A Semiparametric Time Trend Varying Coefficients Model: With An Application to Evaluate Credit Rationing in U.S. Credit Market

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In this paper, we propose a new semiparametric varying coefficient model which extends the existing semi-parametric varying coefficient models to allow for a time trend regressor with smooth coefficient function. We propose to use the local linear method to estimate the coefficient functions and we provide the asymptotic theory to describe the asymptotic distribution of the local linear estimator. We present an application to evaluate credit rationing in the U.S. credit market. Using U.S. monthly data (1952.1-2008.1) and using inflation as the underlying state variable, we find that credit is not rationed for levels of inflation that are either very low or very high; and for the remaining values of inflation, we find that credit is rationed and the Mundell-Tobin effect holds.

Key words: non-stationarity; semi-parametric smooth coefficients; nonlinearity; credit rationing

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1   Introduction

Nonparametric techniques have been widely used in estimation and testing of econometric models. For example, Baltagi and Li (2002) propose to use the nonparametric series method to estimate a semiparametric partially linear fixed effects panel data model, Racine et al (2005) propose using a new smoothing method to estimate a multivariate conditional distribution function, Sun (2005) consider the problem of efficient estimation of partially linear quantile regression model, Fan and Rilstone (2001) propose a model specification test based on nonparametric kernel method. Recently, varying coefficient modeling techniques have attracted much attention among econometricians and statisticians. For theoretical development of varying coefficients model with independent and stationary data, see Cai, Fan and Li (2000), Fan, Yao and Cai (2003), Li, Huang, Li and Fu (2002), among others. The semiparametric varying coefficient model specification has been used in various empirical studies. For example, Chou, Liu and Huang (2004) examined health insurance and savings over the life cycle. Savvides, Mamuneas and Stengos (2006) studied the problem of economic development and the return to human capital. Stengos and Zacharias (2006) investigated the intertemporal pricing and price discrimination of the personal computer market. Jansen, Li, Wang and Yang (2008) studied the impact of U.S. fiscal policy on stock market performance.

In this paper, we propose a new method of estimation and inference that extends the application of semiparametric smooth coefficients models to the case where the dependent variable is non-stationary because it contains a time trend regressor. Let Y_t denote the non-stationary dependent variable, and X_t be the set of stationary regressors. We also define Z_t as a stationary underlying state variable. To capture the time trend behavior of Y_t, we use a time trend, denoted by t, as part of the data generating process. In this paper, we propose two alternative empirical specifications of a semiparametric smooth coefficients model. These specifications vary in their treatment of the time trend.

We consider a semiparametric model which includes a stationary vector variable X_{t1} and a

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time trend as regressors, all of them have varying smooth coefficients. The model is given by

$$Y_t = X_t^T \beta(Z_t) + u_t \equiv X_{t1}^T \beta_{(1)}(Z_t) + t\beta_{(2)}(Z_t) + u_t, \qquad (1)$$

where $X_t^T = (X_{t1}^T, X_{t2}) = (X_{t1}, t)$ is of dimension $1 \times d$, $\beta_{(1)}(\cdot)$ and $\beta_{(2)}(\cdot)$ are smooth functions of Z_t and they are of dimension $(d-1) \times 1$ and 1×1, respectively. We assume that X_{t1}, Z_t and u_t are all stationary variables, while Y_t is non-stationary due to its time trend component.

Equation (1) differs from the varying coefficient model considered by Cai, Li and Park (2009), and Xiao (2009) who consider the case that X_t contains integrated non-stationary regressors (i.e., regressors have unit roots), while our model considers a time trend non-stationary regressor.

We also consider a simpler model in which the trend variable enters the model linearly

$$Y_t = X_{t1}^T \beta_{(1)}(Z_t) + \gamma t + u_t, \qquad (2)$$

where γ is a constant coefficient.

We subsequently discuss and apply this new semiparametric specification to evaluate empirically whether credit are rationed in the U.S. credit market. We start with a simple model with frictions in credit markets. We use general equilibrium techniques and consider a nonlinear structural model that has the micro-foundations required for monetary growth economies. We derive testable implications based on a reduced form model with respect to whether credit is rationed or not in equilibrium. We go directly from the model and its testable implications through estimation and inference.

