Exports, Foreign Technology Imports, and Long-Run Growth

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Abstract

A possible reason for the success of the export-oriented economies such as the East Asian "Tigers" is that exports enabled those countries to finance the accumulation of foreign technology and capital. This paper examines the theoretical foundations of this hypothesis. In an intertemporal optimization framework we divide a developing country's capital accumulation into two parts: traditional home-produced capital and imported foreign capital and technology. Exports are the means of financing the purchase of the latter. We show that an increase in exports leads to more home capital more foreign capital and more output in the long run. In addition export subsidies raise the long-run balanced growth rate while a terms-of-trade deterioration lowers the growth rate.

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I. Introduction

The economic success of many outward-looking economies, especially South Korea, Hong Kong, Singapore and Taiwan, has led economists to examine the links between export expansion and economic growth. The related empirical literature is mounting, with most studies confirming trade as the engine of growth, (see, e.g., World Bank, 1993). In practice, export promotion has become an important goal pursued by many developing countries and export incentive schemes have become increasingly popular.

The reasons why export promotion fosters economic growth are various and can be divided into two broad categories: the externalities argument and the technology argument. To many people, the export sector generates positive externalities to the whole economy; when the export sector expands, the whole economy becomes more efficient and competitive. Thus export growth leads to overall growth. (See Balassa, 1978, and de Melo and Robinson, 1990, among many others.) The export-induced benefits have been empirically modelled and tested in Feder (1983) and Tyler 1981) by introducing positive externalities of the export sector on the nonexport sector, or by introducing positive externalities of export growth on aggregate output.

Others, beginning with Chenery and Bruno (1962) and McKinnon (1964), have focused on the role of exports in generating foreign exchange and introducing advanced technology into poor, developing countries. They argue that: in the early stage of development, many countries cannot produce the needed technology-embodied capital goods; most developing countries even today rely on imports of capital goods in acquiring advanced technology; and exports are the means of earning foreign exchange and financing the desired technology-embodied capital. Therefore, compared to the role of positive externalities exerted by the export sector on the economy, export expansion is much more important in accelerating technology transfer from developed to developing countries and transforming the traditional modes of production into modern ones. In this sense, exports are the vehicle of technology progress and modernization in developing countries.

While these two lines of arguments have been put forward in some simple models and numerous empirical tests, to our knowledge, there does not exist an intertemporal general equilibrium model to address the following questions: What is the nature of foreign capital that gives rise to a positive relationship between exports and income? Does a higher level of exports lead to a higher income level or growth rate? Should exports be subsidized? In this paper, we presents the simplest dynamic general equilibrium model that enables us to answer these and related questions on the link between exports and growth. In section II, we present a two-good model of optimal growth by dividing capital accumulation in a typical developing country into two parts: the accumulation of traditionally, home-produced capital and the accumulation of imported foreign technology. This distinction plays a critical role in our results. Revenues from exports are used for foreign good consumption and foreign technology imports. As long as foreign capital is different from domestic capital, exports can always expand output by enabling the purchase of foreign imports and technology. When foreign demand for the exports of the developing country is inelastic, we formally show that an increase in exports leads to more domestic capital, more foreign technology imports and more output in the long run. These results can be regarded as a confirmation of the Bruno-Chenery-McKinnon argument in a dynamic optimization model.

In section III, we extend our model to the case of endogenous growth by relaxing the assumption of inelastic foreign demand for exports. We assume that a typical developing country can export any amount at the competitive price in the world market. Obviously, this assumption is necessary for a country to have positive endogenous growth rate; otherwise, the inelastic foreign demand will set an exogenous upper bound on the growth rate. In this endogenous growth framework, we find that export subsidies can increase the long-run growth rate.

In section IV. we summarize our results and point out directions of further research.

II. A Traditional Analysis with Inelastic Demand for Exports

There are two economies in this model: the home country and the foreign country. The home country is a developing economy, and foreign country is a developed one. There are two goods --- the home good and the foreign good; and the home good price in the foreign market is p_x.

The home good is produced by a representative agent and the technology is given by the production function, $f(k_h,k_f)$, which is concave, continuously differentiable and homogeneous of degree one in home capital, k_h, and foreign capital, k_f (foreign capital here should be interpreted in a broad sense as both foreign tangible or physical capital and intangible capital such as science and technology). While there is substitution between home capital and foreign capital in production, in general, foreign capital through its embodiment of modern technology is more efficient than home capital.

