Foreign Asset Accumulation, Macroeconomic Policies and Mercantilism*

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Abstract

This paper extends the Viner (1948, 1968)-Zou (1997) model of mercantilism by introducing money and foreign exchanges and reexamines macroeconomic effects of policy shocks. It is shown that in the long run, consumption and foreign asset accumulation increases as a result of stronger mercantilist sentiments, permanent increases in the consumption tax, increases in the monetary growth rate and purchases of foreign bonds; and in the short run, macroeconomic disturbances including the mercantilist sentiments, the monetary growth rate, and the consumption tax have negative effects on current consumption and positive effects on current foreign asset accumulation, while purchasing foreign bonds has positive effects on both current consumption and current foreign asset accumulation.

Keywords: Foreign Asset Accumulation; Mercantilism; Money; Macroeconomic Policies

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1 Introduction

The paper extends the Viner (1948, 1968)-Zou (1997) model of mercantilism by introducing money and foreign exchanges in a framework of the modern theory of international finance, and reexamines the theoretical predictions of the model in the long run and short run.

Even though mercantilism has been examined, criticized, or even ridiculed ever since Adam Smith, some formal models of mercantilism have been developed. In a framework of the strategic trade theory, Irwin (1991) develops a model of mercantilism and proves that it is profitable for the country utilizing trade protection policies (that is, export subsidies). Zou (1997) offers a dynamic model of mercantilism according to the interpretations by Jacob Viner (1937, 1948, 1968), Schmoller (1897), Cunningham (1907, 1968) and Heckscher (1935) and shows that a permanent increase in the mercantilist sentiments or import tariffs leads to more foreign asset holdings and more total consumption in the long run. Even though within a different framework, the positive effect of import tariffs in the Zou (1997) dynamic model corresponds to the positive effect of export subsidies in the Irwin (1991) static model. Furthermore, Zou (1997) tells that it should also be an interesting open question to study fiscal policy, monetary policy and the exchange-rate theory in the Viner-Zou model of mercantilism. Later, Mcdermott (1999) shows that mercantilism does harm to the long-run economic growth of the implementing countries by modeling mercantilism as a channel of public finance. Theoretically, it may be important to develop new models or extend the existing ones of mercantilism.

Moreover, in the policy circles mercantilism has been resurrected over and over again. Governments continue to follow mercantilist policies and become aggravated after the eruption of 2008 financial crisis. In the academic circles, more attentions were paid to economics of mercantilism and its policy implications, for example, Garcia and Soto (2004), Jeanne and Ranciere (2005), Dooley at al. (2005), Aizenman and Lee (2007), Samuelson (2007), Durdu at al. (2009), Krugman (2009), Blanchard and Milesi-Ferretti (2009), Richman at al. (2010), and Stiglitz (2012) etc.. To address the central theme of mercantilism and try to answer the open question proposed by Zou (1997), this paper extends the Viner-Zou model of mercantilism by introducing money and foreign exchanges and reexamines the policy implications of macroeconomic policies.

We organize the study as follows. In section 2, we outline the structure of the model and examine their basic dynamics. We define the utility function of a representative nation not only on both consumption and foreign asset accumulation to capture ‘power vs plenty’ as objectives of mercantilism (Viner, 1948, 1968), but also on money to capture mercantilism as a ‘system of money’ (Heckscher, 1935). In section 3, we first look at how the mercantilist mentality affects long-run consumption, real money balances and foreign asset accumulation and show that a
nation with stronger mercantilist sentiments will have higher long-run consumption and foreign asset holdings. And higher rate of monetary growth leads to more consumption and foreign asset accumulation. Furthermore, consumption tax leads to higher levels of consumption, real money balances and foreign asset holdings, just as what Jacob Viner (1937, 1948, 1968) had said that in order to maximize long-term standard of living mercantilist will surpress the current standard of living. Finally, purchasing foreign bonds from the private sector improves long-run levels of consumption, real money balance holdings and foreign asset accumulation. In section 4, we utilize a technique developed by Judd (1982) and Cui and Gong (2006) to analyze the effects of various exogenous shocks on consumption, real balances and foreign asset accumulation at the initial equilibrium. In section 5, we summarize the main findings and point out directions for future research.

2 The Viner-Zou Model of Mercantilism

We consider a small open economy in a competitive world market, which is populated with many identical agents. Combining Zou (1997)’s modeling strategy for mercantilism and Heckscher’s idea “mercantilism as a system of money”, we define the instantaneous utility function of a representative agent as

$$U(c_t, m_t, b_t) = u(c_t, m_t) + \beta w(b_t),$$

where $c_t$ is per capita consumption, $m_t$ is per capita real money balances, $b_t$ is per capita foreign bonds, and $\beta (> 0)$ measures the mercantilist sentiments in the words of Cunningham (1907) or the mercantilist mentality in the viewpoint of Heckscher (1935). It is assumed that the function $u(c, m)$ is an increasing and concave function of its two arguments, satisfying Edgeworth complementarity (i.e., $u_{cm} > 0$)\(^1\), and $w(b)$ is an increasing and concave function of foreign asset holdings. In the spirits of Jacob Viner (1948, 1968), the utility part $u(c, m)$ can be understood as the utility from plenty (or opulence), whereas the part $\beta w(b)$ can be regarded as the power that people (or a nation) possess and enjoy.\(^2\)

\(^1\)Here money enters the economy by one standard way of “money in the utility function” (MIU) forwarded by Sidrauski (1967). The other way that money enters is through the “cash-in-advance” (CIA) constraint pioneered by Stockman (1981) and Lucas and Stokey (1987). Feenstra (1986) shows that the two ways introducing money in the economy are equivalent under some conditions.

\(^2\)In its abstract form, the mercantilist utility function is similar to the “wealth effect” model as Kurz (1968), the “spirit of capitalism” models as Zou (1994), Baskin and Chen (1996), Smith (2001), and Luo, Smith and Zou (2009), and the “social status” model as Robson (1992) and Luo and Yong (2009). For more mercantilist arguments, Zou (1997) summerizes the viewpoints of Cunningham (1907), Heckscher (1935), Viner (1937, 1948, 1968) and Allen (1987).
The optimization problem of the representative agent with an infinite horizon is maximizing
\[ \int_0^\infty [u(c_t, m_t) + \beta w(b_t)]e^{-\rho t} dt, \]
subject to the budget and stock constraints
\begin{align*}
\dot{a}_t &= y + rb_t + x_t - (1 + \tau)c_t - \pi_t m_t, \\
a_t &= b_t + m_t,
\end{align*}
and the initial condition \( b(0) = b_0 \). Thereinto, \( y \) is the exogenously given real output, \( x_t \) is the real transfers from government, \( a_t \) is the total wealth of the representative agent including foreign bonds \( b_t \) and real money balances \( m_t \), \( r \) is the returns on foreign bonds, \( \pi_t \) is the expected inflation rate, and \( \tau \) is the tax on consumption.

