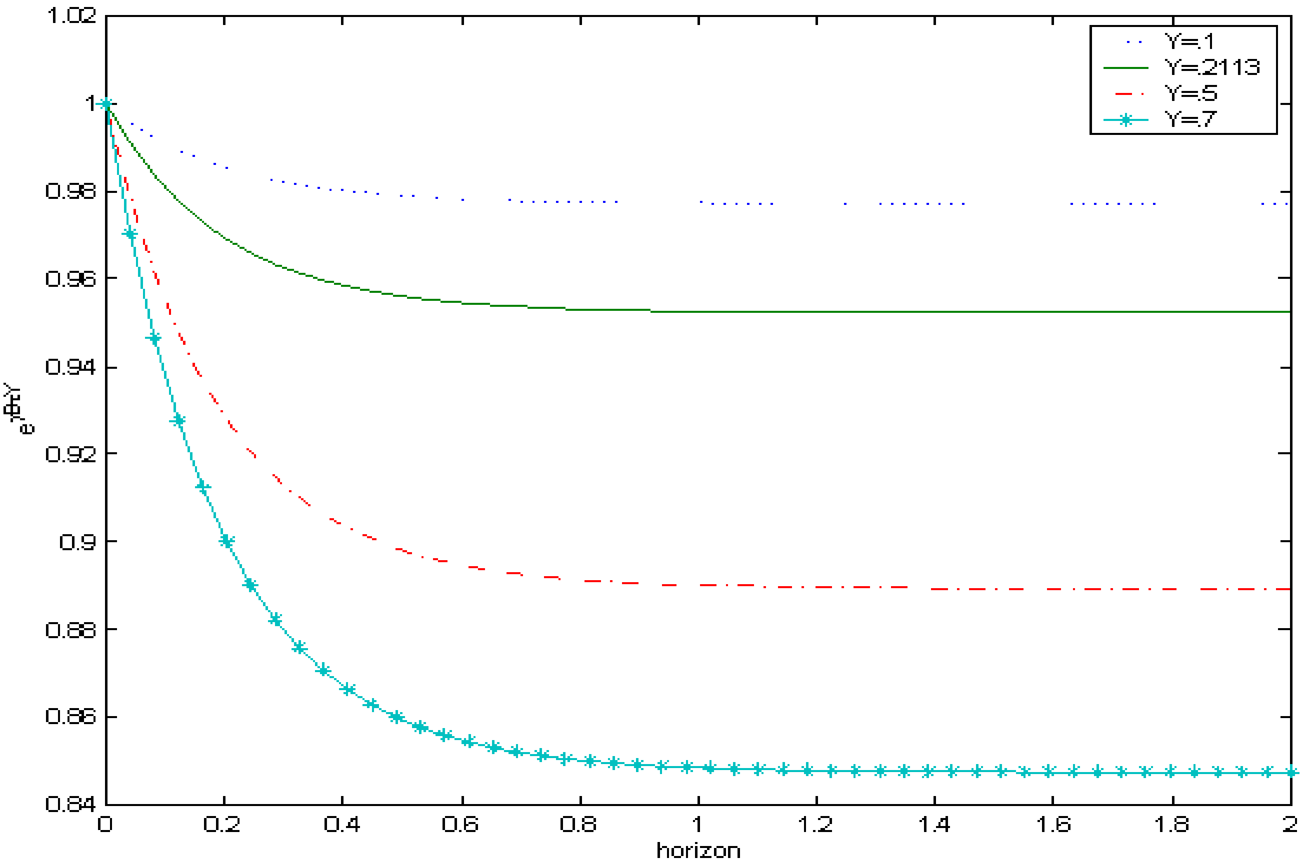

Figure2: the deflator, $e^{\gamma B_\tau Y}$, as a function of horizon for different sizes of variance-jumps