The rest of the paper is organized as follows. In Section 2, we describe our theoretical econometrics model and we propose to use a local linear estimation method to estimate the coefficient functions. We derive the asymptotic distribution for our proposed estimator. In Section 3 we first present a theoretical model, then we study a reduced form model of credit rationing, discuss its testable implication and then use a varying coefficient specification to investigate whether US credit market is rationed. Section 4 concludes the paper. The proof of the asymptotic results is given in an Appendix.

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2 Estimation of A Varying Coefficients Model

Our semiparmetric varying coefficient model is given by

$$Y_t = X_t^T \beta(Z_t) + u_t = X_{t1}^T \beta_{(1)}(Z_t) + t\beta_{(2)}(Z_t) + u_t, \qquad t = 1, ..., n, \tag{3}$$

where Y_t, Z_t and u_t are scalars, and $X_t = (X_{t1}^T, X_{t2})^T = (X_{t1}^T, t)$. We only consider the scalar Z_t case since the extension to multivariate Z_t involves fundamentally no new ideas but only complicated notations.

2.1 Local Linear Estimation

We use a local linear approximation to approximate the unknown coefficient function. When Z_t is close to z, we use $\beta(z) + \beta'(z)(Z_t - z)$ to approximate $\beta(Z_t)$, where $\beta'(z) = d\beta(z)/dz$. The local linear estimator is defined via the following minimization problem.

$$\begin{pmatrix} \widehat{\theta}_0 \\ \widehat{\theta}_1 \end{pmatrix} = \operatorname{argmin}_{\theta_0, \theta_1} \sum_{t=1}^n \left[Y_t - X_t^T \theta_0 - (Z_t - z) X_t^T \theta_1 \right]^2 K_h(Z_t - z), \tag{4}$$

where $K_h(u) = h^{-1} K(u/h)$, $K(\cdot)$ is a kernel function and h is the smoothing parameter. It is well known that $\widehat{\theta}_0 = \widehat{\beta}(z)$ estimates $\beta(z)$ and $\widehat{\theta}_1 = \widehat{\beta}'(z)$ estimates $\beta'(z)$. (4) has the closed form expression for $\widehat{\beta}(z)$ and $\widehat{\beta}'(z)$ and is given by

$$\begin{pmatrix} \widehat{\beta}(z) \\ \widehat{\beta}'(z) \end{pmatrix} = \left[\sum_{t=1}^n \begin{pmatrix} X_t \\ (Z_t - z) X_t \end{pmatrix}^{\otimes 2} K_h(Z_t - z) \right]^{-1} \sum_{t=1}^n \begin{pmatrix} X_t \\ (Z_t - z) X_t \end{pmatrix} Y_t K_h(Z_t - z), \tag{5}$$

where $A^{\otimes 2} = A A^T$. We present the asymptotic theory regarding $\widehat{\beta}(z)$ in the next subsection.

2.2 Asymptotic Properties

Recall that $\widehat{\beta}(z) = (\widehat{\beta}_{(1)}(z)^T, \widehat{\beta}_{(2)}(z))^T$, and that $\widehat{\beta}_{(1)}(z)$ and $\widehat{\beta}_{(2)}(z)$ are the coefficients of X_{1t} and t, respectively. We will show that $\widehat{\beta}_{(1)}(z)$ and $\widehat{\beta}_{(2)}(z)$ have different convergence rates. To establish the asymptotic properties of $\widehat{\beta}(z)$, we define $D_n = \begin{pmatrix} I_{d-1} & 0 \\ 0 & n \end{pmatrix}$, where I_{d-1} is an identity matrix of dimension $d-1$. We also define $M_0(Z_t) = f_z(Z_t) E(X_{t1} X_{t1}^T | Z_t)$, $M_1(Z_t) = (1/2) f_z(Z_t) E(X_{t1} | Z_t)$ and $M_2(Z_t) = (1/3) f_z(Z_t)$, where $f_z(Z_t)$ is the density function of Z_t. Finally we define

$$S(z) = \begin{pmatrix} M_0(z) & M_1(z) \\ M_1(z)^T & M_2(z) \end{pmatrix}. \tag{6}$$

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We also make the following assumptions.