The home price of the consumption good is \( P_t \), and the corresponding world price is \( P^*_t \). Assuming purchasing power parity, we have \( P_t = E_t P^*_t \), where \( E_t \) is the exchange rate. With proper normalization, \( P^*_t \) can be set to one. Then, \( P_t = E_t \).

The Hamiltonian function is defined as follows
\[
H = u(c, m) + \beta w(b) + \lambda[y + rb + x - (1 + \tau)c - \pi m] + \mu(a - b - m),
\]
where \( \lambda \) and \( \mu \) are the Hamiltonian and Lagrangian multipliers of the two constraints, respectively. The necessary conditions for optimization are as follows:
\begin{align*}
&u_c(c, m) - (1 + \tau)\lambda = 0, \\
u_m(c, m) - \lambda \pi - \mu = 0, \\
&\beta w'(b) + r\lambda - \mu = 0, \\
&\mu - \rho\lambda + \dot{\lambda} = 0, \\
&\lim_{t \to \infty} e^{-\rho t} \lambda b = 0.
\end{align*}
Equation (3) shows that the marginal utility of consumption equals the shadow price of the total wealth per capita. From equations (3), (4) and (5), we have
\[
\beta w'(b) = u_m(c, m) - \frac{r + \pi}{1 + \tau}u_c(c, m),
\]
which tells that the marginal benefits of holding foreign assets, i.e., $\beta w'(b)$, is equal to the net marginal benefits of holding money, i.e., $w_m(c, m) - ((r + \pi)/(1 + \tau)) u_c(c, m)$. Equation (6) is the modified consumption Euler equation (or the modified Keynes-Ramsey condition): the marginal rate of substitution between consumption at two points in time equal the rate of substitution plus the marginal rate of substitution of consumption and foreign assets. Combining equations (3), (5) and (6) yields

$$u_{cc}(c, m)\dot{c} + u_{cm}(c, m)\dot{m} = (\rho - r)u_c(c, m) - (1 + \tau)\beta w'(b),$$

which describes explicitly the growth rate of the marginal utility of consumption (notice that $(u_c) = u_{cc}(c, m)\dot{c} + u_{cm}(c, m)\dot{m}$) as a linear first order differential equation with a forcing term (i.e., $[-(1 + \tau)\beta w'(b)/u_c]$), and is also the intertemporal consumption Euler equation.

To fully spell out the dynamics of the mercantilist economy, we need to specify the behavior of government. Government’s revenues come from the inflation tax, consumption tax, and interest earnings from central bank’s foreign reserves, i.e., $M/P + \tau c + rR$, where $M$ denotes nominal money stock and $R$ denotes the amount of foreign reserves. Government consumes goods, $g$, and makes transfers, $x$, to the representative agent. Hence, the budget constraint of government is given by

$$g + x = \frac{\dot{M}}{P} + \tau c + rR.$$  \hspace{1cm} (9)

Let the monetary growth rate be a positive real number $\theta (> 0)$, namely,

$$\frac{\dot{M}}{M} = \theta.$$  \hspace{1cm} (10)

With the help of equation (10) and the definition of real balances (i.e., $m = M/P$), equation (9) turns out to

$$x = \theta m + \tau c + rR - g.$$  \hspace{1cm} (11)

Follow Fischer (1979) and Obstfeld (1981, 1982), we examine the perfect foresight equilibrium path of the mercantilist economy, where the expected and actual inflation rates coincide, and
simultaneously, the expected and actual growth rates of the nominal exchange rate also coincide. For $P_t = E_t$, we know that
\[
\frac{\dot{P}}{P} = \frac{\dot{E}}{E} = e = \pi.
\] (12)
Therefore,
\[
\dot{m} = \left(\frac{\dot{M}}{M} - \frac{\dot{P}}{P}\right) m = (\theta - \pi)m.
\] (13)
For (7), we have $\pi = \frac{(1 + \tau)[u_m(c, m) - \beta w'(b)]}{u_c(c, m)} - r$. Substituting it into (13) leads to
\[
\dot{m} = m\left[(r + \theta)u_c(c, m) + (1 + \tau)(\beta w'(b) - u_m(c, m))\right].
\] (14)
Substituting (14) into (8), and (2), (11) and (13) into (1) result in
\[
\dot{c} = -\frac{1}{u_{cc}} \left\{ \beta(1 + \tau)w'(b) + (r - \rho)u_c(c, m) + \frac{mu_{mc}}{u_c} \left[(r + \theta)u_c + (1 + \tau)(\beta w'(b) - u_m)\right] \right\},
\] (15)
\[
\dot{b} = y + rb + rR - c - g.
\] (16)
Equation (15) is the consumption Euler equation, which is the same equation as (8). Equation (14) gives us the optimal growth rate of real money balance holdings under the rule of optimal portfolio. And equation (16) is the dynamic accumulation equation of foreign assets. Altogether, equations (15), (14), and (16) lay out the whole dynamics of the mercantilist economy.

For the infinite-horizon autonomous system, the economy approaches the steady state in the long run. Because of the nonlinearity of the dynamic system, we need to examine the existence, uniqueness, and stability of the steady state of the economy. Define the steady state $(c^*, m^*, b^*)$ by setting $\dot{c} = \dot{m} = \dot{b} = 0$. We can obtain the following three algebraic equations:
\[
(1 + \tau)\beta w'(b^*) + (r - \rho)u_c(c^*, m^*) = 0,
\] (17)
\[
(r + \theta)u_c(c^*, m^*) + (1 + \tau)[\beta w'(b^*) - u_m(c^*, m^*)] = 0,
\] (18)
\[
y + rb^* + rR - c^* - g = 0,
\] (19)
which pin down the steady state of the economy. Equation (17) can be rewritten as $\frac{(1 + \tau)\beta w'(b^*)}{u_c(c^*, m^*)} = \rho - \tau$, which shows that the marginal rate of substitution of consumption and foreign bonds equals to a positive constant, $\rho - \tau$, and simultaneously tells that the time preference rate of the agent
must be larger than the real interest rate in the economy. Equation (18) is the equilibrium version of the optimality condition (7) with \( \theta = \pi^* \) at equilibrium. It is proved in the appendix 6.1 that a sufficient condition for the existence, uniqueness, and saddle-point stability of

\[
\begin{vmatrix}
-w''(b^*) & (u_{cc}u_{mm} - u_{cm}^2) \\
[(\rho + \theta)u_{cm} - (1 + \tau)u_{mm}] & 1
\end{vmatrix}
\]

\( > r(\rho - r). \) (20)

Then, we have the following proposition.