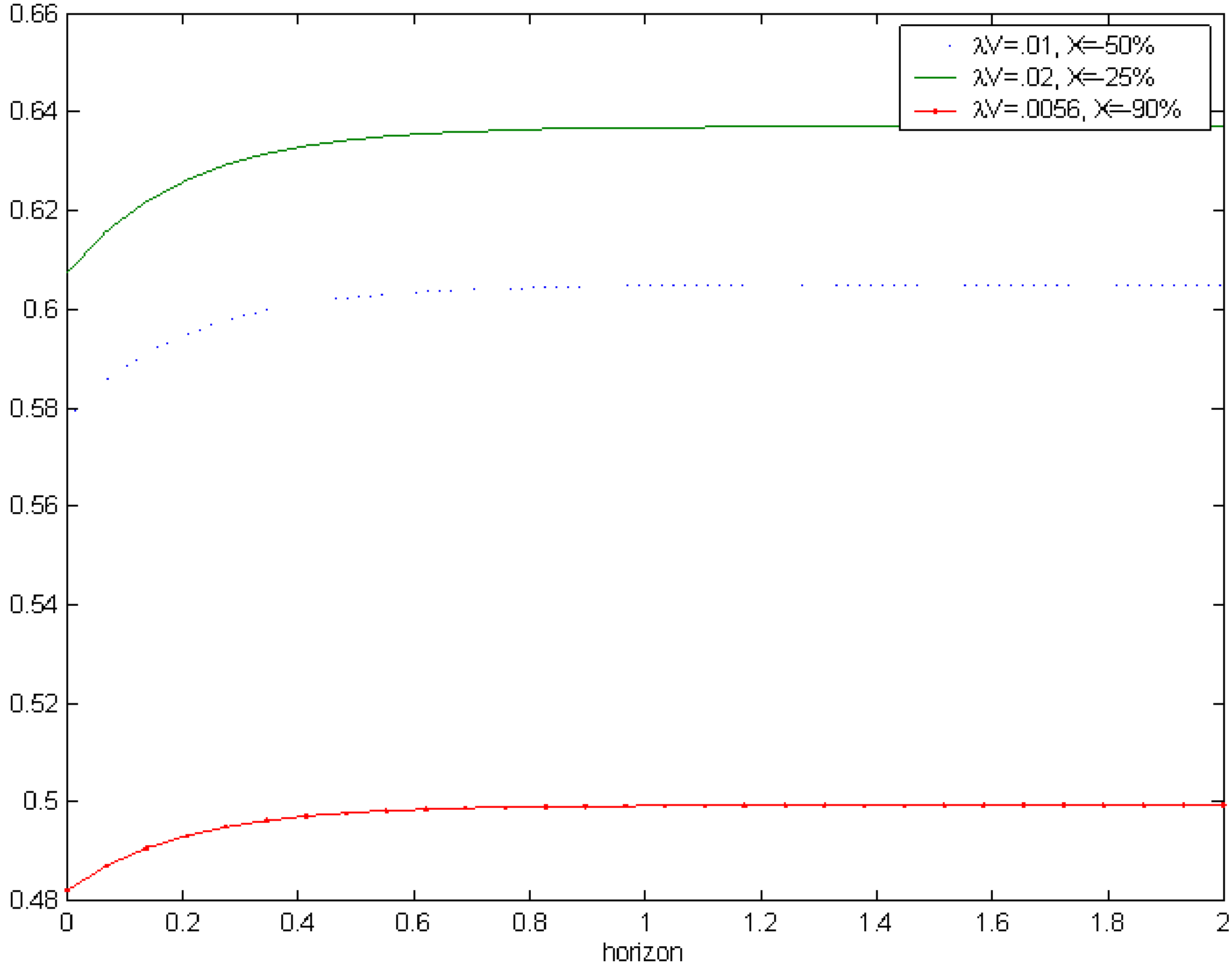

Figure 3: the asymmetric effect of arrival intensity and jump size on the portfolio weight.

