THE SPIRIT OF CAPITALISM, SAVINGS, ASSET PRICES AND GROWTH

Chapter IV

Taxes, Government Spending and Growth

In this chapter, I present some detailed policy analysis in this capitalist-spirit model of growth and compare the results to the ones from the standard optimal growth model in Ramsey (1928), Koopmans (1965) and Cass (1965). It is well-known that long-run capital accumulation in the Ramsey-Koopmans-Cass model is determined by the famous modified golden rule. Without changing the time preference, the population growth rate and other exogenous factors, government spending and consumption tax have no effect on long-run capital accumulation, and they only crowd out equilibrium consumption. In the capitalist-spirit model, I will show that, in the long run, a consumption tax stimulates capital accumulation, and government spending reduces the equilibrium capital stock.

In addition to long-run equilibrium analysis, I utilize a technique developed by Kenneth Judd (1982, 1985, 1987) to analyze the short-run effects of fiscal policies in the capitalist-spirit model. I find that: (1) a future rise in government spending stimulates current private investment; (2) a future increase in the consumption tax raises current private investment; (3) a change in the future output tax has an ambiguous effect on current investment; (4) all current increases in government taxes and government spending reduce current private investment.

IV.1. Long-Run Policy Analysis

In this section, we are going to see some significant differences in their long-run policy implications between the capitalist-spirit model and the Ramsey-Koopmans-Cass growth model. The policy parameters analyzed here are: an output tax τ_y, a consumption tax τ_c, and government spending g. I also assume that all policy changes in this subsection are permanent; I will deal with temporary policy changes in the next section. When these policy instruments are introduced into the model, the optimization program becomes:

$$\text{Max}\int [u(c)+\beta v(b)]e^{-\rho t}dt, \quad \tau_y$$

s.t.

$$\dot{k}=(1-\tau_y)f(k)-(1+\tau_c)c-g. \quad \tau_c$$

Then, the basic motion equations describing the optimal path are modified to be:

$$\dot{c} = \frac{1}{-u''(c)}\left[(1+\tau_c)\beta v'(k) + u'(c)((1-\tau_y)f'(k)-\rho)\right] \tag{4.1}$$

$$\dot{k} = (1-\tau_y)f(k)-(1+\tau_c)c-g. \tag{4.2}$$

The following long-run policy analysis will be made by focusing on the saddle-point equilibria. If there are multiple equilibria in this system, I denote one pair of the steady-state values of consumption and capital as $(\bar{c},\bar{k})$ and linearize the steady-state equations of (4.1) and (4.2) around $(\bar{c},\bar{k})$:

$$\begin{bmatrix} \rho-(1-\tau_y)f'(\bar{k}) & -\dfrac{(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{u''(\bar{c})} \\ -(1+\tau_c) & (1-\tau_y)f'(\bar{k}) \end{bmatrix} \begin{bmatrix} dc \\ dk \end{bmatrix} = \begin{bmatrix} \dfrac{\beta v'(\bar{k})}{u''(\bar{c})}d\tau_c - \dfrac{u'(\bar{c})f'(\bar{k})}{u''(\bar{c})}d\tau_y \\ \bar{c}d\tau_c + f(\bar{k})d\tau_y + dg \end{bmatrix}. \tag{4.3}$$

Denote the 2x2 matrix in equation (4.3) as M' and the the determinant of M' as Δ'. Then Δ' is negative at $(\bar{c},\bar{k})$ by the property of a saddle-point equilibrium.

It has become well known that the steady-state capital stock in the Ramsey-Koopmans-Cass model is uniquely determined by the modified golden rule: $f'(k^{mg}) = \rho$. In this simplest form it tells us that the consumption tax τ_c and government spending g, if introduced to the Ramsey-Koopmans-Cass model as in equation (4.2), cannot affect the long-run capital accumulation k^{mg}. Their long-run effects only appear to crowd out private consumption. But, in this aspect, the capitalist-spirit model stands in sharp contrast to the Ramsey-koopmans-Cass model:

Proposition 4.1: Government spending crowds out both long-run private consumption and long-run **capital stock.**