**Proposition 2.1** *In the Viner-Zou model of Mercantilism, if (20) holds, then the steady state exists uniquely and saddle-point stable.*

### 3 Long-run Policy Analysis

The following two sections study the long-run and short-run effects of an increase of the mercantilist sentiments, an increase in the rate of monetary expansion, an increase in real government consumption, an increase of the consumption tax and the intervention in the foreign exchange market. The macroeconomic disturbances are assumed to take the public by surprise, but they are permanent and lead to no expectation of future policy actions. The economy’s initial position is the stationary state. To execute the long-run analysis, we take total differentials on equations (17), (18) and (19) as follows:

\[
\begin{pmatrix}
(r - \rho)u_{cc} \\
B_{21}
\end{pmatrix}
-1
\begin{pmatrix}
(r - \rho)u_{cm} \\
B_{22}
\end{pmatrix}
(1 + \tau)\beta w''(b^*)
\begin{pmatrix}
dc^* \\
dm^*
\end{pmatrix}
= \begin{pmatrix}
B_1 \\
B_2
\end{pmatrix}
(1 + \tau)\beta w''(b^*) \\
db^*
\begin{pmatrix}
dg - rdR
\end{pmatrix}
\]

where \( B_{21} = (r + \theta)u_{cc} - (1 + \tau)u_{me}, \ B_{22} = (r + \theta)u_{cm} - (1 + \tau)u_{mm}, \ B_1 = -(1 + \tau)w'(b^*)d\beta - \beta w'(b^*)d\tau, \) and \( B_2 = -u_c d\theta - (1 + \tau)w'(b^*)d\beta + (u_m - \beta w'(b^*))d\tau. \)

#### 3.1 The Effect of the Mercantilist Mentality

To examine the long-run effect of a permanent increase of the mercantilist mentality, we should set \( d\theta = dg = dR = d\tau = 0 \) in (21). Then, by Cramer’s Rule, we obtain

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4Roughly speaking, the left-hand side of (20) stands for the relative concavity of the utility parts of \( \beta w(b) \) and \( u(c, m), \) and (20) tells that in order for the saddle-point stability of the steady state, the relative concavity of these two utility parts cannot be too small, with a positive lower bound, \( r(\rho - r). \) Moreover, equation (20) is similar to equation (12) of Zou (1997).
\[
\frac{dc^*}{d\beta} = \frac{(1 + \tau)rw'(b^*)[(1 + \tau)u_{mm} - (\rho + \theta)u_{cm}]}{\Delta} > 0, \\
\frac{dm^*}{d\beta} = \frac{(1 + \tau)rw'(b^*)[(\rho + \theta)u_{cc} - (1 + \tau)u_{mc}]}{\Delta} > 0, \\
\frac{db^*}{d\beta} = \frac{-(1 + \tau)w'(b^*)[(\rho + \theta)u_{cm} - (1 + \tau)u_{mm}]}{\Delta} > 0,
\]

where \( \Delta = \frac{-u_{cc}u_{c}}{m^s} \det(J) < 0 \) holds for (20).

**Proposition 3.1** A permanent increase of the mercantilist sentiments will increase the long-run consumption, real money balances, and foreign asset holdings.

With higher mercantilist sentiments, the agent attaches more importance to her wealth on foreign assets, she saves more (i.e., consumes less) and accumulates more foreign assets in the short run.\(^5\) Therefore, the long-run level of foreign assets will be higher. The preference shocks of the mercantilist sentiments have two opposite effects on consumption in the long run and short run. On one hand, consumption will decrease for more preference for accumulation; on the other hand, more foreign asset holdings means more interest earnings, which displays positive wealth effects on consumption. In the long run, the positive long-run effect dominates the negative one, which leads to more consumption in the long run. Meanwhile, more money must be delivered by consumers for more consumption. Then, the steady state level of real money balance holdings is also raised.

### 3.2 The Effect of the Monetary Growth Rate

Likewise, setting \( d\beta = dg = dR = d\tau = 0 \) in (21) and applying Cramer’s Rule lead to

\[
\frac{dc^*}{d\theta} = \frac{r(r - \rho)u_{c}u_{cm}}{\Delta} > 0, \quad \frac{db^*}{d\theta} = \frac{(r - \rho)u_{c}u_{cm}}{\Delta} > 0, \quad \frac{dm^*}{d\theta} = \frac{u_{c}[r(\rho - r)u_{cc} - (1 + \tau)\beta w'(b^*)]}{\Delta}.
\]

**Proposition 3.2** A permanent increase of the monetary growth rate increases the long-run consumption and foreign asset accumulation; however, the effect on the long-run real balances is ambiguous.

The long-run positive effect on foreign asset accumulation of an increase of the monetary growth rate has two channels: portfolio substitution effect and currency depreciation effect. An increase of the monetary growth rate raises the inflation rate and hence the opportunity cost of

\(^5\)The short-run effect of an increase of the mercantilist sentiments will be verified in section 4.1.
holding money. Thus, consumers will economize real money balances and buy foreign assets in the short run. Meanwhile, equation (12) tells that the exchange rate equals the inflation rate at each instant, also in the steady state (i.e., $e^* = \pi^* = \theta$). Higher equilibrium exchange rate means currency depreciation, which implies that net exports are easier, so is the accumulation of foreign assets. Both channels induce the agent to reduce real balances and increase their holdings of foreign assets. However, an increase of the monetary growth rate has two opposite effects on consumption. On one hand, the increased foreign assets caused by both portfolio substitution effect and currency depreciation effect will bring about more interest earnings and hence a higher level of consumption. On the other hand, higher inflation erodes the total wealth of the private sector. This negative income effect enforces consumers to decrease consumption. Furthermore, with less money balance holdings, consumers consume less due to $u_{cm} > 0$. Altogether, the positive effect of an increase of the monetary growth rate dominates and hence the net effect on consumption is positive.,