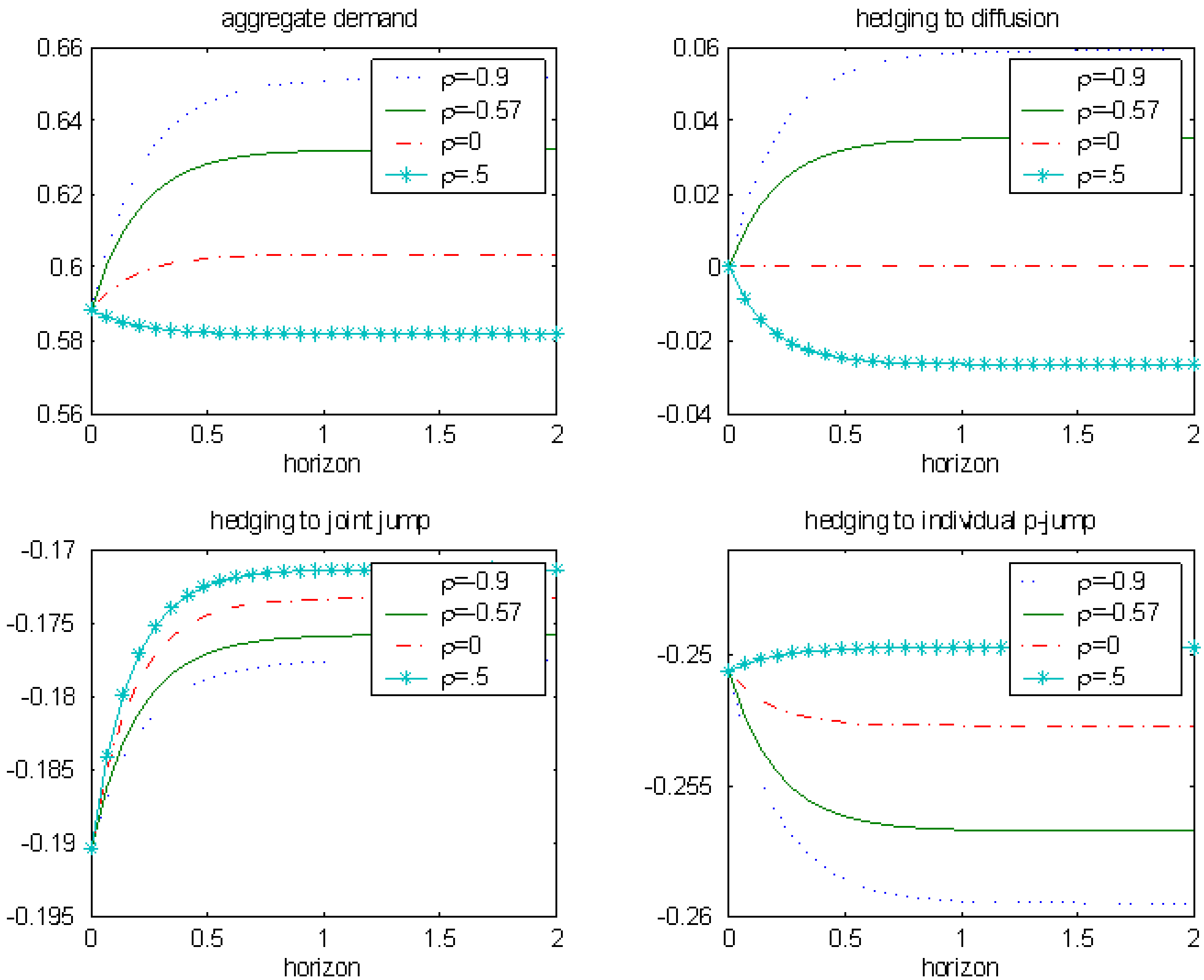

Figure 4: The implications of stochastic variance model for different ρs

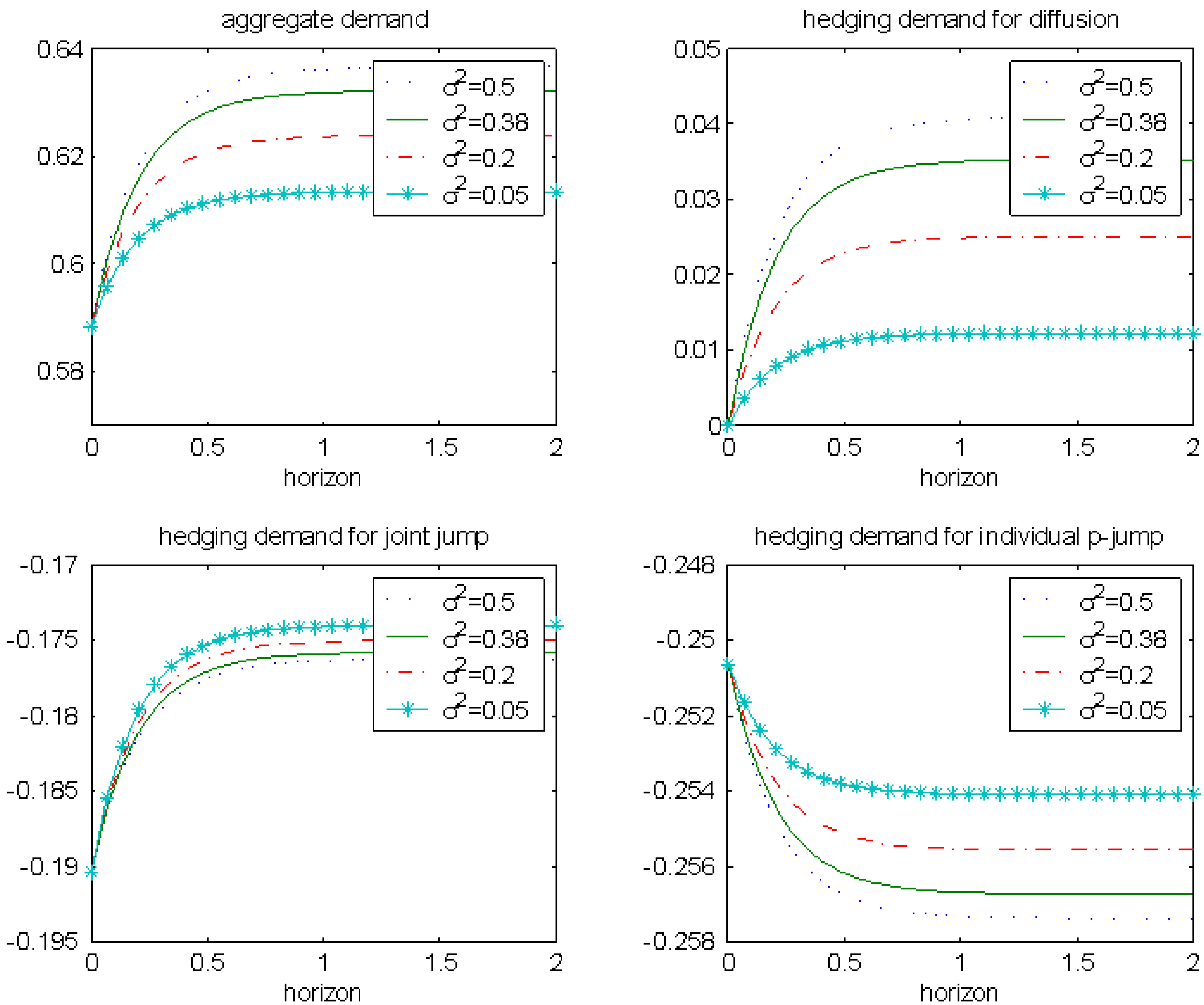

