

A New Approach to Inflation Aversion*

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Abstract

This paper reexamines monetary non-superneutrality and the optimality of the optimum quantity of money in the money-in-utility Sidrauski model with endogenous fluctuations of the time preference by introducing inflation aversion. It is shown that the long-run superneutrality of the standard Sidrauski model does not hold, and Friedman's optimum quantity of money is not optimal.

Keywords: Inflation Aversion, Endogenous Time Preference, Monetary Superneutrality, Optimum Quantity of Money

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1 Introduction

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Inflation, a term familiar to economists, policy makers and common citizens, makes people impatient, anxious, nervous and less confident. Many economists have studied the economic and psychological costs of inflation. Keynes (1936) points out that inflation leads to economic, social and institutional uncertainty and strikes at confidence. Much earlier than Keynes, Bohm-Bawerk (1891) says that inflation increases the time discount rate. Facing high inflation in the late 1960s and 1970s in the United States, Katona (1975) tells us that, with high inflation, even if real income has remained constant or increased substantially, people still feel cheated, and psychologically they regard inflation as a “bad thing”. At the same time, Fabricant (1976) states that the uncertainty and anxiety from inflation makes more impatience and a large time discount rate and that high inflation makes rational calculation more difficult or impossible and makes people possess even less “adequate power to imagine and to abstract” the future. Burns (1978) also writes that “by causing disillusionment and breeding discount, inflation excites doubts among people about themselves, about the competence of their government, and about the free enterprise system itself.” More recently, Shiller (1996) has written that “it was very easy to see why people dislike inflation: people think inflation erodes their standard of living”; and that “this standard of living effect is not the only perceived cost of inflation among non-economists: other perceived costs are tied up with issues of exploitation, political instability, loss of morale, and damage to national prestige.” All these statements and assessments lead to the same conclusion: inflation tends to impair the patience and confidence of the people.

In order to model this negative effect of anticipated inflation on patience, we take the time preference rate as an increasing function of the inflation rate endogenously, and name it “inflation aversion”. The objective of this paper is to investigate the macroeconomic implication of this “stylized” psychological fact. Actually, Stockman (1981) has given some hints on this modelling strategy in the first footnote of his paper. He says that “if inflation affects β (the time preference rate) in the steady state, then any effect of inflation on the capital stock is possible, depending upon how inflation affects this particular aspect of ‘tastes’.” Stockman’s analysis had been anticipated by Keynes (1936) who had attached great importance to this psychological characteristics of human nature and states the endogenous fluctuation of the rate of time-discounting (page 93), “The state of confidence, as they term it, is a matter to which practical men always pay the closest and most anxious attention. But economists have not analysed it carefully and have been content,

as a rule, to discuss it in general terms. In particular it has not been made clear that its relevance to economic problems comes in through its important influence on the schedule of the marginal efficiency of capital. There are not two separate factors affecting the rate of investment, namely, the schedule of the marginal efficiency of capital and the state of confidence. The state of confidence is relevant because it is one of the major factors determining the former, which is the same thing as the investment demand-schedule.”

A large literature has examined the relationship between endogenous time preferences and monetary superneutrality. Uzawa (1968) sets up an infinitely-lived-representative-agent model with an endogenous time preference to replicate the Mundell-Tobin effect. By assuming that the rate of time preference is an increasing and convex function of the current level of utility, he shows that monetary growth raises savings and the capital stock. Using Uzawa’s time preference, Obstfeld (1981) further examines the long-run monetary non-superneutrality in a small open economy. Epstein and Hynes (1983) have also examined monetary superneutrality in Sidrauski (1967) model and concluded that a higher rate of monetary expansion increases the steady-state levels of consumption and capital stock, and reduces the steady-state level of real balances. Recently, in a growth model with the Marshallian time preference, Gootzeit, Schneider and Smith (2002) show that a permanent increase in government expenditure causes “super-crowding-out” of consumption and lowers the steady-state capital stock. By modelling time preference as an increasing function of real wealth, Kam (2005) has also reexamined the existence of the Tobin effect.¹