In the long run, the level of foreign assets and hence consumption will be higher. For real balance holdings, there also exist two opposite effects (i.e., the negative effect of the increased opportunity cost and the positive effect of the increased long-run consumption) and the net effects are ambiguous.\footnote{If the utility is additively separable between consumption and real balance holdings (namely, $u(c, m) = u(c) + v(m)$, which implies $u_{cm} = 0$), then different forces enforced on the economy might cancel each other out. Then, expansionary monetary policies have no long-run effect on the economy, that is, money is superneutrality in the sense of Sidrauski (1967).}

### 3.3 The Effect of Government Consumption

Similar to the standard Ramsey model, if government consumption is wasteful, it will crowd out the private consumption. Setting $d\beta = d\theta = dR = d\tau = 0$ in (21) and applying Cramer’s Rule give rise to

$$
\frac{dc^*}{dg} = \frac{\beta w'' (b^*) [u_{mm} - (\rho + \theta)u_{cm}]}{\Delta} < 0,
$$

$$
\frac{dm^*}{dg} = \frac{\beta w'' (b^*) [(\rho + \theta)u_{cc} - u_{cm}]}{\Delta} < 0,
$$

$$
\frac{db^*}{dg} = \frac{(\rho - r)[u_{cc}u_{mm} - u_{cm}^2]}{\Delta} < 0.
$$

These conclusions are different from Obstfeld (1981), which argues that the government expenditure has no effects on the private consumption and positive effects on foreign asset accumulation.
In our model, government consumption reduces the wealth transfers from government to the private sector and hence decreases the disposable income of consumers. With less income, they must consume less and accumulate less. Furthermore, the negative effect is so high that it can dominate the utility effect of government consumption.\footnote{That is, even if government expenditures enter the utility function (i.e., government expenditures result in the provision of some public goods), the effect of an increase of the government consumption is also negative. That is to say, even if the utility function is $U(c, g, m, b) = u(c, g) + v(m) + \omega w(b)$ with $u_g > 0$ and $u_{cg} > 0$, the effect is still negative.}

**Proposition 3.3** A permanent increase of government consumption, whether or not into the private utility, always reduces the long run levels of consumption, real money balances and foreign asset holdings.

### 3.4 The Effect of the Consumption Tax

Generally, taxes mean higher prices. The imposition of the consumption tax is likely to decrease the levels of consumption and welfare. But, the converse conclusions are drawn in our model. The effect of the consumption tax can be seen by applying Cramer’s rule to equation (21)

\[
\begin{align*}
\frac{dc^*}{d\tau} &= \frac{r\beta(1+\tau)u_mw'(b^*)}{\Delta} > 0, \\
\frac{dm^*}{d\tau} &= \frac{r\beta(\rho-\tau)u_tw'(b^*) + \beta(1+\tau)[u_mw''(b^*) - ru_mcw'(b^*)]}{\Delta} > 0, \\
\frac{db^*}{d\tau} &= \frac{\beta(1+\tau)u_{mm}w'(b^*)}{\Delta} > 0.
\end{align*}
\]

**Proposition 3.4** A permanent increase in the consumption tax raises consumption, real money balances and foreign asset accumulation in the long run.

With a higher price on the consumption good, people consume less and invest more on the foreign assets currently. Gradually, they will accumulate more and more foreign assets. In the long run, they will attain higher level of foreign assets and more interest payments. With higher income, their long-run levels of consumption are also increased. Hence, in the nations with mercantilism, consumption tax is an effective way to suppress current consumption, stimulate savings and investment, and hence increase their wealth and power of the nations in the long run. Just as Jacob Viner (1937, 1948, 1968) had explained that mercantilist appears to be maximizing a country’s power through accumulation of foreign assets and maximizing the long-term standard of living by suppressing the current standard of living. Meanwhile, similar to Zou
(1997), proposition (3.4) provides support for the mercantilist policy of protection, namely, the ‘fear of goods’ (Heckscher, 1935), if attainment of higher long-run consumption is the objective of a nation. Both Proposition 3.1 and 3.4 indicate the long-run harmony between wealth and power. Indeed, from the mercantilist perspective, ‘there is long-run harmony between these two ends, although in particular circumstances it may be necessary for a time to make economic sacrifices in the interest of ...long-run prosperity’ (Viner, 1968). Following an increase in the consumption tax, the short-run consumption will be cut because people invest more in foreign assets. But in the long run, the increased foreign asset accumulation gives rise to more consumption and more power for the nation.

3.5 The Effect of Purchasing Foreign Bonds

Another interesting comparisons between Obstfeld (1981)’s model and ours are the different effects of the central bank’s foreign exchange intervention. In Obstfeld’s model, if the central bank intervenes in the foreign exchange market by purchasing foreign bonds from the public with domestic currency, the total real assets in the economy are not affected, and, as the central bank’s reserves also earn real income and wealth remains the same. Therefore, the central bank’s intervention does not have real effects on foreign asset accumulation, consumption and real money balance holdings. It only occasions a rise in the price level exactly proportional to an increase in money supply. However, since foreign bonds are directly valued in the utility in our model, the symmetry of foreign bonds and foreign reserves in Obstfeld’s model disappears. Shortly after the purchase of the central bank, the reduction of foreign bonds held by the private sector results in higher marginal utility of foreign assets, hence the optimality condition (7) and the equilibrium condition (18) no longer hold. When the initial equilibrium foreign assets are reduced by $dR$ and real balances are increased by $dR$, (7) and (18) become

$$\alpha w'(b - dR) + (r + \theta)u_c(c, m + dR) - u_m(c, m + dR) > 0,$$

$$\alpha w'(b^* - dR) + (r + \theta)u_c(c^*, m^* + dR) - u_m(c^*, m^* + dR) > 0.$$

To restore equilibrium, the agent will increase consumption and buy more foreign bonds in the short run.\textsuperscript{8} And at the new equilibrium, private consumption, real money balances, and foreign asset holdings will reach higher levels. By utilizing Cramer’s Rule in (21), we obtain

\textsuperscript{8}The short-run positive effects on foreign asset accumulation of purchasing foreign bonds can be checked in section 4.5.
\[
\frac{dc^*}{dR} = \frac{\beta ru''(b^*)[(\rho + \theta)u_{cm} - u_{mm}]}{\Delta} > 0,
\]
\[
\frac{dm^*}{dR} = \frac{\beta ru''(b^*)[u_{mc} - (\rho + \theta)u_{cc}]}{\Delta} > 0,
\]
\[
\frac{db^*}{dR} = \frac{r(\rho - r)[u_{cc}u_{mm} - u_{cm}^2]}{\Delta} > 0.
\]

**Proposition 3.5** The central bank’s purchase of foreign claims from the public with domestic currency will lead to more foreign asset accumulation (the sum of central bank’s reserve and private holdings), more consumption and more real money balances in the long run.