Figure 5: The implications of stochastic variance model for different σs

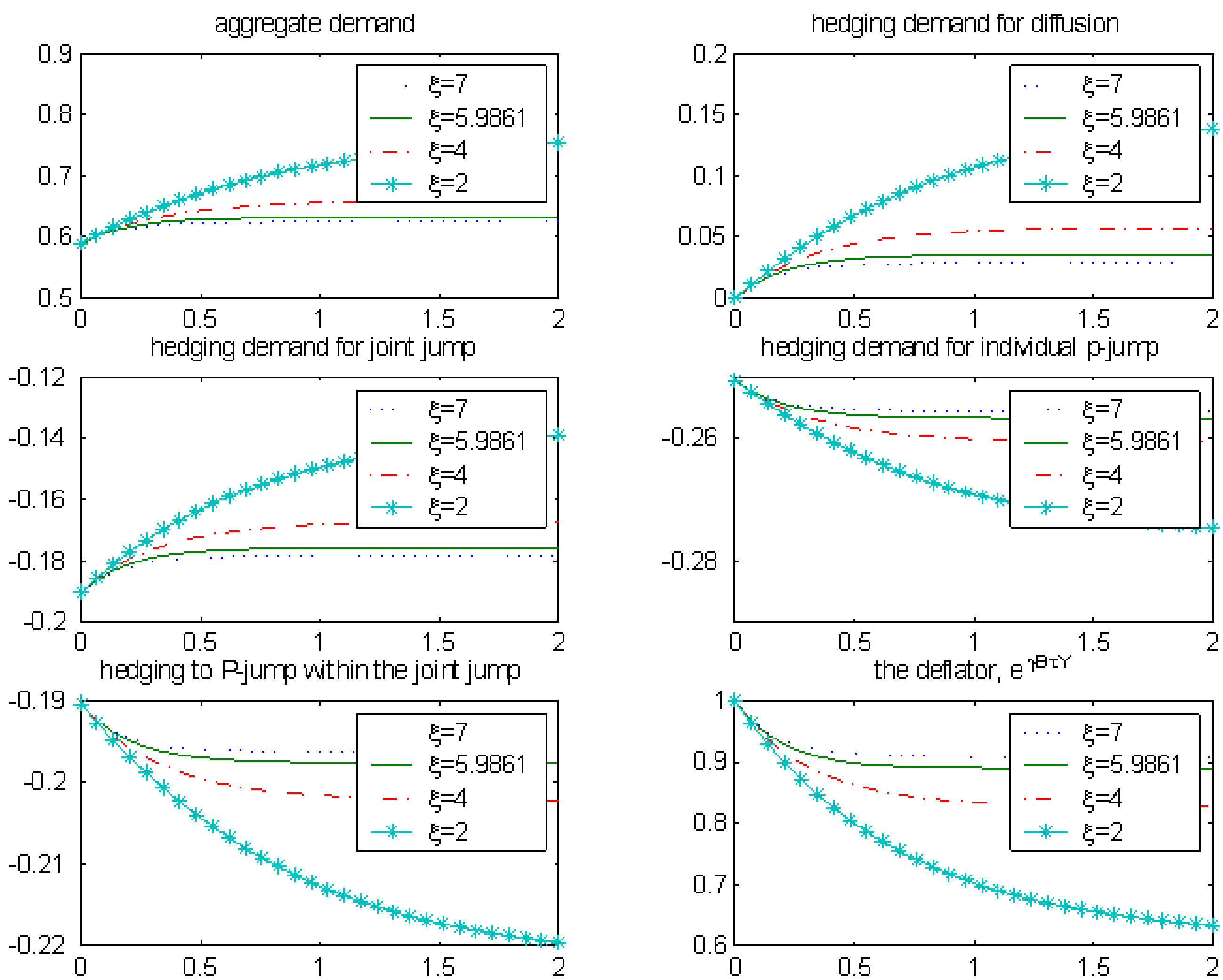

Figure 6: The implications of stochastic variance model for different ξs