We further assume that there is no foreign direct investment in the home country. To obtain foreign technology, the representative agent relies on her export earnings $p_x x$, where x is the representative agent's exports. There are many ways to specify foreign demand for the home country's exports. One popular approach pioneered by Chenery and Bruno (1962) and McKinnon (1964) among others is to posit an inelastic demand for the developing country's exports. That is to say, in each time period, the home country's exports are given by fixed foreign demand:

$$x = x^*. \qquad (1)$$

Let c_h and c_f be the representative agent's home good consumption and foreign good consumption, respectively. Let δ_h and δ_f be the capital depreciation rates for home capital and foreign, capital respectively. Then the dynamic equations for the accumulation of home and foreign capital are:

$$k_h = f(k_h, k_f) - c_h - \delta_h k_h - x^*, \qquad (2)$$

$$\dot{k}_f = p_x x^* - c_f - \delta_f k_f. \qquad (3)$$

Let the representative agent in the home country have an instantaneous utility function specified as:

$$u(c_h) + \theta v(c_f). \qquad (4)$$

The separability of the utility function is purely for analytical simplicity. The constant θ is positive and measures the preference for foreign good consumption. As usual, the functions u and v have the following standard properties: $u' > 0, v' > 0, u'' < 0,$ and $v'' < 0.$

The representative agent in the home country maximizes a discounted utility stream over an infinite time horizon subject to the dynamic constraints (1) and (9):

$$\int_0^{\infty}\{u(c_h)+\theta v(c_f)\}e^{-\rho t}dt \qquad (5)$$

The initial values of home capital and foreign capital are given by $k_h(0)$ and $k_f(0)$ respectively.

The current value Hamiltonian function is

$$H(c_h, c_f, k_h, k_f, \lambda_h, \lambda_f) = u(c_h) + \theta(c_f) + \lambda_h[f(k_h, k_f) - c_h - x - \delta_h k_h]$$

$$+\lambda_f[p_x x^* + c_f - \delta_f k_f] \qquad (6)$$

The necessary conditions for maximization are

$$u'(c_h) = \lambda_h \qquad (7)$$

$$\theta v'(c_f) = \lambda_f \qquad (8)$$

$$\lambda_h[(\partial f/\partial k_h) - (\delta_h + \rho)] = -\dot{\lambda}_h \qquad (9)$$

$$\lambda_h(\partial f/\partial k_h) - \lambda_f(\delta_f + \rho) = -\dot{\lambda}_f \qquad (10)$$

plus the dynamic constraints (2) and (3), the initial conditions, and the transversality conditions.

Substituting (7) and (8) into (9) and (10), we obtain a dynamic system of c_h, c_f, and k_f:

$$\dot{c}_h = \frac{u'(c_h)}{-u''(c_h)}[\partial f/\partial k_h - (\delta_h + \rho)] \qquad (11)$$

$$\dot{c}_f = \frac{1}{-\theta v''(c_f)}[u'(c_h)\partial f/\partial k_f - \theta v'(c_f)(\delta_f + \rho)] \qquad (12)$$

$$\dot{k}_h = f(k_h, k_f) - c_h - x^* - \delta_h k_h \qquad (13)$$

$$\dot{k}_f = p_x x^* - c_f - \delta_f k_f \qquad (14)$$

In the steady state,

$$\dot{c}_h = \dot{c}_f = \dot{k}_d = \dot{k}_f = 0 \qquad (15)$$

so the necessary conditions for optimization in equilibrium are

$$(\partial f/\partial k_h) - (\delta_h + \rho) = 0 \qquad (16)$$

$$u'(\bar{c}_h)\,\partial f \big/ \partial k_f - \theta v'(\bar{c}_f)(\delta_f + \rho) = 0 \qquad (17)$$

$$f(\bar{k}_h, \bar{k}_f) - \bar{c}_h - x^* - \delta_h \bar{k}_h = 0 \qquad (18)$$

$$p_x x^* - \bar{c}_f - \delta_f \bar{k}_f = 0 \qquad (19)$$

where a bar over a variable denotes its steady state value and all derivatives are evaluated at the steady state.