The above result has another logic which establishes that purchasing foreign bonds from the private sector as an effective method of protection utilized by mercantilist. That is, in order to purchase more foreign assets, the central bank must pay domestic currency. Probably the central bank releases money into the economy. With more money in the economy, domestic currency will depreciate and the exchange rate will be higher. Then, net exports will be much easier and so is the accumulation of foreign assets.

## 4 Short-run Policy Analysis

The short-run effects of macroeconomic policies will be examined in this section. It is assumed that at \( t = 0 \) the economy is in the steady state \((e^*, m^*, b^*)\) and these policy parameters follow the following rule of changes:

\[
x' = x + \varepsilon h_i(t), \ i = \beta, \theta, g, \tau, R,
\]

where \( \varepsilon \) is a scalar parameter, initially equal to zero, and functions \( \{h_i(t), i = \beta, \theta, g, \tau, R\} \) are bounded and eventually constant. For simplicity, we take the separable log utility: \( U(c, m, b) = \log c + \log m + \beta \log b \). By utilizing the method of Laplace transform developed by Judd (1982) and Cui and Gong (2006), we can derive the dynamic system for short-run analysis:9

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9The derivation details can be found in appendix 6.2.
\( c_\varepsilon(0) = (r - \lambda_3) [H_R(\lambda_2) - H_g(\lambda_2)] - \frac{(\rho - r)^2 (y + r R - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))] [(1 + \tau) H_\beta(\lambda_2) + \beta H_\tau(\lambda_2)]}, \)

\[
m_\varepsilon(0) = \begin{cases} 
- \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} (r - \lambda_3) [r H_R(\lambda_2) - H_g(\lambda_2)] + \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} \frac{(\rho - r)^2 (y + r R - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} \\
\{}[(1 + \tau) [H_\beta(\lambda_2) - H_\beta(\rho + \theta)] + \beta [H_\tau(\lambda_2) - H_\tau(\rho + \theta)]\} \\
- \frac{(\rho - r)(1 + \tau)(y + r R - g)}{\beta (\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]} H_\theta(\rho + \theta) - \frac{(\rho - r)(1 + \tau)(y + r R - g)}{\beta (\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]} H_\tau(\rho + \theta) + \\
\frac{(\rho - r)^2 (1 + \tau)(y + r R - g)}{\beta (\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]} H_\beta(\rho + \theta) - \frac{\omega_2 (r - \lambda_2) - \omega_2 (r - \lambda_3)}{\lambda_2 - \lambda_3} [r H_R(\rho + \theta) - H_g(\rho + \theta)] \end{cases}
\]

\( b_\varepsilon(0) = -c_\varepsilon(0) + r h_R(0) - h_g(0). \)

To examine the short-run effects of permanent policy shocks, we define the permanent positive changes in macroeconomic policies by

\( h_i(t) = 1, \ i = \beta, \theta, g, \tau, R, \text{ and } t > 0. \)

The Laplace transform of \( h_i(t) \) with the parameter \( \lambda_j, j = 1, 2 \) is as follows:

\( H_i(\lambda_j) = \frac{1}{\lambda_j}. \)

Equipped with these definitions, equations (23)-(25) provide the short-run effects of all sorts of permanent changes of macroeconomic policies.

### 4.1 The Effect of the Mercantilist Sentiments

Let \( i = \beta \) in equations (23)-(25). We have

\( c_\varepsilon(0) = -\frac{(\rho - r)^2 (y + r R - g)}{\beta \lambda_2 [\rho - r (1 + \beta (1 + \tau))]} < 0, \)

\( m_\varepsilon(0) = \frac{(\rho - r)^2 (y + r R - g)}{\beta \lambda_2 (\rho + \theta)^2} \left[ \frac{(\rho - r) (1 + \beta (1 + \tau))}{\beta \lambda_3} \right] + \frac{(\rho - r) (1 + \beta (1 + \tau)) (\lambda_2 - \theta)}{[\rho - r (1 + \beta (1 + \tau))]}, \)

\( b_\varepsilon(0) = -c_\varepsilon(0) > 0. \)

**Proposition 4.1** A permanent increase in the mercantilist mentality decreases current consumption, increases current foreign asset accumulation, but its effect on current real balance holdings is ambiguous.
The negative effects on current consumption and positive effects on current asset holdings have been pointed out in section 3.1. A permanent increase of the mercantilist sentiments tells that people prefer more foreign assets. People will increase their holdings of foreign assets. As is pointed out, there are two opposite effects on consumption: a negative effect due to the preference shock and a positive effect because of the wealth effects. In the short run, the negative effect on consumption dominates the wealth effect for the increased interest payments, and hence current consumption decreases. Furthermore, since the wealth effect on current consumption is so small that less money is needed for purchase in the short run than in the long run. Then, the short run effects on current real balance holdings are ambiguous.

Combining Proposition 3.1 with 4.1, a permanent increase in the mercantilist sentiments brings out more foreign asset accumulation both in the short run and long run; however, their effects on consumption are different: current consumption decreases and long-run consumption increases. This divergence may explain why Smith’s criticism on mercantilists’ total disregard of consumption is unfair since Smith missed an important fact: just like what the theory has predicted, the mercantilist country only misses out on consumption for a while and the victim country only gets increased consumption for a while. Eventually the growth of industry and income in the mercantilist country and the loss of industry and income in the victim country reverses the tide.\footnote{It is well known that Adam Smith rejected the mercantilist focus on production, arguing that consumption was the only way to grow an economy. In his 1776 book, Wealth of Nations, Adam Smith first laid out the theory that mercantilism hurts the economy of the country practicing it because it hurts consumers in order to benefit producers. He correctly wrote: “consumption is the sole end and purpose of all production; and the interest of the producer ought to be attended to only so far as it may be necessary for promoting that of the consumer. The maxim is so perfectly self-evident that it would be absurd to attempt to prove it. But in the mercantile system the interest of the consumer is almost constantly sacrificed to that of the producer; and it seems to consider production, and not consumption, as the ultimate end and object of all industry and commerce. (iv.8.49)”}

4.2 The Effect of the Monetary Growth Rate

Setting \( i = \theta \) in equations (23)-(25) gives us

\[
c_\varepsilon(0) = b_\varepsilon(0) = 0, \quad m_\varepsilon(0) = -\frac{(\rho - r)(1 + \tau)(y + rR - g)}{(\rho + \theta)^2 \left[ \rho - r (1 + \beta (1 + \tau)) \right]} < 0.
\]