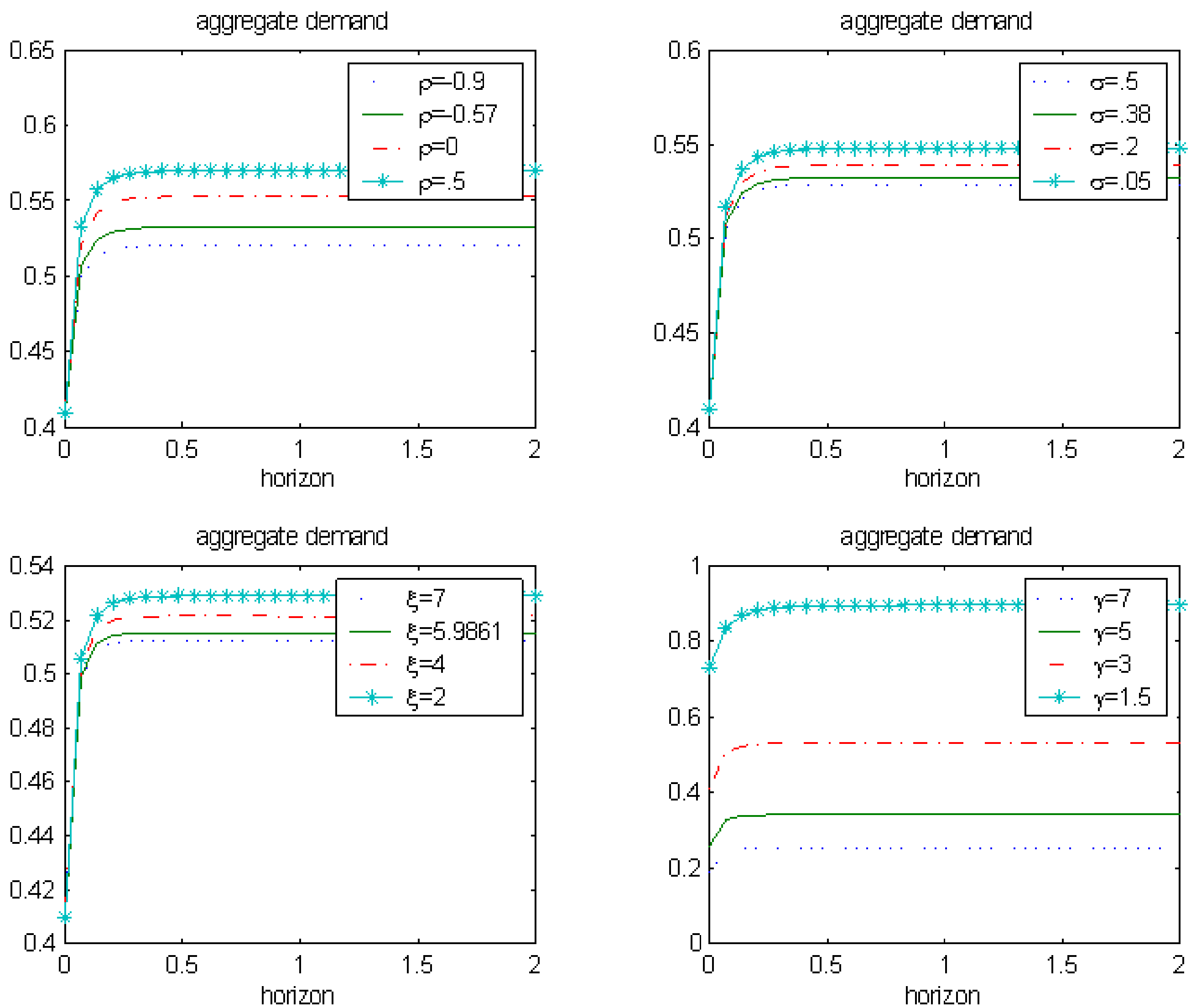

Figure 7: the aggregate demand under alternate calibrations when the stock price drops by 50% and stock variance rises up to 70% under various parameter values.