This proposition is much closer to the policy discussions in the real life when people talk about how government spending crowds out private investment and capital formation. From the perspective of the Ramsey-Koopmans-Cass model, these kinds of policy discussions do not hold tight because government spending does not crowd out the equilibrium capital accumulation. Here the capitalist spirit model provides a firm foundation for the policy debate on the effect of government spending on capital formation. To show proposition 4.1, I apply Cramer's rule in equation (4.3):

$$\frac{dc}{dg} = \frac{(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{\Delta' u''(\bar{c})} < 0. \tag{4.4}$$

$$\frac{dk}{dg} = \frac{[\rho-(1-\tau_y)f'(\bar{k})]}{\Delta'} < 0. \tag{4.5}$$

The reason for proposition 4.1 is the following: as an increase in government spending reduces the representative agent's income, the representative agent responds to the increase in the very short run by cutting his consumption. Since the capital stock is fixed in the short run, the marginal utility of the reduced consumption u'(c) (here c is less than the initial steady state consumption $\bar{c}$ as a result of an increase in g) weighted by $(1-\tau_y)(\rho-f'(k))$ will be higher than the marginal utility of the capital stock $\beta v'(\bar{k})$ weighted by the constant $(1+\tau_c)$. So the equilibrium condition,

$$(1-\tau_y)(\rho-f'(k))u'(c) = \beta v'(k)(1+\tau_c),$$

can no longer be maintained. The representative agent will react to lower his investment in the short run until eventually the capital stock is reduced and the above equilibrium condition is restored when both c and k are smaller than the initial equilibrium values $(\bar{c},\bar{k})$.

Turning to a consumption tax τ_c, I find another significant difference between the capitalist-spirit model and the Ramsey-Koopmans-Cass model:

Proposition 4.2: In the long run, a consumption tax has an ambiguous effect on consumption but it always increases capital accumulation.

Unlike in the Ramsey-Cass-Koopmans model where a consumption tax has no effect on capital accumulation and it only reduces consumption, the capitalist-spirit model gives rise to a positive connection between ~~the~~ consumption tax and capital accumulation. The ambiguous ~~result~~ effect of a consumption tax on consumption is also very interesting. To show this proposition, I simply apply Cramer's rule again in equation (4.3):

$$\frac{dc}{d\tau_c} = \frac{\beta v'(\bar{k})(1-\tau_y)f'(\bar{k}) + \bar{c}(1+\tau_c)\beta v''(\bar{k}) + \bar{c}(1-\tau_y)u'(\bar{c})f''(\bar{k})}{\Delta' u''(\bar{c})}, \tag{4.6}$$

$$\frac{dk}{d\tau_c} = \frac{[\rho-(1-\tau_y)f'(\bar{k})]u''(\bar{c})\bar{c} + (1+\tau_c)\beta v'(\bar{k})}{\Delta' u''(\bar{c})} > 0. \tag{4.7}$$

The sign of $dk/d\tau_c$ is always positive, but the sign $dc/d\tau_c$ is ambiguous since the first term in the numerator of the right-hand side of equation (4.6) is positive while the second and the third terms are negative. We also note that Δ' is negative, and $[\rho-(1-\tau_y)f'(\bar{k})]$ is negative.

This proposition sounds strange especially when a consumption tax may lead to a higher consumption. I offer the following explanation: when a consumption tax at a higher rate is imposed ~~on consumption~~, the representative agent will save more and invest more to increase his capital stock because the effective cost of consumption is now higher than before, and the capital stock is more attractive in generating utility. With more capital stock and with the assumption that the net marginal productivity of capital is always positive, the newly accumulated capital will produce more output for consumption and for investment. Of course when capital depreciation and population growth are introduced into the equation of capital accumulation, we have to assume that the gross marginal productivity of capital is greater than the rate of capital accumulation and the rate of population growth. This has often been done in recent endogenous-growth models such as Barro (1990), Rebelo (1991), Jones and Manuelli (1991), among many others. If the gross marginal productivity of capital is less than the sum of the depreciation rate and the population growth rate, then a consumption tax will always lead to more capital accumulation and possibly more consumption when the initial steady-state capital stock is more than the modified-golden-rule one (this is always true in the capitalist spirit model as in Chapter I) but less than the golden-rule one; see Phelps (1961).

As for the output tax, its effect on capital accumulation does not show qualitative difference ~~in both~~ between the capitalist-spirit model and the Ramsey-Koopmans-Cass model. In both models, an output tax reduces the long-run capital stock. So I will not analyze it here in any detail. But this similarity should not obscure the significant differences between these two models in their ~~effects of~~ predictions of a consumption tax and government spending.