**Proposition 4.2** An increase of the monetary growth rate just reduces the demand for real money balances, and has no effect on the current consumption and foreign asset accumulation.
The neutrality of money in the short run corresponds to the superneutrality result in the long run with the separable utility case, since we have chosen a separable utility case in this section. Here we have also drawn an interesting conclusion that in the Viner-Zou monetary model with separable utility, monetary policies have no real effects on the economy in the short run and long run. Similar to the analysis in section 3.2, the positive wealth effect for the increased foreign assets and the negative effect of the inflation tax exactly cancel each other out. Hence, the net short-run effects on the economy of expansionary monetary policies are zero. However, if the utility between consumption and real money balances is nonseparable, the result must be as follows: current consumption decreases and current asset accumulation increases, which have been conjectured in section 3.2. Encountering the increased monetary growth rate, the agent with perfect foresight will expect that the equilibrium inflation rate will be higher and domestic currency will also depreciate. Hence, she will reduce her current holdings of real money balances and hence current consumption and buy more foreign bonds.

4.3 The Effect of Government Consumption

Substituting $i = g$ in equations (23)-(25) leads to

$$c_x(0) = -\frac{r - \lambda_3}{\lambda_2} < 0, \quad m_x(0) = -\frac{(\omega_2 - \omega_3)(r - \lambda_3)}{\lambda_2 (\lambda_2 - \lambda_3)} + \frac{\omega_3(r - \lambda_2) - \omega_2(r - \lambda_3)}{(\rho + \theta) (\lambda_2 - \lambda_3)}, \quad b_x(0) = -\frac{\rho - r}{\lambda_2} < 0.$$

Proposition 4.3 A permanent increase of government consumption decreases current consumption and foreign asset accumulation, and its effect on current real money balances is ambiguous.

Except for the ambiguous effect on real money balances, government consumption crowds out current private consumption and foreign asset accumulation. Since more government consumption means the reduction of the agent’s disposable income, a permanent increase of government consumption decreases consumption and foreign asset accumulation all the time, just as Proposition 3.3 and Proposition 4.3 have shown.
4.4 The Effect of the Consumption Tax

Setting $i = \tau$ in equations (23)-(25) gives rise to

\[
    c_\varepsilon(0) = -\frac{(\rho - r)^2 (y + rR - g)}{\lambda_2 (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]},
\]
\[
    m_\varepsilon(0) = \frac{(\omega_2 - \omega_3) (\rho - r)^2 (y + rR - g) [(\rho + \theta) - \lambda_2]}{\lambda_2 (\lambda_2 - \lambda_3) (\rho + \theta) (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]},
\]
\[
    b_\varepsilon(0) = -c_\varepsilon(0) > 0.
\]

**Proposition 4.4** A permanent increase in the consumption tax decreases current consumption, increases current asset accumulation, and its effect on current real balances is ambiguous.

Higher consumption tax means higher price on consumption, and consumers will consume less in the short run. Since the current income of consumers keeps constant, consumers will raise their holdings of foreign assets. As a mercantilist policy, the sole shortcoming of the consumption tax is its negative effect on current consumption. In the long run, it does not matter, especially for small developing countries. Therefore, much criticism on mercantilism may be unfair.

4.5 The Effect of Purchasing Foreign Bonds

Setting $i = R$ in equations (23)-(25) gives us

\[
    c_\varepsilon(0) = -\frac{r (r - \lambda_3)}{\lambda_2} > 0, \quad m_\varepsilon(0) = \frac{r (r - \lambda_2) (\omega_2 - \omega_3)}{\lambda_2 (\lambda_2 - \lambda_3)} - \frac{r [\omega_3 (r - \lambda_2) - \omega_2 (r - \lambda_3)]}{(\rho + \theta) (\lambda_2 - \lambda_3)}, \quad b_\varepsilon(0) = \frac{r (\rho - r)}{\lambda_2} > 0.
\]

**Proposition 4.5** The central bank’s purchase of foreign claims from the public with domestic currency will increases current consumption and current asset accumulation, however, its effect on current real balances is ambiguous.

Once the foreign reserves held by the central bank are increased, their effects on both consumption and foreign asset accumulation are positive in the short run and long run. The logic has been given in section 3.5. In order to hold more foreign bonds, the central bank must release more domestic currency into the economy, which is equivalent to raise the monetary growth rate. And the agent expects that inflation will increase and the exchange rate will rise. Hence, both portfolio substitution effect and currency depreciation effect will induce the agent to increase consumption and foreign asset accumulation both in the short run and long run.
5 Conclusion

This paper extends the Viner-Zou model of mercantilism by introducing money and foreign exchanges, and reexamines the long-run and short-run effects of macroeconomic disturbances. The most interesting, and perhaps surprising fact about our model of mercantilism is its theoretical predictions. We have shown that a nation with stronger mercantilist sentiments has higher consumption and larger foreign asset accumulation in the long run; a permanent rise in the consumption tax brings about more foreign asset holdings and more consumption in the long run; an increase in the monetary growth rate and purchasing foreign bonds from the private sector increase the long-run levels of consumption and foreign asset accumulation. In the short-run analysis, macroeconomic policy shocks including the mercantilist sentiments, the monetary growth rate, and the consumption tax, have negative effects on current consumption and positive effects on current foreign asset accumulation, while purchasing foreign bonds has positive effects on both current consumption and foreign asset accumulation in the short run.

In future research, it is desirable to extend the endowment-economy and small-economy model in this paper into a big-country model with both capital accumulation and foreign asset holdings. And it should be very interesting to study risk-taking, global diversification, growth and welfare in the Viner-Zou model of mercantilism. Furthermore, the mercantilist model can be extended to consider relative wealth or relative power status in a many-country world as in Bakshi and Chen (1996), where the utility function from wealth and power is modified to be $\beta w(b/B)$ with $B$ as the average wealth level in the rest of the world.