And ever since Friedman puts forward his famous rule for the optimum quantity of money², many economists have examined its optimality. It has been shown to be optimal in monetary economies with monopolistic competition (Ireland, 1996) and, under certain circumstances, in a variety of monetary economies where government levies other distorting taxes (Chari, Christiano and Kehoe (1996), Gahvari (2007), Ireland (2003) and Da Costa and Werning (2008)). However, there exist several cases where the Friedman rule is not optimal. These include economies with cash-in-advance constraints

¹Actually, many papers have examined the recursive structure of the endogenous time preferences, such as Obstfeld (1990), Epstein (1983, 1987).

²Friedman (1969) argues that a positive nominal interest rate represents a distortionary tax on real money balances. To reach the first-best, the distortion should be removed and the nominal interest rate should be set to zero. This prescription is known as the Friedman rule for the optimum quantity of money.

(Stockman (1981), Abel (1985), and Ellison & Rankin, 2007); economies with time inconsistency of monetary and fiscal policy (Alvarez, Kehoe and Neumeyer, 2004), economies with intergenerational wealth effects of monetary growth (Gahvari, 1988, 2007); economies with redistributive effects of monetary growth (Bhattacharya, Haslag and Martin, 2005), and economies with strong Tobin effects (Bhattacharya, Haslag and Martin, 2009).

The paper incorporates “inflation aversion” into the standard Sidrauski (1967) model and reexamines monetary superneutrality and the optimality of Friedman’s rule for optimum quantity of money. Again, “inflation aversion” means that inflation causes people to become more impatient and they increase their subjective discount rate. The formal model of inflation aversion is presented in section 2. In Section 3, we show the dynamics of the system and study the properties of the steady state. Comparative dynamics are analyzed in section 4, and a summary of our main findings concludes the paper.

2 The Model

2.1 The Endogenous Time Preference with Inflation Aversion

As is well known, the time preference rate is a measure of the agent’s patience in common sense. And in the continuous-time model, the larger the time discount rate, the less patience the agent. Usually the time discount rate is assumed to be an exogenously given, positive constant. In order to investigate the possible economic effects of the psychological aversion of inflation, we assume that the time preference rate of the representative individual is a strictly increasing, strictly concave function of the expected inflation rate. That is,

$$\rho_t = \rho(\pi_t), \tag{1}$$

which satisfies

$$\rho'(\pi_t) > 0, \rho''(\pi_t) < 0, \rho(0) = \rho_f. \tag{2}$$

Assumptions (1) and (2) make the time preference rate endogenous, and they imply the higher the inflation rate, the less patience the individual. But notice that the decrease in the patience is at a decreasing rate. Moreover, the discount rate is a positive constant if the inflation rate is zero, just like a

“Fisherian” consumer with a constant rate of time preference, i.e., $\rho(0) = \rho_f$. Furthermore, it is also assumed that the time discount factor of the individual at time t depends not only on the current level of inflation, but also on the entire path of past inflation $\{\pi_v\}_{v=0}^t$, namely,

$$\Delta_t = \int_{v=0}^t \rho(\pi_v) dv. \quad (3)$$

Then the modelling strategy has generated a new state variable, the real time discount factor Δ_t . Differentiating Δ_t with respect to t in equation (3), we obtain the dynamic accumulation equation of the time discount factor, namely,

$$\dot{\Delta}_t = \rho(\pi_t). \quad (4)$$

With these new elements introduced, this paper will reexamine the Sidrauski model and the long-run effects of the monetary policy.³

2.2 The Sidrauski Model with Inflation Aversion

2.2.1 Consumer’s Behavior

The representative individual’s optimization problem is to maximize

$$W = \int_{t=0}^{\infty} [u(c_t, m_t)] e^{-\Delta_t} dt \quad (5)$$

subject to the budget constraint

$$\dot{a}_t = r_t k_t + w_t - c_t - \pi_t m_t + \tau_t, \quad (6)$$

and wealth constraint

$$a_t = k_t + m_t, \quad (7)$$

plus the no-Ponzi-game condition

$$\lim_{t \rightarrow \infty} a_t \exp\left(-\int_{v=0}^t r_v dv\right) = 0, \quad (8)$$