Condition (16) resembles the modified golden rule in the Cass (1965) model. Condition (17), the optimal condition for investing in foreign technology and capital, says that an increase in investment of foreign capital brings about the benefit, $u'(c_h)\,\partial f \big/ \partial k_f$, and its associated cost, $\theta v'(.)(\delta_f + \rho)$; at equilibrium, these two effects are equal. Condition (18) implies that aggregate output is used as home good consumption c_h, exports x^* and home good investment $\delta_h k_h$. Condition (19) says that total exports are used as foreign good consumption and foreign good investment.

Next, we will study the stability of the model and the effects of an increase in exports on capital accumulation, output and consumption.

Proposition 1: The equilibrium is saddle-point stable.

Proof: Linearize equations (11), (12), (13) and (14) around the steady state values $\bar{c}_h, \bar{c}_f, \bar{k}_h, \bar{k}_f$:

$$\begin{bmatrix} \dot{c}_h \\ \dot{c}_f \\ \dot{k}_h \\ \dot{k}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & (u'/-u'')\partial^2 f / \partial k_h^2 & (u'/-u'')\partial^2 f / \partial k_h \partial k_f \\ \dfrac{u''}{-\theta v''}\partial f / \partial k_f & \delta_f + \rho & \dfrac{u'}{-\theta v''}\partial^2 f / \partial k_h \partial k_f & \dfrac{u'}{-\theta v''}\partial^2 f / \partial k_f^2 \\ -1 & 0 & \partial f / \partial k_h - \delta_h & \partial f / \partial k_f \\ 0 & -1 & 0 & -\delta_f \end{bmatrix} \begin{bmatrix} c_h - \bar{c}_h \\ c_f - \bar{c}_f \\ k_h - \bar{k}_h \\ k_f - \bar{k}_f \end{bmatrix}$$

(20)

Let $\gamma_1, \gamma_2, \gamma_3,$ and γ_4 be the four eigenvalues of the dynamic system. Then the trace of the 4x4 matrix is the sum of the four eigenvalues:

$$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 = \delta_f + \rho + [\partial f / \partial k_h - \delta_h] - \delta_f = 2\rho > 0 \qquad (21)$$

for $(\partial f / \partial k_h - \delta_h) = \rho$ by (16). Expression (21) says that at least one eigenvalue is positive.

Next the product of the four eigenvalues is given by the determinant of the 4x4 matrix:

$$\gamma_1\gamma_2\gamma_3\gamma_4 = \frac{(u')^2}{u''\theta v''}[(\partial^2 f / \partial k_h^2)(\partial^2 f / \partial k_f^2) - (\partial^2 f / \partial k_h \partial k_f)^2] + \frac{u'}{u''}\delta_f(\delta_f + \rho)\partial^2 f / \partial k_h^2$$

$$+ \frac{u'}{\theta v''}(\partial f / \partial k_f)^2(\partial^2 f / \partial k_h^2) - \frac{\rho u'}{\theta v''}(\partial f / \partial k_f)(\partial^2 f / \partial k_h \partial k_f) > 0. \qquad (22)$$

Each term on the right hand side of (22) is positive as $f(k_h, k_f)$, $u(c_h)$ and $v(c_f)$ are concave, $(\partial^2 f / \partial k_h \partial k_f) > 0$. (22) together with (21) implies that there exist either two positive eigenvalues or four positive eigenvalues. We need to show that the former is true.

Suppose that there exist four positive eigenvalues. Then the term $(\gamma_1\gamma_2\gamma_3 + \gamma_1\gamma_2\gamma_4 + \gamma_2\gamma_3\gamma_4 + \gamma_1\gamma_3\gamma_4)$ is positive. But straightforward calculation shows that this term (which equals the sum of the four third-order principal minors of the 4x4 matrix) is negative:

$$\gamma_1\gamma_2\gamma_3 + \gamma_1\gamma_2\gamma_4 + \gamma_2\gamma_3\gamma_4 + \gamma_1\gamma_3\gamma_4 = \rho(u'/-u'')\partial^2 f / \partial k_h^2 - \delta_f(\delta_f + \rho)(\partial f / \partial k_h - \delta_h)$$

$$-(u'/\theta v'')(\partial f / \partial k_h - \delta_h)\partial^2 f / \partial k_f^2 < 0. \qquad (23)$$

which contradicts the assumption that the system has four positive eigenvalues. Therefore, the system has two negative roots and two positive ones. As the number of predetermined variables k_h and k_f is equal to the number of negative roots, and the number of jumped variables c_h and c_f is equal to the number of positive roots, the system has a unique convergent, saddle-point path to the steady state. QED.