IV.2. Short-run policy analysis

To find out how short-run policy changes at the present time and in the future affect current consumption and investment, I follow the technique pioneered by Judd (1982, 1986, 1987). Assume that the economy is initially ~~namely~~, at time t = 0, in the equilibrium $(\bar{c}, \bar{k})$ corresponding to the policy parameters τ_y, τ_c and $\bar{g}$. Also at time t = 0, government policies change as follows:

$$\tau_y' = \tau_y + \varepsilon h_{\tau y}(t), \tag{4.8}$$

$$\tau_c' = \tau_c + \varepsilon h_{\tau c}(t), \tag{4.9}$$

$$g' = \bar{g} + \varepsilon g(t), \tag{4.10}$$

where ϵ is a parameter. Functions $h_{\tau y}(t)$, $h_{\tau c}(t)$ and g(t) describe the intertemporal policy changes in a magnitude-free fashion since ϵ can represent different magnitude of changes. For example, a change in the output tax during time $T_1 < t < T_2$ can be represented by setting $h_{\tau y}(t)$ to be one for $T_1 < t < T_2$ and zero otherwise.

$h_{\tau y}(t)$

Substituting τ_y', τ_c' *and* g' for τ_y, τ_c and g in equations (4.1) and (4.2):

$$\dot{c} = \frac{1}{-u''(c)}[(1+\tau_c+\varepsilon h_{\tau c}(t))\beta v'(k) + u'(c)((1-\tau_y-\varepsilon h_{\tau y}(t))f'(k) - \rho)], \tag{4.11}$$

$$\dot{k} = (1-\tau_y-\varepsilon h_{\tau y}(t))f(k) - (1+\tau_c+\varepsilon h_{\tau c}(t))c - \bar{g} - \varepsilon g(t) \tag{4.12}$$

The solutions for k and c depend on both t and ϵ. I write the solutions as $k(t,\epsilon)$ and $c(t,\epsilon)$. Since $\epsilon = 0$ implies that the system remains at the initial position, the effects of policy changes can be seen from the impact on the paths of c and k as ϵ shifts from zero to a small positive or negative value. Formally, I define the initial impacts of ϵ on c and k here:

$$c_\varepsilon(t) = \frac{\partial c(t,0)}{\partial \varepsilon} \qquad k_\varepsilon(t) = \frac{\partial k(t,0)}{\partial \varepsilon}$$

$$\dot{c}_\varepsilon(t) = \frac{\partial\left[\dfrac{\partial c(t,0)}{\partial \varepsilon}\right]}{\partial t} \qquad \dot{k}_\varepsilon(t) = \frac{\partial\left[\dfrac{\partial k(t,0)}{\partial \varepsilon}\right]}{\partial t}$$

Differentiating equations (4.11) and (4.12) with respect to ϵ while evaluated at $\epsilon = 0$ yields a pair of differential equations in the variables c_ϵ and k_ϵ:

$$\begin{bmatrix} \dot{c}_\varepsilon \\ \dot{k}_\varepsilon \end{bmatrix} = M' \begin{bmatrix} c_\varepsilon \\ k_\varepsilon \end{bmatrix} + \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} \tag{4.13}$$

here M' is the 2x2 Jacobian matrix in (4.3), and

$$w_1(t) = \frac{1}{-u''(\bar{c})}\left[h_{\tau c}(t)\,\beta v'(\bar{k}) - u'(\bar{c})h_{\tau y}(t)f'(\bar{k})\right], \tag{4.14}$$

$$w_2(t) = -h_{\tau y}(t)f(\bar{k}) - h_{\tau c}(t)\bar{c} - g(t). \tag{4.15}$$

As in Judd (1987), the Laplace transform can be used to solve equation (4.13). For sufficiently large positive s, the Laplace transform of a function f(t) ($t > 0$) is another function F(s), where

$$F(s) = \int_0^\infty f(t)e^{-st}dt.$$

change "r" to "τ"

Let $C_\epsilon(s)$, $K_\epsilon(s)$, $H_{\tau y}(s)$, $H_{\tau c}(s)$, $G(s)$, $W_1(s)$ and $W_2(s)$ be the Laplace transforms of $c_\epsilon(t)$, $k_\epsilon(t)$, $h_{\tau y}(t)$, $h_{\tau c}(t)$, $g(t)$, $w_1(t)$ and $w_2(t)$, respectively. Then

o.K

$$\begin{bmatrix} C_\varepsilon \\ K_\varepsilon \end{bmatrix} = (s\Lambda - M')^{-1} \begin{bmatrix} W_1(s) + c_\varepsilon(0) \\ W_2(s) \end{bmatrix} \tag{4.16}$$

o.K

here Λ is the identity matrix. Write out $(s\Lambda - M')^{-1}$ explicitly in (4.16):

o.K

$$(s\Lambda - M')^{-1} = \frac{1}{-[(s-\mu)(s-\omega)]} \begin{bmatrix} s-(1-\tau_y)f'(\bar{k}) & \dfrac{-(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{u''(\bar{c})f''(\bar{k})} \\ -(1+\tau_c) & s-\rho+(1-\tau_y)f'(\bar{k}) \end{bmatrix},$$