³For simplicity, we just consider the case without population growth.

where c_t , m_t , k_t , and a_t are consumption, real money balances, physical capital stock, and total wealth, respectively; r_t and w_t are the real interest rate and real wages; Δ_t and π_t are the time discount factor and the expected rate of inflation; and τ_t denotes lump-sum real money transfer payments. The stock constraint requires that the total wealth a_t be allocated between capital k_t and real balances m_t . And the no-Ponzi-game condition rules out unlimited borrowings. The instantaneous utility function $U_t = u(c_t, m_t)$ is assumed to be well-behaved, satisfying $u_c > 0$, $u_m > 0$, $u_{cc} < 0$, $u_{mm} < 0$, $u_{cc}u_{mm} - u_{cm}^2 > 0$ and the Inada conditions. Following Sidrauski (1967) and Fischer (1979), we assume that both commodities are not inferior⁴. Furthermore, to reach a definitive conclusion, following Calvo (1979), we assume that consumption and real money balances are Edgeworth-complementary, i.e., $u_{cm} > 0$.⁵ Intuitively, an increase in real balances raises the marginal valuation of consumption and increases consumption; and a lower level of money holdings decreases the marginal valuation of consumption and lowers consumption. Hence, in the steady state, consumption and real money balances move in the same direction.

To proceed, the optimization problem of the representative consumer is to maximize (5), subject to (6), (4), (7) and (8). The Hamiltonian associated with this problem is

$$H = u(c_t, m_t)e^{-\Delta t} + \tilde{\lambda}_t[r_t k_t + w_t - c_t - \pi_t m_t + \tau_t] + \tilde{\mu}_t \rho(\pi_t) + \tilde{q}_t(k_t + m_t - a_t), \quad (9)$$

where $\tilde{\lambda}_t$ and $\tilde{\mu}_t$ are the multiplier associated with the constraints (6) and (4), representing the shadow values of wealth and time discount factor, respectively; \tilde{q}_t is the Lagrangian multiplier attached to the stock constraint (7), representing the marginal value of total wealth.⁶

The first-order conditions for a maximum are given by equations (10)-(13) together with the transversality conditions:

$$u_c(c, m)e^{-\Delta} = \tilde{\lambda}, \quad (10)$$

⁴It is not hard to prove that the normality of the two goods is equivalent to the following two conditions, respectively, $u_{mm} - \frac{u_{cm}u_m}{u_c} < 0$, $\frac{u_{cc}u_m}{u_c} - u_{cm} < 0$.

⁵Wang and Yip (1992) called the assumption pareto complementarity between consumption and money.

⁶For notional simplicity, we will omit the time subscript in the following mathematical presentations.

$$u_m(c, m)e^{-\Delta} = (r + \pi)\tilde{\lambda}, \quad (11)$$

$$\dot{\tilde{\lambda}} + r\tilde{\lambda} = 0, \quad (12)$$

$$u(c, m)e^{-\Delta} = \tilde{\mu}. \quad (13)$$

$$\lim_{t \rightarrow \infty} e^{-\Delta} \tilde{\lambda} k = 0, \quad \lim_{t \rightarrow \infty} e^{-\Delta} \tilde{\mu} \Delta = 0. \quad (14)$$

Equations (10) and (11) are two intratemporal optimality conditions, implying that the marginal utility of consumption and (or) real balances equals the real marginal valuation of wealth; equations (12) and (13) are two Euler equations, which determine the intertemporal choices of consumption and real money balances; and equation (12) is the Keynes-Ramsey condition, which implicitly shows that the marginal rate of substitution between consumption at two points of time must equal the marginal rate of transformation.