To find out the effects of an export increase on the home country's accumulation of home capital and foreign capital, and the home country's consumption of home product and foreign product, we totally differentiate (16), (17), (18) and (19), and evaluate all expressions at the steady state values:

$$\begin{bmatrix} \partial^2 f/\partial k_h^2 & \partial^2 f/\partial k_h \partial k_f & 0 & 0 \\ (\partial^2 f/\partial k_f \partial k_h)u' & (\partial^2 f/\partial k_f^2)u' & u''\partial f/\partial k_f & -\theta v''(\delta_f+\rho) \\ \partial f/\partial k_h - \delta_h & \partial f/\partial k_f & -1 & 0 \\ 0 & \delta_f & 0 & -1 \end{bmatrix} \begin{bmatrix} dk_h \\ dk_f \\ dc_h \\ dc_f \end{bmatrix} = \begin{bmatrix} 0 \\ v'(\delta_f+\rho)d\theta \\ dx^* \\ p_x dx^* \end{bmatrix}$$

(24)

The determinant of the 4x4 matrix can be calculated by using the Laplace expansion on the fourth column. Denote the determinant as Δ', (note, in the steady state, $\partial f/\partial k_h - \delta_h = \rho$.)

$$\Delta' = \delta_f(\delta_f+\rho)\theta v''(.)\partial^2 f/\partial k^2 + u'(.)x^*[(\partial^2 f/\partial k_h^2)(\partial^2 f/\partial k_f^2) - (\partial^2 f/\partial k_h \partial k_f)^2]$$

$$+u''(.)x^*(\partial^2 f/\partial k_h^2)(\partial f/\partial k_f)^2 - \rho x^* u''(.)(\partial^2 f/\partial k_h \partial k_f)(\partial f/\partial k_f) > 0$$

(25)

Δ' is positive because $f(k_h, k_f), u(.)$ and $v(.)$ are concave, and $\partial f/\partial k_h$, $\partial f/\partial k_f$, and $\partial^2 f/\partial k_h \partial k_f$ are positive.

Proposition 2: In the long run, an increase in the home country's exports leads to more foreign capital, more home capital and more output in the home country.

Proof: Use Cramer's rule in (24):

$$dk_f/dx^* = \{[\theta(\delta_f+\rho)v'' p_x + u''\partial f/\partial k_f]\partial^2 f/\partial k_h^2\}/\Delta' > 0 \qquad (26)$$

So an increase in the home country's exports leads to more foreign capital in the home country.

The effect on home capital accumulation is easily seen from (16) and (26):

$$dk_d/dx^* = [-(\partial^2 f/\partial k_h \partial k_f)/(\partial^2 f/\partial k_h^2)]dk_f/dx^* > 0 \qquad (27)$$

As more exports increase both foreign and home capital accumulation in the long run, more output will be produced. QED.

Proposition 2 highlights the importance of developing countries' exports in technology absorption and economic growth. In this model, it is exports that finance the means for the developing country to acquire advanced foreign technology and capital from the developed country, and it is foreign technology which improves the productivity of home capital and accelerate home capital accumulation and output growth. In this sense, exports can be regarded as the engine of economic growth in the developing country.

Note that this proposition cannot be obtained from the traditional Solow model with a fixed saving rate. To see this, let σ be the saving rate. The total gross savings are $\sigma f(k_h, k_f)$ and the gross savings for foreign good investment are assumed to be $\sigma' x,\ 0 \le \sigma' \le 1$. Then the capital accumulation equations are:

$$\dot{k}_h = \sigma f(k_h, k_f) - \delta_h k_h - x^*,$$

$$\dot{k}_f = \sigma' p_x x^* - \delta_f k_f .$$

Now if exports increase, $\sigma' x$ will increase. In the steady state, k_f will if rise. But an increase in exports directly reduces investment resources for home good investment. If the rise in output due to increased foreign capital falls short of the direct reduction in home good investment, k_h will be lower and the total output may also be lower. This can happen because the saving rate is fixed both in the short run and the long run. In our optimization model, the possibility of a lower home capital as a result of export increase is avoided because, in the short run, more foreign capital improves the productivity of home capital and people respond to this by saving more for home good investment. So in the long run, there will be more home capital and more foreign capital.