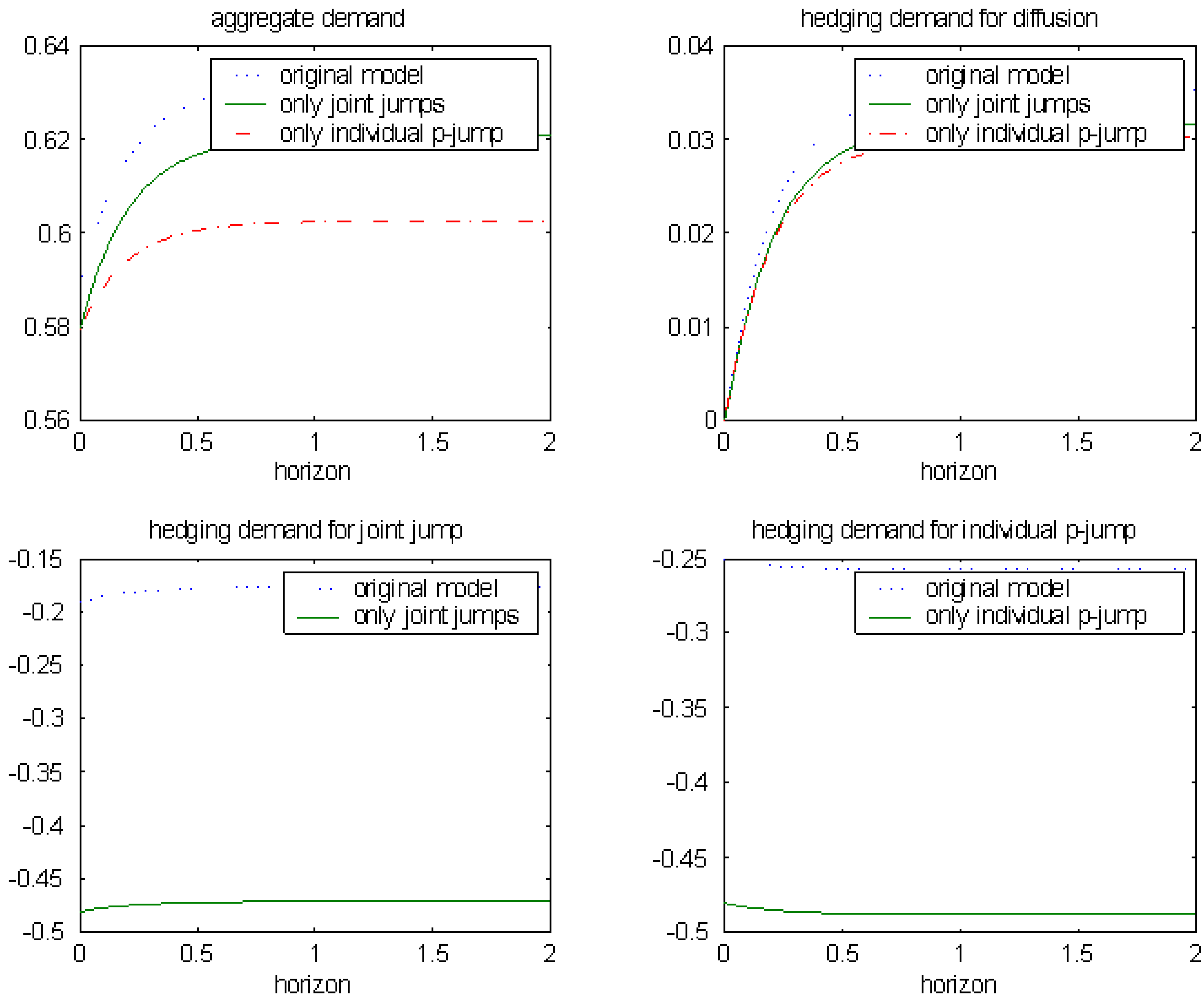

Figure 8: the difference between our original model and the two jump-misspecification models for the aggregate demand and its three hedging componets.