Now let us define the current-value Hamiltonian multipliers λ and μ as a product of their corresponding present-value Hamiltonian multipliers and e^{Δ} :

$$\lambda = e^{\Delta} \tilde{\lambda}, \quad \mu = e^{\Delta} \tilde{\mu}. \quad (15)$$

Taking the derivative of equation (15) with respect to t , we have:

$$\dot{\lambda} = [\dot{\tilde{\lambda}} - \rho(\pi)\tilde{\lambda}]e^{-\Delta}, \quad \dot{\mu} = [\dot{\tilde{\mu}} - \rho(\pi)\tilde{\mu}]e^{-\Delta}. \quad (16)$$

Substituting equation (10) into equation (15) leads to

$$u_c(c, m) = \lambda. \quad (17)$$

Putting equations (16), (15) and (17) into equation (10) gives rise to

$$\dot{\lambda} = -[r - \rho(\pi)]\lambda. \quad (18)$$

Taking the derivative of equation (17) with respect to t , and using (17) and (18) lead to

$$\dot{c} = -[r - \rho(\pi)] \frac{u_c(c, m)}{u_{cc}(c, m)} - \frac{u_{cm}(c, m)}{u_{cc}(c, m)} \dot{m}. \quad (19)$$

Equations (10) and (11) imply that:

$$\frac{u_m(c, m)}{u_c(c, m)} = (r + \pi). \quad (20)$$

Hence, at optimum the marginal rate of substitution between consumption and real money balances is equal to the nominal interest rate, which is the price of monetary services or the opportunity cost of holding money.

Finally, equations (13) and (16) together imply

$$\dot{\mu} = u(c, m) + \rho(\pi)\mu. \quad (21)$$

2.2.2 Behavior of the Firm

It is assumed that the production function of the firm is well behaved, namely, $f(0) = 0$, $f(\infty) = \infty$, $f(0 < k < \infty) > 0$, $f'(0) = \infty$, $f'(\infty) = 0$, $f'(0 < k < \infty) > 0$, $f''(k) < 0$, and that factor markets are competitive.⁷ Accordingly,

$$r = f'(k), w = f(k) - kf'(k). \quad (22)$$

That is to say, the market interest rate equals the marginal productivity of capital and the market wage rate equals the marginal productivity of labor.

2.2.3 Macroeconomic Equilibrium

In order to complete the system, we introduce the government's behavior. It is assumed that the government maintains a constant rate of monetary expansion

$$\frac{\dot{M}}{M} = \theta \quad (23)$$

and keeps its budget balanced:

$$\tau + g = \frac{\dot{M}}{P}, \quad (24)$$

⁷For simplicity, we assume that the rate of depreciation for capital is zero.

where θ and g are two constants denoting the monetary growth rate and government expenditure, respectively. By the definition of real money balances, $m = \frac{M}{P}$. Substituting equation (23) into equation (24) results in

$$\tau + g = \theta m. \quad (25)$$

We impose the assumption of perfect foresight which says that the expected rate of inflation is equal to the real rate of inflation, namely,

$$\frac{\dot{P}}{P} = \pi. \quad (26)$$

Taking the derivative of $m = \frac{M}{P}$ with respect to t , rearranging, and substituting equations (23) and (26) into it, we have

$$\dot{m} = (\theta - \pi)m. \quad (27)$$

Putting equation (22) into (20) and rearranging them,

$$\pi = \frac{u_m(c, m)}{u_c(c, m)} - f'(k). \quad (28)$$

From equation (28), we solve π as a function of $c, m,$ and k , i.e., $\pi_t = \pi(c, k, m)$. And it is easy to show that

$$\pi_c = \frac{u_{mc}u_c - u_{cc}u_m}{u_c^2} > 0, \pi_m = \frac{u_{mm}u_c - u_{cm}u_m}{u_c^2} < 0, \pi_k = -f''(k) > 0. \quad (29)$$

Putting $\pi_t = \pi(c, k, m)$ into equation (27) gives the dynamics of real money balances

$$\dot{m} = (\theta - \pi(c, k, m))m. \quad (30)$$

Substituting equations (7), (22), (25), and (27) into equation (6) results in the dynamic equation of physical capital accumulation

$$\dot{k} = f(k) - c - g. \quad (31)$$

Putting equations (22), (30), and $\pi_t = \pi(c, k, m)$ into equation (19) gives the dynamic equation of consumption

$$\dot{c} = -[f'(k) - \rho(\pi(c, k, m))] \frac{u_c(c, m)}{u_{cc}(c, m)} - \frac{u_{cm}(c, m)}{u_{cc}(c, m)} (\theta - \pi(c, k, m))m. \quad (32)$$

Therefore, equations (30)-(32) describe the whole dynamics of the model.