Proposition 3: The greater the preference for foreign goods, the lower are foreign capital, home capital, home good consumption in the long run.

In the model, we defined a parameter θ in the utility function to measure the home country's preference for foreign good consumption. This is an attempt to capture the cross-country difference in the tastes for home and foreign products. Recent empirical studies about developing countries' debt crisis have often emphasized the importance of channeling export revenue and foreign borrowing into investment instead of imported consumption goods; see Sachs (1986). This point is reflected in our model by the fact that a foreign good lover with a higher value of θ accumulates less capital produces less output and consumes less home good.

To prove this, we apply Cramer's rule to (24):

$$dk_d / d\theta = (\delta_f + \rho) v' [\partial^2 f / \partial k_h^2] / \Delta' < 0 \qquad (28)$$

$$dk_h / d\theta = [-(\partial^2 f / \partial k_h \partial k_f)/(\partial^2 f / \partial k_h^2)] dk_k / d\theta < 0 \qquad (29)$$

$$dc_h / d\theta = [\partial f / \partial k_h - \delta_h] \partial k_h \partial \theta + \partial f / \partial k_f) \partial k_f / \partial \theta < 0. \qquad (32)$$

QED.

The reason for proposition 3 is clear: If foreign good consumption is preferred by the representative agent in the home country, more export revenue will be used to import consumption goods, leaving less for investment in foreign technology. In the long run, there will be less foreign capital and technology available in home country, and thus the productivity of home capital will be lower and output smaller.

III. An Endogenous Growth Model with Competitive Foreign Markets

The inelastic foreign demand, x^*, adopted in the Bruno-Chenery-McKinnon approach in the last section, prevents the developing home country's economy from expanding faster that a certain positive rate. If foreign demand is growing at some exogenously given rate, then the home country's output, consumption and capital can at most grow at the same rate. In reality, developing countries can often expand their exports through pricing policy and trade policy, and they may even have some market power over their exports. In this section, while relaxing the very restrictive assumption in the last section, we make another extreme or a small-developing-country assumption that the home country can export at any amount at a market given price p_x .

With this assumption of perfectly elastic demand for the home country's exports, a positive, endogenous growth rate can be generated. To derive an explicit solution for the endogenous growth rate in our model, we specialize our production function and the utility function. We will take the technology to be Cobb-Douglas: $f(k_h, k_f) = k_h^{\alpha} k_f^{(1-\alpha)}$. We also define the representative agent in the home country to have an instantaneous utility function specified as $u(c_h, c_f) = \log c_h + \theta \log c_f$.

We still assume that there is no foreign direct investment or foreign borrowing in the home country. To obtain foreign capital, the representative agent relies on her export earning $p_x x$, here x is the agent's exports. To make our model more realistic, we introduce a few policy parameters here. For each unit of exports, the home country's government levies a tax or provides subsidy at the rate τ_x. The government also taxes output at the rate τ_y. Let c_h and c_f continue to be the representative agent's home good consumption and foreign good consumption, respectively. Then the dynamic equations for the accumulation of home capital and foreign capital are:

$$\dot{k}_h = (1-\tau_y)k_h^{\alpha}k_f^{(1-\alpha)} - c_h - (1+\tau_x)x, \tag{33}$$

$$\dot{k}_f = p_x x - c_f, \tag{34}$$

where we have, for simplicity, assumed away capital depreciation.

The representative agent in the home country maximizes:

$$\int_0^{\infty}[\log c_h + \theta \log c_f]e^{-\rho t}dt,$$

subject to constraints (33) and (34). The initial values of home capital and foreign capital are again given by $k_h(0)$ and $k_f(0)$, respectively.