6 Appendix

6.1 Proof of Proposition 2.1

In this appendix, we will prove that if (20) holds, the steady state exists uniquely and saddle-point stable. From (19), we have $b^* = \frac{c^*}{r} + \left(\frac{\bar{y} - y}{r} - R\right) = \frac{c^*}{r} + \kappa$. Putting $b^*$ into equations (17) and (18) yields:

\begin{align*}
\beta (1 + \tau) w' \left(\frac{c^*}{r} + \kappa\right) + (r - \rho) u_c(c^*, m^*) &= 0, \quad (r + \theta) u_c(c^*, m^*) + (1 + \tau) [\beta w' \left(\frac{c^*}{r} + \kappa\right) - u_m(c^*, m^*)] = 0.
\end{align*}

Taking total differentials on both equations yields:

\begin{align*}
\frac{dm^*}{dc^*} &= \frac{\beta (1 + \tau) w''(b^*)}{(r - \rho) ru_{mc}^*} + \frac{1}{(r - \rho) ru_{mc}^*}, \quad \frac{dm^*}{dc^*} = \frac{-\left(1 + \tau\right) \left[\beta w''(b^*)/r - u_{cm}^*\right] - (r + \theta) u_{cc}^*}{(r + \theta) u_{mc}^* - (1 + \tau) u_{mm}^*} > 0.
\end{align*}
If the slope of the first curve is larger than the one of the second curve at each point (i.e., (20) holds),\footnote{Note that (20) holds if and only if \( \frac{\beta(1+\tau)\beta''(b^*)}{\rho-\nu} \left( \frac{(r-\rho)u^*_{cc}}{u_{cc}} \right) > \frac{(r-\rho)u^*_{cc}}{(r-\rho+1+\tau)u_{cc}} \), where \( \beta''(b^*) > 0 \).} then they cross only once in the space of \((c^*, m^*)\). That is to say, if (20) holds, then the steady state exists uniquely.

We will confirm that if (20) holds, then the steady state is saddle-point stable. We linearize the dynamic equations (15), (14), and (16) around the steady state as follows:

\[
\begin{pmatrix}
\dot{c} \\
\dot{m} \\
\dot{b}
\end{pmatrix} =
\begin{pmatrix}
-A_{11} & -A_{12} & -A_{13} \\
m^*\left[ (r+\theta)u_{cc} - (1+\tau)u_{mm} \right] & m^*\left[ (1+\tau)\beta w''(b^*) \right] & 0 \\
-1 & 0 & r
\end{pmatrix}
\begin{pmatrix}
c - c^* \\
m - m^* \\
b - b^*
\end{pmatrix},
\]

where \( A_{11} = (r-\rho)u_{cc} + \frac{m^*u_{cc}}{u_{cc}}[(r+\theta)u_{cc} - (1+\tau)u_{mm}] \), \( A_{12} = (r-\rho)u_{cm} + \frac{m^*u_{mc}}{u_{cc}}[(r+\theta)u_{cm} - (1+\tau)u_{mm}] \), and \( A_{13} = (1+\tau)\beta w''(b^*) + \frac{m^*u_{mc}}{u_{cc}}(1+\tau)\beta w''(b^*) \).

It is easy to know that the sign of the trace of \( J \) is positive, namely,

\[ \text{trace}(J) = \rho - \frac{(1+\tau)m^*}{u_{cc}u_{cc}}[u_{cc}u_{mm} - u_{cm}^2] > 0, \]

which shows that there exists at least an eigenvalue with a positive real part. If (20) holds, then the determinant of the \( J \) is negative, namely,

\[ \det(J) = -\frac{m^*}{u_{cc}u_{cc}}\left\{ r(\rho - r)\left[ u_{cc}u_{mm} - u_{cm}^2 \right] + \beta w''(b^*) \left[ (\rho + \theta)u_{cm} - (1+\tau)u_{mm} \right] \right\} < 0, \]

which implies that the Jacobian matrix has a negative real eigenvalue or three eigenvalues with negative real parts. Combining them, we know that the Jacobian matrix has just one negative eigenvalue. Since there is only one state variable in the system, the steady state is locally saddle-point stable.

### 6.2 Deriving the Dynamic System for Short-Run Analysis

In this appendix, we derive the dynamic equations for short-run analysis. Substituting (22) and the utility form into Eqs. (14)-(16) yields

\[
\text{(22)}
\]

\footnote{Note that the partial derivatives in the Jacobian matrix \( J \) are evaluated at the steady state.}
\[
\dot{c} = c^2 \left( \frac{1}{b} (\beta + \varepsilon h_\beta(t)) (1 + \tau + \varepsilon h_\tau(t)) + \frac{(r - \mu)}{c} \right),
\]
\[
\dot{m} = cm \left( \frac{1}{c} (r + \theta + \varepsilon h_\theta(t)) + (1 + \tau + \varepsilon h_\tau(t)) \left( \frac{\beta + \varepsilon h_\beta(t)}{b} - \frac{1}{m} \right) \right),
\]
\[
\dot{b} = y + r(b + R + \varepsilon h_R(t)) - c - (g + \varepsilon h_g(t)),
\]
with boundary conditions \(|\lim_{t \to \infty} b(t)| < \infty, b(0) = b_0\). The steady state can be derived as

\[
(c^*, m^*, b^*) = \left( \frac{(\rho - r)(y + rR - g)}{(\rho - r)(1 + \beta (1 + \tau))}, \frac{(\rho - r)(1 + \tau)(y + rR - g)}{(\rho - r)(1 + \beta (1 + \tau))}, \frac{\beta (1 + \tau)(y + rR - g)}{(\rho - r)(1 + \beta (1 + \tau))} \right).
\]

The positivity of consumption requires that the parameter values satisfy \(\rho - r (1 + \beta (1 + \tau)) > 0\), from which we conclude that \(b^*\) is positive (i.e., foreign assets).

The optimal solutions for \(c, m, b\) depend on both \(t\) and \(\varepsilon\). Define \(x_\varepsilon(t) = \partial x(t, 0)/\partial \epsilon\), \(\dot{x}_\varepsilon(t) = \partial[\partial x(t, 0)/\partial t] / \partial t, x = c, m, b\). Differentiating the above three equations w.r.t \(\varepsilon\) and evaluating them at \(\varepsilon = 0\) yield the following system:

\[
\begin{pmatrix}
\dot{c}_\varepsilon \\
\dot{m}_\varepsilon \\
\dot{b}_\varepsilon
\end{pmatrix} =
\begin{pmatrix}
\frac{\rho - r}{\rho + \theta} & 0 & -\frac{(\rho - r)^2}{\beta (1 + \tau)} \\
-\frac{(r + \theta)(1 + \tau)}{\rho + \theta} & \rho + \theta & -\frac{(\rho - r)^2}{\beta (r + \theta)} \\
0 & 0 & r
\end{pmatrix}
\begin{pmatrix}
c_\varepsilon(t) \\
m_\varepsilon(t) \\
b_\varepsilon(t)
\end{pmatrix} +
\begin{pmatrix}
u_1(t) \\
u_2(t) \\
u_3(t)
\end{pmatrix},
\]

where

\[
u_1(t) = \frac{(\rho - r)^2 (y + rR - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} (1 + \tau) h_\beta(t) + \beta h_\tau(t),
\]
\[
u_2(t) = \frac{(\rho - r)(y + rR - g)}{(\rho + \theta) [\rho - r (1 + \beta (1 + \tau))]} [(1 + \tau) h_\theta(t) - (r + \theta) h_\tau(t) + \frac{1}{\beta} (1 + \tau) (r - \mu) h_\beta(t)],
\]
\[
u_3(t) = rh_R(t) - h_\theta(t).
\]