and μ is the positive characteristic root and ω is the negative characteristic root of M' (note that Δ' is negative for the saddle-point equilibrium):

$$\mu = \frac{\rho + (1-\tau_y)f'(\bar{k}) + \{[\rho + (1-\tau_y)f'(\bar{k})]^2 - 4\Delta'\}^{1/2}}{2} > 0$$

$$\omega = \frac{\rho + (1-\tau_y)f'(\bar{k}) - \{[\rho + (1-\tau_y)f'(\bar{k})]^2 - 4\Delta'\}^{1/2}}{2} < 0;$$

and

$$W_1(s) = \frac{1}{-u''(\bar{c})}[H_{\tau c}(s)\beta v'(\bar{k}) - u'(\bar{c})H_{\tau y}(s)f'(\bar{k})],$$

$$W_2(s) = H_{\tau y}(s)f(\bar{k}) - H_{\tau c}(s)\bar{c} - G(s),$$

where $H_{\tau y}(s) = \int_0^\infty h_{\tau y}(t)e^{-st}dt, H_{\tau c}(s) = \int_0^\infty h_{\tau c}(t)e^{-st}dt$, and $G(s) = \int_0^\infty g(t)e^{-st}dt$. Naturally these Laplace transforms can be regarded as the present values of different policy changes discounted at the rate s (> 0).

In (4.16), $c_\epsilon(0)$ is the initial jump in consumption immediately following policy changes. As usual in dynamic analysis, this jump is necessary to assure the convergence of the variables along the perfect-foresight path. To determine $c_\epsilon(0)$, it is noted that the existence of a saddle-point equilibrium in the capitalist-spirit model implies a bounded, steady-state capital stock for any ϵ. Therefore, $K_\epsilon(0)$ must be finite for all $s > 0$, even for $s = \mu$ (the positive characteristic root of the dynamic system). However, when $s = \mu$, the matrix (sΛ - M') is singular. To remove this singularity, implicitly, the numerator on the right hand side of (4.16) has to be zero [see the appendix in Judd (1987) for technical details]. That is to say,

$$(W_1(\mu) + c_\epsilon(0))[\mu - (1-\tau_y)f'(\bar{k})] - W_2(\mu)\frac{(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{u''(\bar{c})} = 0,$$

or,

$$c_\epsilon(0) = -W_1(\mu) + W_2(\mu)\frac{(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{u''(\bar{c})[\mu - (1-\tau_y)f'(\bar{k})]}, \tag{4.17}$$

and

$$W_1(\mu) = \frac{1}{-u''(\bar{c})}[H_{\tau c}(\mu)\beta v'(\bar{k}) - u'(\bar{c})H_{\tau y}(\mu)f'(\bar{k})],$$

$$W_2(\mu) = -H_{\tau y}(\mu)f(\bar{k}) - H_{\tau c}(u)\bar{c} - G(\mu).$$

0.K

To see how these policy changes affect current investment, I substitute (4.17) into (4.13) and set $t = 0$ (also note that $k_\varepsilon(0)$ is zero because the initial capital stock is given and cannot jump):

$$\dot{k}_\varepsilon(0) = -(1+\tau_c)c_\varepsilon(0) - h_{\tau y}(0)f(\bar{k}) - h_{\tau c}(0)\bar{c} - g(0) \tag{4.18}$$

where $c_\varepsilon(0)$ is given in (4.17).

Combining (4.17) and (4.18), I have:

Proposition 4.3: A future increase in government spending represented by $G(\mu)$ stimulates current private investment.

The economic intuition for this proposition is the following. Anticipating a rise in the future government spending, the representative agent will expect a lower income in the future because government spending acts like a lump-sum tax in this model. Therefore, current consumption will be reduced in response to a lower future income and as a measure to smooth consumption over time. This reduced consumption is given by

$$\frac{dc_\varepsilon(0)}{dG(\mu)} = \frac{-(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{u''(\bar{c})[\mu - (1-\tau_y)f'(\bar{k})]} < 0.$$

Since current income is not changed, the reduction in current consumption is saved for current investment. In fact, from (4.18), current investment will increase by the amount of reduced consumption plus the savings in the consumption tax:

$$\frac{d\dot{k}_\varepsilon(0)}{dG(\mu)} = -(1+\tau_c)\frac{dc_\varepsilon(0)}{dG(\mu)} > 0.$$

Therefore, comparing to proposition 4.1 the short-run effect of a future increase in government spending on capital accumulation is just the opposite of the long-run effect of a permanent increase in government spending.