The current value Hamiltonian is

$$H(c_h, c_f, k_h, k_f, \lambda_h, \lambda_f) = \log c_h + \theta \log c_f + \lambda_h[(1-\tau_y)k_h^{\alpha}k_f^{(1-\alpha)} - c_h - (1+\tau_x)x] + \lambda_f[p_x x - c_f] \tag{35}$$

The necessary conditions for an optimum are

$$c_f = \frac{\theta p_x}{(1+\tau_x)}c_h, \tag{36}$$

$$\frac{\dot{c}_h}{c_h} = (1-\tau_y)\alpha[\frac{k_f}{k_h}]^{1-\alpha} - \rho, \tag{37}$$

$$\frac{\dot{c}_f}{c_f} = p_x(1+\tau_x)^{-1}(1-\tau_y)(1-\alpha)[\frac{k_h}{k_f}]^{\alpha} - \rho, \tag{38}$$

$$\dot{k}_h = (1-\tau_y)k_h^{\alpha}k_f^{(1-\alpha)} - c_h - (1+\tau_x)x, \tag{33}$$

$$\dot{k}_f = p_x x - c_f. \tag{34}$$

We proceed to analyze our model in the rest of this section. In the optimal condition (36), we log-differentiate both sides with respect to time t:

Proposition 4: The home-good consumption and the foreign-good consumption grow at the same rate.

We denote this growth rate as $\gamma = \dot{c}_h/c_h = \dot{c}_f/c_f$.

To find an explicit solution to this consumption growth rate γ, we substitute

$\gamma = \dot{c}_h / c_h = \dot{c}_f / c_f$ into optimal conditions (37) and (38), and denote the ratio of home capital over foreign capital as $z = k_h / k_f$:

$$\gamma = (1-\tau_y)\alpha z^{\alpha-1} - \rho, \tag{39}$$

$$\gamma = p_x(1+\tau_x)^{-1}(1-\tau_y)(1-\alpha)z^{\alpha} - \rho. \tag{40}$$

That is to say,

$$(1-\tau_y)\alpha z^{\alpha-1} = p_x(1+\tau_x)^{-1}(1-\tau_y)(1-\alpha)z^{\alpha}. \tag{41}$$

Solving *z* from equation (41):

$$z = \alpha(1-\alpha)^{-1} p_x(1+\tau_x). \tag{42}$$

Then substituting *z* of equation (42) into (40), we have:

Proposition 5: The growth rate of both home and foreign good consumption is given by:

$$\gamma = \alpha^{\alpha}(1-\alpha)^{1-\alpha} p_x^{(1-\alpha)}(1-\tau_y)(1+\tau_x)^{\alpha-1} - \rho. \tag{43}$$

Proposition 6: Given the constant consumption growth rate γ in (43), the growth rates of foreign capital and home capital are the same; and the ratio $z = k_h / k_f$ is a constant.

To show that this is true, we rewrite (40) as

$$\gamma + \rho = p_x(1+\tau_x)^{-1}(1-\tau_y)(1-\alpha)z^{\alpha}.$$

Log-differentiate both sides of the above equation and note that $z = k_h / k_f$:

$$\dot{k}_h / k_h = \dot{k}_f / k_f. \tag{44}$$

We will denote this common growth rate for these two capital stock as

$\gamma' = \dot{k}_h / k_h = \dot{k}_f / k_f$.

Proposition 7: If the capital growth rate, γ', is a constant, then γ'= γ, namely, the growth rate of capital stocks is the same as the growth rate of consumption.

Proof: Divide equations (33) and (34) by k_h and k_f, respectively and note that $z = k_h / k_f$:

$$\gamma' = \dot{k}_h / k_h = (1-\tau_y) z^{\alpha-1} - (c_h / k_h) - (1+\tau_x) x / k_h, \qquad (45)$$

$$\gamma' = \dot{k}_f / k_f = (p_x x / k_f) - c_f / k_f. \qquad (46)$$

Since $c_f = \theta p_x (1+\tau_x)^{-1} c_h$ from (36), we can rewrite (46) as

$$\gamma' = \dot{k}_f / k_f = (p_x x / k_f) - \theta \rho_x (1+\tau_x)^{-1} c_h / k_f$$

$$= [(p_x x / k_k) - \theta \rho_x (1+\tau_x)^{-1} c_h / k_h] z. \qquad (47)$$

From (47), we solve (x / k_h) :

$$(x / k_h) = [(\gamma' / z) + \theta p_x (1+\tau_x)^{-1} c_h / k_h] / p_x. \qquad (48)$$

Substituting (x/k_h) into (45) and collecting terms:

$$c_h / k_h = [(1-\tau_y) z^{2-\alpha} - \gamma' (z + (1+\tau_x) p_x^{-1}] / (1+\theta) z. \qquad (49)$$

We know that the right hand side of (49) is constant because z is a constant as given in proposition 3 and γ' is a constant by assumption. Thus log-differentiate both sides of (49):

$$\dot{c}_h / c_h = \dot{k}_h / k_h;$$

or,

$$\gamma = \dot{c}_h / c_h = \dot{c}_f / c_f = \dot{k}_h / k_h = \dot{k}_f / k_f = \gamma'. \qquad (50)$$

QED.