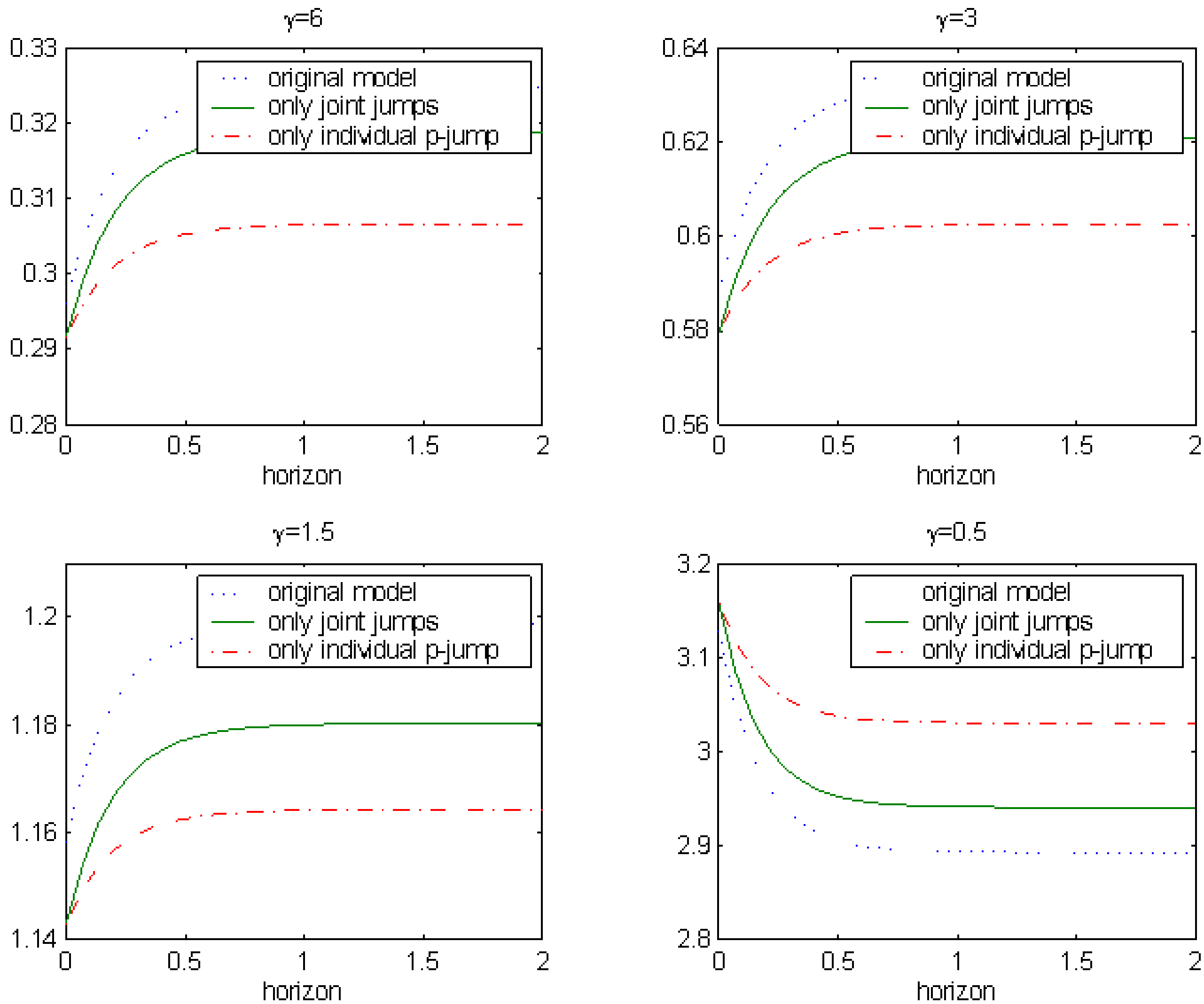

Figure 9: the effect of jump-misspecifications on the aggregate demand for different γs.