(A1) (i) (X_{t1}, Z_t) is a strictly stationary β-mixing process with size $-2(2 + \delta)/\delta$ for some $\delta > 0$, u_t is a martingale different process satisfying $E(u_t^2|\mathcal{F}_t) = E(u_t^2) = \sigma_u^2$, and $E(u_t^4|\mathcal{F}_t) < \infty$, where $\mathcal{F}_t$ is the sigma field generated by $\{X_{s1}, Z_s\}_{s=-\infty}^t$. (iii) $\beta(\cdot)$ has a bounded and continuous third order derivative function.

(A2) (i) $K(\cdot)$ is a bounded symmetric density function with $\int K(v)v^2 dv = \mu_2(K)$ being a finite positive constant. (ii) $h \to 0$, $nh^2 \to \infty$ and $nh^7 = o(1)$ as $n \to \infty$.

The above regularity conditions are quite standard and provide sufficient conditions to establish our Theorem 1 below. However, they are not the weakest possible conditions. For example, the conditional homoskedastic error assumption can be relaxed to allow for conditional heteroskedastic errors.

THEOREM 1 *Under Assumptions A1 - A2 given above, we have*

$$\sqrt{nh}D_n \left[\widehat{\beta}(z) - \beta(z) - h^2\mu_2(K)\beta''(z)\right] \to N(0, \Sigma_\beta(z)) \text{ in distribution,}$$

where $\mu_2 = \int K(v)v^2 dv$, $\beta''(z) = d^2\beta(z)/dz^2$, $N(0, \Sigma_\beta(z))$ denotes a normal distribution with mean zero and variance matrix given by $\Sigma_\beta(z) = \sigma_u^2 v_0(K)S(z)^{-1}$, $v_0(K) = \int K^2(v)v^2 dv$, and $S(z)$ is defined in (6).

A detailed proof of the above Theorem is provided in Appendix A.

Note that Theorem 1 shows that while the coefficient of X_{t1} has the standard rate of convergence: $\hat{\beta}_{(1)}(z) - \beta_{(1)}(z) = O_p(h^2 + (nh)^{-1/2})$ because $var(\hat{\beta}_{(1)}(z)) = O((nh)^{-1})$, the coefficient function of t has a much faster rate of convergence: $\hat{\beta}_{(2)}(z) - \beta_{(2)}(z) = O_p(h^2 + (n^3h)^{-1/2})$ because $var(\hat{\beta}_{(2)}(z)) = O((n^3h)^{-1})$ (due to the extra n factor at the lower diagonal position in matrix D_n).

3   An Empirical Application

3.1   Theory Background of Credit Rationing

In this section, we introduce the theory background of credit rationing. This is a simplification and generalization of Hernandez-Verme (2004). In this economy, there is an adverse selection

problem in the credit market. We let r_t denote the real gross interest rate on loans. Borrowers and Lenders each take r_t as given.

We introduce reserve requirement as a first building block of the monetary policy in this economy. In general, these required reserves must be held in the form of currency, either domestic or foreign. It seems reasonable to assume that the reserve requirement is binding, so henceforth we suppose that this is the case.

The second building block is the evolution of the money supply. The monetary authority directly control over the domestic money supply. The evolution of the money supply M_t is given by

$$M_t = (1 + \sigma)M_{t-1}, \tag{7}$$

where $\sigma > -1$ is the rate of money growth set exogenously by the Federal Reserve System. We use $\pi_t = \frac{p_t - p_{t-1}}{p_{t-1}}$ to denote the domestic rate of inflation at date t.