Proposition 4.4: A future increase in the consumption tax represented by $H_{\tau c}(\mu)$ will encourage current private investment.

$H_{\tau c}(\mu)$

I can offer a similar explanation for this proposition as the one for proposition 4.3 above. With a future rise in the consumption tax, the representative agent will become poorer in terms of actual consumption. He responds today by reducing some consumption and saving more for the future. This consumption smoothness over time leads to a reduction in current consumption:

$$\frac{dc_{\varepsilon}(0)}{dH_{\tau c}(\mu)} =$$

$$\left[\frac{\beta v'(\bar{k})}{u''(\bar{c})}\right] - \frac{(1+\tau_c)\beta v''(\bar{k})+(1+\tau_y)u'(\bar{c})f''(\bar{k})}{[u''(\bar{c})[\mu-(1-\tau_y)f'(\bar{k})]}\bar{c} < 0.$$

Again, since the savings as a result of a reduced consumption are not subject to a consumption tax, the actual savings and investment will increase by, from (4.18), $(1 + \tau_c)$ times of the reduced consumption:

$$\frac{d\dot{k}_{\varepsilon}(0)}{dH_{\tau c}(\mu)} = -(1+\tau_c)\frac{dc_{\varepsilon}(0)}{dH_{\tau c}(\mu)} > 0.$$

Thus, combining this proposition with proposition 4.2, I have found that a future consumption tax works in the same direction as a permanent rise of consumption tax in favoring investment.

Proposition 4.5: A future rise in the output tax represented by ~~$H_y\mu$~~ has an ambiguous effect on current investment.

$H_{\tau y}(\mu)$

From (4.18),

$$\frac{d\dot{k}_\varepsilon(0)}{dH_{\tau y}(\mu)} =$$

d. f.t.

$$\left[\frac{-u'(\bar{c})}{u''(\bar{c})}\right] - \frac{(1+\tau_c)\beta v''(\bar{k}) + (1-\tau_y)u'(\bar{c})f''(\bar{k})}{u''(\bar{c})[\mu-(1-\tau_y)f'(\bar{k})]} f(\bar{k}).$$

The first term on the right hand side is positive while the second term is negative; the net effect is ambiguous. This is because a higher future output tax directly reduces the incentive to save (invest) more and produce more. On the other hand, the reduced income in the future will force the representative agent to save more today to compensate a future loss in his income . This works as a counter force to the direct disincentive of an output tax on savings.

Proposition 4.6: All current increases in government taxes and spending reduce current investment.

This can be seen from (4.18) that an increase in current output tax represented by $h_{ty}(0)$ reduces current investment by $f(\bar{k})$:

$$\frac{d\dot{k}_\varepsilon(0)}{dh_{\tau y}(0)} = -f(\bar{k}) < 0;$$

similarly, the negative effect of a current rise in consumption tax on current investment is given by:

$$\frac{d\dot{k}_\varepsilon(0)}{dh_{\tau c}(0)} = -\bar{c} < 0;$$

finally, an increase in current government spending reduced current investment dollar by dollar:

$$\frac{d\dot{k}_g(0)}{dg(0)} = -1.$$

The economic explanation for this proposition can be stated as follows. A current, momentary increase in government taxes and government spending, with future government taxes and spending held constant, will have no effect on current consumption because the representative agent endeavors to have a steady state level of consumption [see Judd (1985) for a similar explanation for the effect of a current increase in government spending on current consumption and current investment]. This is why all current taxes and current government spending have not appeared as the determinants of current consumption in equation (4.17).

These propositions indicate that the timing of taxes and government spending has a significant impact on the short-run investment. While a future increase in both consumption tax and government spending stimulates current investment, a current increase in consumption tax and government spending discourages current investment. Therefore, as a corollary to these propositions, an increase in consumption tax and government spending both today and in the future will have an ambiguous effect on the short-run investment. Similar result also holds for a rise in output tax both today and in the future. To stimulate current investment, the government can either announce a future increase in the consumption tax and government spending or cut all current taxes and government spending.

IV.3. Conclusion

This chapter served two purposes. First, it makes two striking comparisons between the effects of a consumption tax and government spending on long-run capital accumulation. In the Ramsey-Koopmans-Cass model, these policy changes have no effect on the equilibrium capital stock; in the capitalist-spirit model, a permanent increase in consumption tax raises the equilibrium capital stock while a permanent increase in government spending reduces the equilibrium capital stock.

Second, this chapter has derived many interesting results regarding the effects of various policy changes on short-run investment. In particular, it is shown that a future increase in consumption tax and government spending stimulates current investment but a current rise in consumption tax, output tax and government spending discourages the short-run investment.