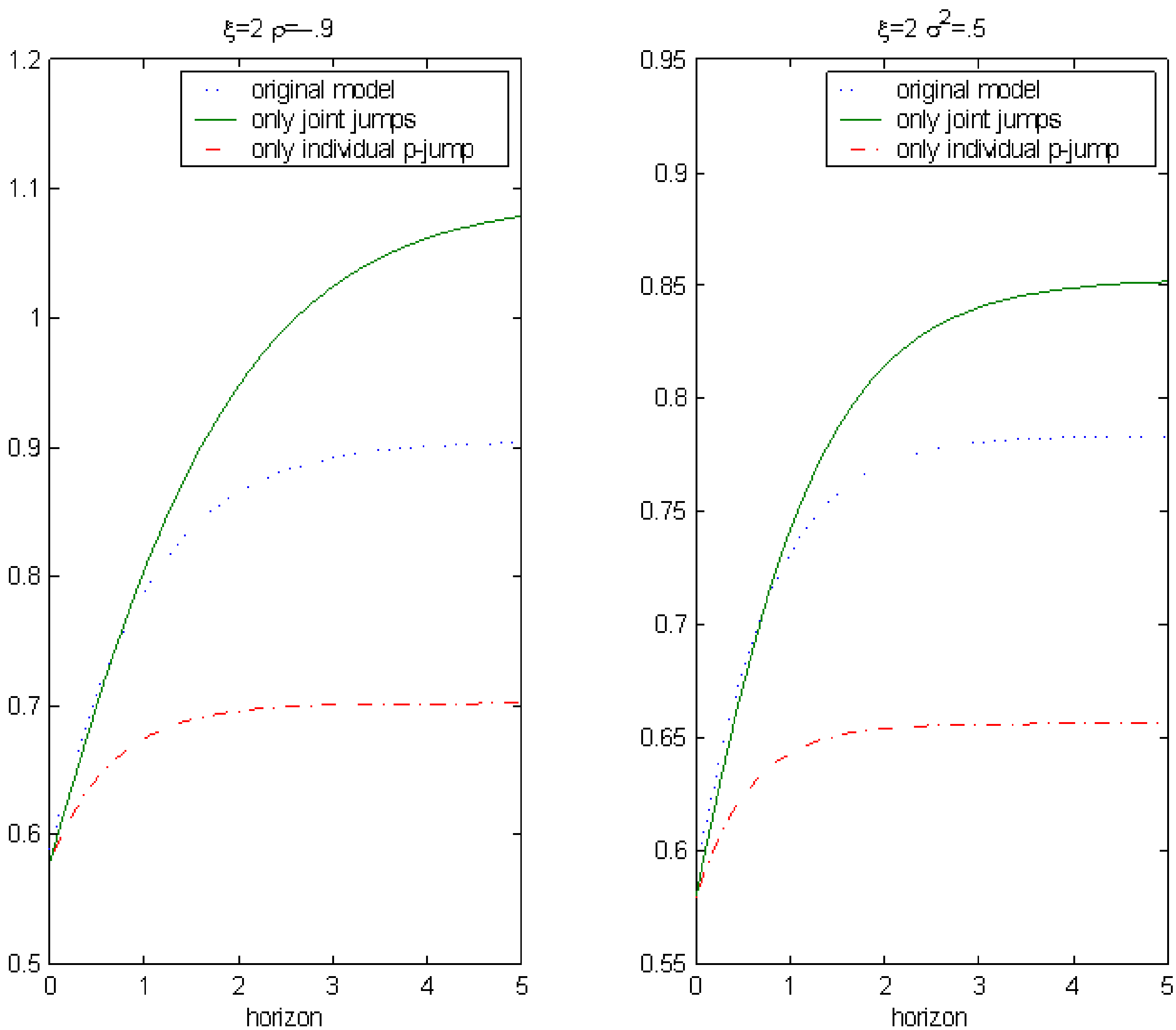

Figure 10: the large effect of jump-misspecifications on the aggregate demand under certain parameter values.

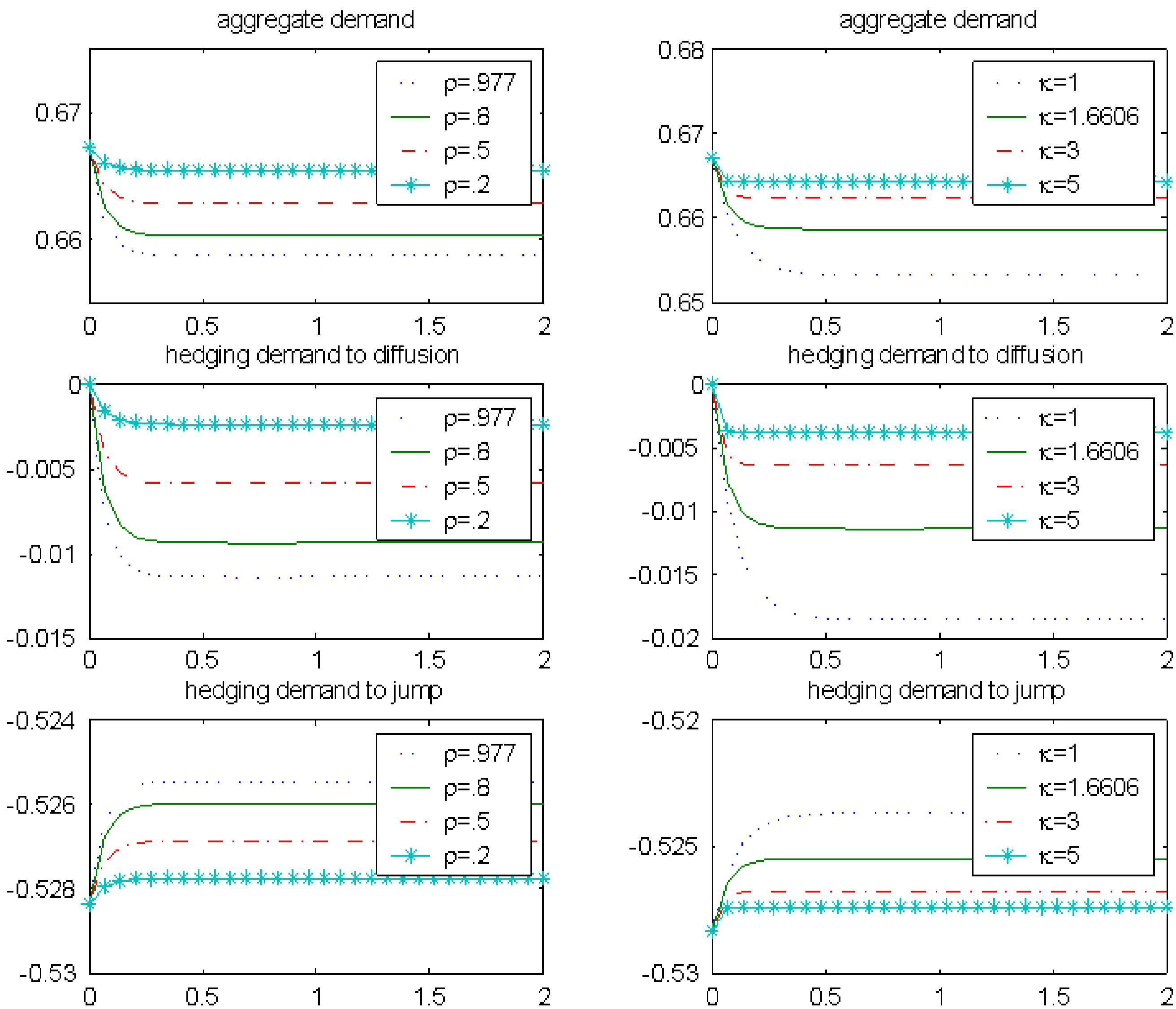

Figure 11: the implications of the model with the stochastic equity premium as the state.