2.3 Dynamics and the Steady State

2.3.1 The Steady State

In the steady state (c^*, k^*, m^*) , $\dot{c} = \dot{k} = \dot{m} = 0$, namely,

$$f'(k^*) = \rho(\pi(c^*, k^*, m^*)), \quad (33)$$

$$f(k^*) = c^* + g, \quad (34)$$

$$\theta = \pi(c^*, k^*, m^*). \quad (35)$$

Equation (33) gives the familiar modified golden-rule level of capital accumulation, which shows that, in the steady state, the marginal product of physical capital equals the subjective time preference rate; equation (34) tells that the steady-state production can be divided into two parts: one is the steady-state level of consumption, and the other is the exogenous level of government expenditure; and equation (35) shows that the steady-state level of inflation is equal to the exogenous level of monetary growth.

Furthermore, it is easy to see the existence and uniqueness of the steady state from the steady-state equations (33)-(35) and the basic assumptions of the model.

2.3.2 Stability of the Steady State

To examine the local stability of the steady state, we linearize equations (30)-(32) around the steady state (c^*, k^*, m^*)

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ -1 & f'(k^*) & 0 \\ -\pi_c^* m^* & -\pi_k^* m^* & -\pi_m^* m^* \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \\ m - m^* \end{bmatrix}, \quad (36)$$

where

$$\begin{aligned}
a_{11} &= \frac{\pi_c^*[u_c^*\rho'(\theta) + u_{cm}^*m^*]}{u_{cc}^*} < 0, \\
a_{12} &= \frac{-u_c^*[f''(k^*) - \pi_k^*\rho'(\theta)]}{u_{cc}^*} + \frac{u_{cm}^*\pi_k^*m^*}{u_{cc}^*} < 0, \\
a_{13} &= \frac{\pi_m^*[u_c^*\rho'(\theta) + u_{cm}^*m^*]}{u_{cc}^*} > 0.
\end{aligned}$$

Let us define the Jacobian matrix of the linearized system as J . It is not hard to find that

$$\prod_{i=1}^3 \lambda_i = \det(J) = \frac{u_c^*f''(k^*)\pi_m^*m^*}{u_{cc}^*} < 0. \quad (37)$$

Equation (37) implies that there exists one negative real eigenvalue or three eigenvalues with negative real parts. The trace of the Jacobian matrix is

$$\sum_{i=1}^3 \lambda_i = \text{tr}(J) = f'(k^*) + \frac{(u_{mc}^*u_c^* - u_{cc}^*u_m^*)\rho'(\theta) + (u_{mc}^{*2} - u_{cc}^*u_{mm}^*)m^*}{u_c^*u_{cc}^*}, \quad (38)$$

and we cannot decide its sign on the basis of the assumptions of the model. In order to guarantee the saddle-point stability of the steady state, we impose the following assumption:

$$\text{tr}(J) > 0. \quad (39)$$

If condition (39) holds, then there exists a unique negative eigenvalue corresponding to the unique predetermined variable k . Hence, the steady state is a saddle point.

Notice that, the condition (39) is not stringent at all. In addition, the curvature of the time preference function plays no role in the determination of stability, since the second derivative of the time preference function does not enter the Jacobian matrix J . For sure, let us see three numerical examples.