Clearing in the Credit Market with a binding reserve requirement and the evolution of the money supply then requires that the equilibrium real interest rate on loans r_t is an *increasing function of π_t*, the inflation rate at time t highlighting the role of the reserve requirement. [1] The intuition behind this result is as follows: higher inflation rates reduce the return that banks receive from their currency-reserves holdings, and r_t must increase for banks to be able to compete for deposits in the market.

3.1.1 General Equilibrium and Alternative Credit Regimes

There are two possible credit regimes that we discuss in detail below: a Walrasian regime - where credit is not rationed - and a Private Information regime - where credit is rationed.

A Walrasian Regime

We say that the economy is in a Walrasian regime at a particular point in time when a Walrasian equilibrium occurs. Let k_t^W denote the per capita capital stock when the economy is in a

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[1] See equation (3) in Hernandez-Verme (2004) for more details.

Walrasian equilibrium at date t. The economy is in a Walrasian equilibrium when

$$f'(k_t^W) = r_t, \tag{8}$$

where $f'(k) = df(k)/dk$. This condition is fairly common in standard economic theory. In this case, we say that credit is not rationed, since borrowers may borrow as much as they can at the equilibrium interest rate r_t.

In terms of comparative statics, we observe that when credit is not rationed, increases in r_t translate into increases in the marginal product of capital. Given standard decreasing marginal products, then, $k_t^W = k^W(r_t)$ is a decreasing nonlinear function of r_t. This means also that $y_t^W = f\left[k^W(r_t)\right]$, and, thus, output per capita in a Walrasian equilibrium is also decreasing in r_t. In summary, an increase in the equilibrium interest rate on loans reduces output per capita in equilibria where credit is not rationed.

A Private Information Regime

When a Private Information equilibrium occurs at a particular date, we say that the economy is in a Private Information regime, and because of adverse selection problem we observe that the link between the marginal product of capital and the market interest rate on loans is broken. Let k_t^P denote the capital stock per capita when the economy is in a Private Information equilibrium at date t. The economy is in a Private Information equilibrium when the following inequality holds:

$$f'(k_t^P) > r_t. \tag{9}$$

When Condition (9) holds, borrowers are willing to borrow arbitrarily large amounts at the market interest rate on loans r_t. In such a situation, lenders keep interest rate lower to reduce the risk and avoid potential default problems, and this causes Credit Rationing.

Under the circumstances mentioned above, an increase in r_t increases the amount of credit available and borrowed and, thus, k_t^P. Thus, $k_t^P = k^P(r_t)$ is an increasing nonlinear function of r_t. This means that $y_t^P = f\left[k^P(r_t)\right]$, and output in a Private Information equilibrium is also increasing in r_t. In summary, an increase in the equilibrium interest rate on loans increases output when credit is rationed, and a short-run version of Mundell-Tobin effect prevails.

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3.1.2 Testable Implications of the Model

We can use a reduced-form equation that is consistent with the model presented above and that can also be used to evaluate whether credit is rationed or not. In particular, for the sake of parsimony, we use the following semi-parametric equation:

$$y_t = \beta_1(\pi_t) + \beta_2(\pi_t)\,r_t + \beta_3(\pi_t)\,t + u_t, \tag{10}$$

where the underlying state variable is the inflation rate, while $\beta_1(\pi_t), \beta_2(\pi_t)$ and $\beta_3(\pi_t)$ are smooth coefficient functions that depend on the inflation rate π_t. By using this flexible specification, we can evaluate whether credit rationing is present or not, together with the region of the state-space for which this is true. In particular, let $\widehat{\beta_2}(\pi_t)$ denote the estimated function of $\beta_2(\pi_t) = \partial y_t / \partial r_t$. Then, the regions in which $\widehat{\beta_2}(\pi_t) > 0$ is associated with Private Information equilibria and, thus, credit will be rationed. The complementary regions in which $\widehat{\beta_2}(\pi_t) < 0$ is associated with Walrasian equilibria and credit will not be rationed.