Proposition 8: Given $\gamma = \gamma'$, exports grow at the constant rate γ.

To see this, we log-differentiate both sides of (48):

$$\dot{x} / x = \dot{k}_h / k_h = \gamma' = \gamma. \qquad (51)$$

To sum up, all economic variables in our model, home goods consumption, foreign goods consumption, home capital, foreign capital and exports, grow at the same rate γ,

$$\gamma = \alpha^{\alpha} (1-\alpha)^{1-\alpha} p_x^{(1-\alpha)} (1-\tau_y)(1+\tau_x)^{\alpha-1} - \rho. \qquad (43)$$

Proposition 9: Export subsidies raise the long-run economic growth rate; a favorable terms of trade shift (a rise in p_x) raises the growth rate; and a output tax reduces the growth rate.

To show this, we differentiate γ with respect to different parameters in (43):

$$d\gamma / d\tau_x = \alpha^{\alpha}(1-\alpha)^{1-\alpha} p_x^{(1-\alpha)}(1-\tau_y)(\alpha-1)(1+\tau_x)^{\alpha-2} < 0;$$

$$d\gamma / dp_x = \alpha^{\alpha}(1-\alpha)^{1-\alpha}(1-\alpha) p_x^{-\alpha}(1-\tau_y)(1+\tau_x)^{\alpha-1} > 0;$$

$$d\gamma / d\tau_y = -\alpha^{\alpha}(1-\alpha)^{1-\alpha} \rho_x^{(1-\alpha)}(1+\tau_x)^{\alpha-1} < 0.$$

Proposition 9 has some strong implications for both empirical studies and policy discussion. Recall that our model is set up in an environment of perfect competition: there exists no distortion in factor demand at home and abroad; the production function is the typical Cobb-Douglas one with two capital inputs. Still export subsidies can increase the long-run growth rate. It is not difficult to justify this observation. As export subsidies lead to more exports, which in turn result in more foreign technology imports, the accumulation of foreign technology in the home country is accelerated and more output is produced. These two effects are combined to give rise to more domestic capital accumulation because on the one hand, foreign technology improves the efficiency of domestic capital, and, on the other, more output simply provides more resources for home capital accumulation.

It should be emphasized that, if export subsidies are financed through an output tax, then, as seen in proposition 9, the output tax has a negative impact on the balanced growth rate. Thus, if we take the government budget constraint into our consideration, we have two offsetting effects at work.

The negative effect of a terms-of-trade shock on the balanced growth rate is easy to understand. As the shock reduces the foreign exchange available for the home country to import foreign technology, productivity at the home country is lowered and total domestic production shrinks. The result is slower growth.

Just for comparison, we note that the consumption preference parameter for foreign good, θ, does not have any effect on the long-run growth rate while it is negatively related to the steady state capital accumulation as shown in proposition 3 in the last section. This is due to the specific utility function used in our analysis of endogenous growth.

To complete our analysis, we have to check a few things. First, we need to determine the initial values of our variables. We are given the initial stock variables $k_h(0)$ and $k_f(0)$. Since all variables are growing at the same constant rate γ, we have:

$$k_h(t) = k_h(0)e^{\gamma t},$$

$$k_f(t) = k_f(0)e^{\gamma t},$$

$$c_h(t) = c_h(0)e^{\gamma t},$$

$$c_f(t) = c_f(0)e^{\gamma t},$$

$$x(t) = x(0)e^{\gamma t}.$$

The initial home capital investment and initial foreign capital investment can be easily shown to be:

$$\dot{k}_h(0) = k_h(0)\gamma, \qquad (52)$$

$$\dot{k}_f(0) = k_f(0)\gamma. \qquad (53)$$

Substituting these two values in equations in (33) and (34), and noting that $c_f(0) = \theta p_x(1+\tau_x)^{-1}c_h(0)$:

$$k_h(0)\gamma = (1-\tau_y)k_h(0)^{\alpha}k_f(0)^{(1-\alpha)} - c_h(0) - (1+\tau_x)x(0),$$

$$k_f(0)\gamma = p_x x(0) - \theta p_x(1+\tau_x)^{-1}c_h(0).$$

Then,

$$c_h(0) = (1+\theta)^{-1}\{(1-\tau_y)k_h(0)^{\alpha}k_f(0)^{(1-\alpha)} - k_h(0)\gamma - (1+\tau_x)k_f(0)\gamma p_x^{-1}\} \qquad (54)$$

$$c_f(0) = \theta p_x(1+\tau_x)^{-1}(1+\theta)^{-1}\{(1-\tau_y)k_h(0)^{\alpha}k_f(0)^{(1-\alpha)} - k_h(0)\gamma - (1+\tau_x)k_f(0)\gamma p_x^{-1}\} \qquad (55)$$

$$x(0) = [k_f(0)\gamma p_x^{-1} + \theta(1+\tau_x)^{-1}](1+\theta)^{-1}\{(1-\tau_y)k_h(0)^{\alpha}k_f(0)^{(1-\alpha)} - k_h(0)\gamma - (1+\tau_x)k_f(0)\gamma p_x^{-1}\}. \qquad (56)$$

Therefore, all those parameter changes not only affect the long-run growth rate, they also affect the initial optimal choices of consumption, investment and exports.

Next, we need to check the boundedness of the discounted utility, which is given by:

$$\int_0^\infty \{\log c_h) + \theta \log c_f\} e^{-\rho t} dt$$

$$= \int_0^\infty \{\log c_h(0) + \theta \log c_f(0) + (1+\theta)\gamma t\} e^{-\rho t} dt$$

$$= [\log c_h(0) + \theta \log c_f(0)]\rho^{-1} + \rho^{-2}(1+\theta)\gamma < \infty.$$

Which is finite.

Finally, it is necessary to impose a balanced government budget constraint. Since we have excluded all borrowing possibility, the government budget has to be balanced in each time t (here we take τ_x to be an export subsidy):

$$\tau_y k_h^\alpha k_f^{1-\alpha} = \tau_x x.$$

Or,

$$\tau_y = k_h(0)^{-\alpha} k_f(0)^{\alpha-1} x(0)\tau_x, \tag{57}$$

and x(0) is given by

$$x(0) = [k_f(0)\gamma p_x^{-1} + \theta(1+\tau_x)^{-1}](1+\theta)^{-1}\{(1-\tau_y)k_h(0)^\alpha k_f(0)^{(1-\alpha)} - k_h(0)\gamma - (1+\tau_x)k_f(0)\gamma p_x^{-1}\}. \tag{56}$$

$$\gamma = \alpha^\alpha(1-\alpha)^{1-\alpha} p_x^{(1-\alpha)}(1-\tau_y)(1+\tau_x)^{\alpha-1} - \rho. \tag{43}$$

Combining (43), (56) and (57), we can implicitly solve τ_y as a function of τ_x and, then, substituting $\tau_y(\tau_x)$ into (43) and get the long-run growth rate

$$\gamma = \alpha^\alpha(1-\alpha)^{1-\alpha} p_x^{(1-\alpha)}[1-\tau_y(\tau_x)](1+\tau_x)^{\alpha-1} - \rho. \tag{58}$$

IV. Conclusions

In this paper, we have formulated in an intertemporal framework the connection between exports and foreign technology imports in a typical developing country. If foreign demand for a developing country's exports is inelastic, and if the foreign exchange constraints on technology are acute as in many poor developing countries, then, an increase in exports leads to more domestic capital accumulation, more foreign technology imports and more output in the long run. Our analysis, while based on dynamic optimization, provides a full support for the early Bruno-Chenery-McKinnon approach.

Even when we totally abandon the assumption of inelastic demand but retain the assumption on the difference between foreign technology and domestic technology in a typical developing country, our endogenous growth model shows how export promotion can lead to a higher balanced growth rate. While we show that export subsidies can increase the country's long-run growth rate, the costs of financing these subsidies can dampen and even undermine this effect. The policy implications of these results will therefore have to await further empirical work.

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