Example 1, Assume the utility function is separable in consumption and real balances for simplicity: $u(c, m) = \log c + \log m$. Let the production be a Cobb-Douglas technology: $f(k) = k^{0.35}$. And define the time preference as a concave function of the inflation rate: $\rho(\pi) = \log(\pi + 1.2)$. With $\theta = 0.001$,

the unique steady state is: $k^* = 2.7083$, $c^* = 1.4172$, $m^* = 7.6959$, $\pi^* = 0.001$, $\rho^* = 0.1832$ and the corresponding eigenvalues are: -0.2921 , 0.4103 , 0.0957 . Then $tr(J) = 0.2139 > 0$, and condition (39) is satisfied.

Example 2, Let the utility function, the production function and θ be the same as in example 1. Let the time discount rate be: $\rho(\pi) = \log(\pi + 0.01)$. Then, the unique steady state is given by: $k^* = 205.0257$, $c^* = 6.4437$, $m^* = 536.9720$, $\pi^* = 0.001$, $\rho^* = 0.011$ and the corresponding eigenvalues are: -0.0194 , 0.0248 , 0.0056 . Now $tr(J) = 0.0100 > 0$, and condition (39) holds again.

Example 3, Keep everything the same as in example 1 expect for the time discount rate: $\rho(\pi) = \exp(\pi) - 0.998$. Then, the unique steady state is given by: $k^* = 1312.8879$, $c^* = 12.9698$, $m^* = 3242$, $\pi^* = 0.001$, $\rho^* = 0.0056$ and the three eigenvalues are: -0.0054 , 0.0065 , 0.00519 . It is obvious the sum of the three eigenvalues is positive $tr(J) = 0.00629 > 0$ as required by condition (39).

Therefore, we have the following Proposition.

Proposition 1 *In the Sidrauski model with inflation aversion, if $tr(J) > 0$, the steady state is locally saddle-point stable.*

3 Macroeconomic Policy Analysis

3.1 Long-run Effects of Monetary Policy

3.1.1 Monetary Non-superneutrality

Totally differentiating equations (33)-(35) gives us a three-dimensional linear system as follows:

$$\begin{bmatrix} \rho'(\theta)\pi_c^* & \rho'(\theta)\pi_k^* - f''(k^*) & \rho'(\theta)\pi_m^* \\ 1 & -f'(k^*) & 0 \\ \pi_c^* & \pi_k^* & \pi_m^* \end{bmatrix} \begin{bmatrix} dc^* \\ dk^* \\ dm^* \end{bmatrix} = \begin{bmatrix} 0 \\ -dg \\ d\theta \end{bmatrix}. \quad (40)$$

Setting $dg = 0$ and applying Cramer's rule, we obtain

$$\frac{dc^*}{d\theta} = \frac{\rho'(\theta)f'(k^*)}{f''(k^*)} < 0, \quad (41)$$

$$\frac{dk^*}{d\theta} = \frac{\rho'(\theta)}{f''(k^*)} < 0, \quad (42)$$

$$\frac{dm^*}{d\theta} = \frac{f''(k^*) - \rho'(\theta)\pi_k^* - \rho'(\theta)\pi_c^*f'(k^*)}{\pi_m^*f''(k^*)} < 0. \quad (43)$$

Therefore, we have the following proposition.

Proposition 2 *A permanent increase in the monetary growth rate decreases the steady-state consumption, capital accumulation and real balance holdings. That is to say, money is not superneutral in the Sidrauski model with inflation aversion.*

In the standard Sidrauski model with a constant time preference rate, the steady-state levels of capital stock and consumption are given by the same conditions as those in the nonmonetary Ramsey model, and, they are independent of the monetary growth rate. That is to say, money is superneutral in the long run. However, in the Sidrauski model with inflation aversion, the time preference rate depends on the inflation rate endogenously, and hence, the steady-state level of capital depends on the inflation rate (or money growth rate). The logic for the failure of superneutrality is as follows: an increase in the rate of money growth raises the rate of time preference in the steady state ($\rho'(\theta) > 0$), the steady-state interest rate increases ($\frac{dr^*}{d\theta} = \rho'(\theta) > 0$), and then the steady-state stock of capital falls because of diminishing returns and higher rental costs. In turn, the fall in the capital stock reduces net output and consumption. These conclusions are just what Burns (1978) had said: inflation “weakens the willingness to save. It drives up the level of interest rates. It affects adversely both stock prices and bond prices.”