3.2 Econometric Methodology

3.2.1 Model Specification

We start from the simple linear regression model

$$Y_t = X_t^T \beta + Z_t^T \gamma + u_t, \quad t = 1, 2, ..., n, \tag{11}$$

where X_t^T is $1 \times d$ vector with one component being 1, Z_t^T is a $1 \times q$ vector, and β and γ are constant parameter vectors with dimensions $d \times 1$ and $q \times 1$, respectively. Equation (11) will be the benchmark against which we will compare our results. The credit rationing example, the specific linear model can be found in equation (16).

Our choice of specification of the empirical model is consistent with the simple theoretical framework that we presented in the previous section. Thus, we propose to use the following semi-parametric varying coefficient specification:

$$Y_t = X_t^T \beta(Z_t) + u_t, \quad t = 1, 2, ..., n, \tag{12}$$

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where the coefficient function $\beta(Z_t)$ is a $d \times 1$ vector of unspecified smooth functions of the underlying state variable Z_t. For credit rationing example, the varying coefficient models we used can be found in equation (14) and (15).

This model specification allows for a more flexible functional form and also avoids the "curse of dimensionality" associated with a fully nonparametric model. Under the assumption that model (12) is correctly specified, $E(u_t|X_t, Z_t) = 0$. Pre-multiplying both sides of (12) with X_t, taking conditional expectation $E(\cdot|Z_t = z)$, and then solving for $\beta(z)$ yields

$$\beta(z) = \left[E(X_t X_t^T | Z_t = z)\right]^{-1} E(X_t Y_t | Z_t = z). \tag{13}$$

We next replace the conditional mean function in (13) by some nonparametric estimator, say by the local linear kernel estimator, and we obtain a feasible estimator of $\beta(z)$.

In our model, the dependent variable is the industrial production per capita, which we denote as Y_t. Since the industrial production per capita has an obvious time trend, the explanatory variable X_t includes the time trend t. X_t also contains the growth rate of the real gross interest rate on loans $\Delta ln(r_t)$, since the real interest rate is nonstationary. The explanatory state variable Z_t is the inflation rate π_t. Since the non-stationarity of industrial production per capita is caught by the time trend, we redefine the coefficient smooth function of π_t associated with the time trend t as $\beta_3(\pi_t)$. So, we can rewrite the model in (12) as

$$Y_t = \beta_1(\pi_t) + \beta_2(\pi_t) \Delta ln(r_t) + \beta_3(\pi_t) t + u_t. \tag{14}$$

The coefficient for the intercept, $\beta_1(\pi_t)$, is a function of the underlying state-variable π_t (inflation rate), and so is the coefficient $\beta_2(\pi_t)$ that measures the effect of the real interest rate on the industrial production per capita at date t.

We obtain an alternative model specification when the time trend t enters the model linearly, which means that the effect of the time trend is constant and independent of the state variable π_t. So, the smooth coefficient function of t, $\beta_3(\pi_t)$, reduces to a constant parameter γ. Under these conditions, the alternative nonlinear model with constant time trend becomes

$$Y_t = \gamma t + \beta_1(\pi_t) + \beta_2(\pi_t) \Delta ln(r_t) + u_t \tag{15}$$

The corresponding linear regression model (11) is given by,

$$Y_t = \beta_{1,0} + \beta_{2,0} \, \Delta ln(r_t) + \beta_{3,0} \, t + \beta_{4,0} \, \pi_t + u_t, \tag{16}$$

where $\beta_{j,0}$s ($j = 1, 2, 3, 4$) are constant coefficients.

In the remainder of the paper we will refer the simple linear model in (16) as *model 1*, the partially linear varying coefficient model (15) as *model 2*, and the general varying coefficient model in (14) as *model 3*.

3.2.2 Model Specification Testing

As is standard in the literature, we first test whether the varying coefficient models 2 and 3 represent the data significantly better than the standard linear OLS model or model 1.