Denote the Laplace transforms by the upper case letters of the associated variables being in lower case. Taking the Laplace transform with parameter \(s\) in the matrix system leads to

\[
(sI - J)
\begin{pmatrix}
C_\varepsilon(s) \\
M_\varepsilon(s) \\
B_\varepsilon(s)
\end{pmatrix} =
\begin{pmatrix}
U_1(s) + c_\varepsilon(0) \\
U_2(s) + m_\varepsilon(0) \\
U_3(s)
\end{pmatrix},
\]

18
where \( U_1(s) = \frac{(\rho-r)^2 (y+R-g)}{\beta(1+\tau)(\rho-r(1+\beta(1+\tau)))} \) \([1 + \tau) H_\beta(s) + \beta H_\tau(s)\], \( U_3(s) = rH_R(s) - H_g(s) \), and
\[ U_2(s) = \frac{(\rho-r)(y+R-g)}{(\rho+\theta)(\rho-r(1+\beta(1+\tau)))} \left[(1 + \tau) H_{\theta}(s) - (r + \theta) H_{\tau}(s) + \frac{(1+\tau)(\rho-r)}{\beta} H_\beta(s)\right]. \]
13 The eigenvalues and eigenvectors of the Jacobian matrix are solved as follows:

\[
\lambda_1 = \rho + \theta, \quad \lambda_{2,3} = \frac{1}{2} \left\{ \rho \pm \sqrt{\rho^2 + \frac{4(\rho-r)}{\beta(1+\tau)} [\rho - r(1 + \beta (1 + \tau))] } \right\};
\]

\[ v_1 = (0, 1, 0)', v_2 = (r - \lambda_2, \omega_2, 1)', v_3 = (r - \lambda_3, \omega_3, 1)', \]

where \( \omega_i = \frac{1}{\rho+\theta-\lambda_i} \left[ (\rho-r)^2 + (r-\lambda_2) \frac{r+\theta}{\rho+\theta} (1 + \tau) \right], i = 2, 3. \) It is easy to know that \( \lambda_2 > 0, \lambda_3 < 0 \) by the assumption \( \rho - r(1 + \beta (1 + \tau)) > 0 \). By (20), the saddle-point stability condition \( \det J = -\frac{4(\rho+\theta)(\rho-r)}{\beta(1+\tau)} [\rho - r(1 + \beta (1 + \tau))] < 0 \) holds. Since the Jacobian matrix \( J \) is nonsingular, there exists an invertible matrix \( V = (v_1, v_2, v_3) \) such that \( \Lambda = V^{-1} J V \), where

\[
\Lambda = \begin{pmatrix}
\rho + \theta & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{pmatrix}, \quad V^{-1} = \begin{pmatrix}
\frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} & 1 & \frac{\omega_3(r-\lambda_2) - \omega_2(r-\lambda_3)}{\lambda_2 - \lambda_3} \\
\frac{1}{\lambda_2 - \lambda_3} & 0 & \frac{r-\lambda_3}{\lambda_2 - \lambda_3} \\
\frac{1}{\lambda_2 - \lambda_3} & 0 & \frac{r-\lambda_2}{\lambda_2 - \lambda_3}
\end{pmatrix}.
\]

Then,

\[
(sI - \Lambda) V^{-1} \begin{pmatrix}
C_\epsilon(s) \\
M_\epsilon(s) \\
B_\epsilon(s)
\end{pmatrix} = V^{-1} \begin{pmatrix}
U_1(s) + c_\epsilon(0) \\
U_2(s) + m_\epsilon(0) \\
U_3(s)
\end{pmatrix}.
\]

Setting \( s = \rho + \theta, \lambda_2 \) in the above matrix equation gives us two equations:

\[
0 = \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} [U_1(\rho + \theta) + c_\epsilon(0)] + U_2(\rho + \theta) + m_\epsilon(0) + \frac{\omega_3(r-\lambda_2) - \omega_2(r-\lambda_3)}{\lambda_2 - \lambda_3} U_3(\rho + \theta),
\]

\[
0 = -\frac{1}{\lambda_2 - \lambda_3} [U_1(\lambda_2) + c_\epsilon(0)] + 0 + \frac{r-\lambda_3}{\lambda_2 - \lambda_3} U_3(\lambda_2),
\]

from which \( c_\epsilon(0) \) and \( m_\epsilon(0) \) can be derived

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13 We dropped \( b_0(0) \) in the transformed matrix system because \( b \) is a state variable and the initial foreign asset \( b_0 \) cannot be changed immediately.
\[ c_\varepsilon(0) = (r - \lambda_3) [H_R(\lambda_2) - H_g(\lambda_2)] - \frac{(\rho - r)^2 (y + rR - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} \left[ (1 + \tau) H_\beta(\lambda_2) + \beta H_\tau(\lambda_2) \right]; \]

\[ m_\varepsilon(0) = \begin{cases} 
  \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} (r - \lambda_3) [rH_R(\lambda_2) - H_g(\lambda_2)] + \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} \frac{(\rho - r)^2 (y + rR - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} \left[ (1 + \tau) H_\beta(\lambda_2) + \beta H_\tau(\lambda_2) \right] \\
  \frac{\omega_2 - \omega_3}{\lambda_2 - \lambda_3} \frac{(\rho - r)^2 (y + rR - g)}{\beta (1 + \tau) [\rho - r (1 + \beta (1 + \tau))]} \left[ (1 + \tau) H_\beta(\lambda_2) + \beta H_\tau(\lambda_2) \right] \end{cases} \]

By substituting \( c_\varepsilon(0) \) and \( m_\varepsilon(0) \) into the system about \( \dot{c}_\varepsilon, \dot{m}_\varepsilon, \dot{b}_\varepsilon \), we have

\[ \dot{b}_\varepsilon(0) = -c_\varepsilon(0) + rH_R(0) - h_g(0). \]

References