As inflation rises, the opportunity cost of holding money is higher. Hence, the steady-state level of real balances decreases. In the Sidrauski model, the money demand function is $m^* = m^*(\rho, \theta)$ and the effect of a positive monetary disturbance is negative, i.e., $\frac{dm^*}{d\theta} = m_\theta^*(\rho, \theta) < 0$. But, in the model with inflation aversion, the money demand function is $m^* = m^*(\rho(\theta), \theta)$ and a negative item ($m_\rho^*(\rho, \theta)\rho'(\theta) = -\frac{\rho'(\theta)[\pi_c^*f'(k^*) + \pi_k^*]}{\pi_m^*f''(k^*)}$) is added to $\frac{dm^*}{d\theta}$, namely,

$$\frac{dm^*}{d\theta} = m_\rho^*(\rho, \theta)\rho'(\theta) + m_\theta^*(\rho, \theta) = -\frac{\rho'(\theta)[\pi_c^*f'(k^*) + \pi_k^*]}{\pi_m^*f''(k^*)} + m_\theta^*(\rho, \theta) < 0.$$

Hence, the total effect includes both the original Keynesian part, $m_\theta^*(\rho, \theta)$ and the new part coming from “inflation aversion”, $m_\rho^*(\rho, \theta)\rho'(\theta)$ ($= -\frac{1}{\pi_m^*}[\pi_c^* \frac{f'(k^*)}{f''(k^*)} + \pi_k^* \frac{1}{f''(k^*)}] \rho'(\theta)$). Therefore, the negative effect on real money balances of a positive monetary disturbance is stronger.

The negative effect of inflation is strong enough so that both consumption and physical capital decrease, which is similar to Stockman (1981) and Abel (1985), but is different from the positive effect of inflation in Tobin (1965), Uzawa (1968), Epstein & Hynes (1983), Obstfeld (1981), and Kam (2005). And these conclusions affirm the theoretical conjecture of Stockman (1981) and the empirical evidences provided by Fischer (1993)⁸.

3.1.2 The Optimum Quantity of Money

To examine the optimality of Friedman’s rule for optimum quantity of money, let us write down the steady-state utility:

$$W^* = \int_{t=0}^{\infty} e^{-\rho(\theta)t} u(c^*, m^*) dt = \frac{u(c^*, m^*)}{\rho(\theta)}. \quad (44)$$

Taking the derivative of W^* with respect to θ in equation (44) yields

$$\frac{dW^*}{d\theta} = \frac{[u_c^* \frac{dc^*}{d\theta} + u_m^* \frac{dm^*}{d\theta}] \rho(\theta) - \rho'(\theta) u(c^*, m^*)}{\rho^2(\theta)} < 0. \quad (45)$$

It is easy to find that the total effect of a permanent increase in monetary growth on the equilibrium welfare can be divided into three negative parts: a decrease in utility owing to a lower consumption, $u_c^* \frac{dc^*}{d\theta} \rho(\theta)$; a decrease in utility due to lower real balances, $u_m^* \frac{dm^*}{d\theta} \rho(\theta)$; and a decrease in utility due to increased impatience, $-\rho'(\theta) u(c^*, m^*)$. Altogether, equation (45) tells us that an increase in the monetary growth rate cuts the steady-state welfare. Therefore, the equilibrium welfare can be improved by reducing the rate of monetary growth. That is to say, Friedman’s rule for optimum quantity of money is not optimal in the economy. In fact, we can explain this in another way. Suppose that the Friedman rule still holds, that is, the nominal interest rate is equal to zero. From equation (20), we have $u_m^* = 0$. Putting it into equation (45) leads to $\frac{dW^*}{d\theta} (= \frac{u_c^* \frac{dc^*}{d\theta} \rho(\theta) - \rho'(\theta) u(c^*, m^*)}{\rho^2(\theta)}) < 0$. This inequality

⁸Fischer (1993) demonstrates a 1% rise in inflation can cost an economy on the order of 0.1% in its rate of growth.

implies that the steady-state level of welfare can be improved all along by reducing the monetary growth rate. The optimum quantity of money may be setting $\theta = -\infty$, which is unreasonable and impossible. This implies that the optimum quantity of money in Friedman's sense does not hold in our model.

3.2 Long-run Effects of Fiscal Policy

3.2.1 Purely Crowding Out of Consumption

Similar to section 3.1.1, setting $d\theta = 0$ in equation (40) and applying Cramer's rule lead to

$$\frac{dc^*}{dg} = -1, \quad (46)$$

$$\frac{dk^*}{dg} = 0, \quad (47)$$

$$\frac{dm^*}{dg} = \frac{\pi_c^*}{\pi_m^*} < 0. \quad (48)$$

Proposition 3 *An increase in government expenditure reduces the steady-state consumption and real balance holdings, whereas it has no effect on the steady-state capital stock.*

It is easy to see from equations (46) and (47) that the long-run effects of positive government disturbances are the same as the nonmonetary Ramsey model: an increase of government expenditure crowds out private consumption one-to-one, and it has no effect on the long-run capital accumulation. And the negative effect on real money balances of an increase in government expenditures can be explained intuitively. The budget constraint of the government says that the income of inflation tax $\frac{\dot{M}}{P} (= \theta m)$ can be divided into two parts: the expenditure on the government consumption, g , and lump-sum transfers to the private sector, τ . If keeping τ and m constant and increasing g , the monetary authorities must increase θ and, hence, the inflation rate π . Then, a private individual with inflation aversion becomes more impatient, and he increases current consumption and cuts real money balances.

3.2.2 The Effect on the Steady-State Welfare

Taking the derivative of W^* with respect to θ in equation (44) gives rise to

$$\frac{dW^*}{dg} = \frac{u_c^* \frac{dc^*}{dg} + u_m^* \frac{dm^*}{dg}}{\rho(\theta)} = \frac{-[u_c^* - u_m^* \pi_c^* f''(k^*)]}{\rho(\theta)} < 0. \quad (49)$$

The equation above implies that an increase of government expenditure reduces the steady-state welfare. The negative effects can be divided into two parts: one is from the decrease in consumption ($\frac{-u_c^*}{\rho(\theta)}$), and the other is from a lower real balance holdings ($\frac{u_m^* \pi_c^* f''(k^*)}{\rho(\theta)}$). Then, with less patience and higher opportunity costs of holding money, consumers increase consumption, reduce savings and lower the holdings of money. In the long run, the steady-state levels of consumption and real money balances are reduced, and so is the welfare.

Furthermore, it may be interesting to examine the long-run effects of a fiscal expansion on the equilibrium inflation rate. Totally differentiating $\pi_t^* = \pi(c^*, k^*, m^*)$ with respect to g results in

$$\frac{d\pi^*}{dg} = \pi_c^* \frac{dc^*}{dg} + \pi_k^* \frac{dk^*}{dg} + \pi_m^* \frac{dm^*}{dg} = -\pi_c^* + \pi_m^* \frac{dm^*}{dg}. \quad (50)$$

First, we notice that the term $-\pi_c^*$ is negative from equation (29). Next the second term $\pi_m^* \frac{dm^*}{dg}$ is positive from equations (29) and (46). Hence, the total effect is ambiguous.

4 Summary

In this short paper, by introducing the inflation rate into the representative agent's time preference rate, we have reexamined the effects of monetary and fiscal policies in the money-in-utility model. The comparative static analysis has demonstrated: neither monetary superneutrality nor Friedman's rule for optimum quantity of money holds. Specifically, with an increase of the money growth rate, the steady-state consumption, physical capital stock, real money balance holdings, and welfare all decrease. In addition, with a rise in government expenditure, the steady-state consumption, real money balances, and welfare will be reduced, whereas the steady-state capital stock remains unchanged.

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