We start from the benchmark model 1, a linear regression model with time trend, as described in (16). We use the Generalized Likelihood Ratio (GLR) test as suggested by Cai, Fan and Yao (2000) to conduct model specification tests. Particularly, we test whether the linear specification model is adequate for the data, with the linear model as the null hypothesis and one of the varying coefficient models as the alternative. We do so first with model 3, and next with model 2. The test is based on the difference of the sums of squared residuals between the two competing models as follows:

$$GLR = \frac{\sum_{t=1}^{n} \widehat{u}_t^2 - \sum_{t=1}^{n} \tilde{u}_t^2}{\sum_{t=1}^{n} \tilde{u}_t^2} \tag{17}$$

where $\widehat{u}_t$ is the residual from the null hypothesis linear model, and $\tilde{u}_t$ is the residual from the alternative smooth coefficient model. Typically, one rejects the null hypothesis of linearity when large values for the *GLR* statistic are obtained.

We now turn to explain the multiple steps involved in this test. Cai, Fan and Yao (2000) suggest using a bootstrap approach to evaluate the p-value of the test. In particular, they boot-strapped the centralized residuals from the nonparametric fit instead of the linear fit, because the nonparametric estimate of the residuals is consistent under both the null and alternative hypotheses. We use u_t^* to denote the bootstrap error - which is obtained following the fitted residual from the varying coefficient model. The bootstrap error u_t^* follows the 'wild' bootstrap distribution

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conditions (see Cai et al (2000) for more details). We then obtain the GLR statistics and critical values via the following five steps:

Step 1: For each $t = 1, 2, ..., n$, we generate values for u_t^* that satisfies the 'wild' bootstrap distribution conditions. We then compute $y_t^* = X_t^T \widehat{\beta}(\pi_t) + u_t^*$, where $X_t^T = (1, \Delta ln(r_t), t)$ for $t = 1, 2, ..., n$.

Step 2: We obtain the least square estimator by using the bootstrap sample

$$\widehat{\beta_{ols}^*} = \left(\sum_{t=1}^{n} \tilde{X}_t \tilde{X}_t^T \right)^{-1} \sum_{t=1}^{n} \tilde{X}_t y_t^*, \tag{18}$$

where $\tilde{X}_t^T = (1, \Delta ln(r_t), t, \pi_t)$ for $t = 1, 2, ..., n$. Next, we obtain the estimated bootstrap OLS residuals by using $\widehat{u_t^*} = y_t^* - \tilde{X}_t^T \widehat{\beta_{ols}^*}$.

Step 3: We obtain the kernel estimator of $\widehat{\beta^*}(\pi_t)$ using the bootstrap sample, as

$$\widehat{\beta^*}(\pi_t) = \left(\sum_{j=1}^{n} X_j X_j^T K\left(\frac{\pi_j - \pi_t}{h} \right) \right)^{-1} \sum_{j=1}^{n} X_j y_j^* K\left(\frac{\pi_j - \pi_t}{h} \right). \tag{19}$$

Then, we proceed to calculate the estimated bootstrap residuals using $\widetilde{u_t^*} = y_t^* - X_t^T \widehat{\beta^*}(\pi_t)$.

Step 4: We compute the bootstrap statistic using

$$GLR_n^* = \frac{\sum_{t=1}^{n} \widehat{u_t^*}^2 - \sum_{t=1}^{n} \widetilde{u_t^*}^2}{\sum_{t=1}^{n} \widetilde{u_t^*}^2} \tag{20}$$

Step 5: We repeat steps 1- 4 a number of times, say B times, and obtain the empirical distribution of the B test statistics of $\{GLR_{n,j}^*\}_{j=1}^{B}$. Let $GLR_{n,(\alpha)}^*$ denote the αth percentile of the bootstrap statistics. We then reject the null hypothesis at the significance level α if $GLR_n > GLR_{n,(\alpha)}^*$ obtains.

3.3   Empirical Results

3.3.1   Data

In order to focus on the short run relationships described by our theoretical model, we use monthly data. The following variables were obtained from the FRED data set of the Federal Reserve Bank of St. Louis: the industrial production index (IP_t), the bank prime loan rate (I_t), the CPI (P_t), and population (POP_t). The data spans from January 1952 to January 2008 with

a total of 673 monthly observations. We calculate the series of industrial production per capita, IPP_t, by using the industrial production index and population series as follows,

$$IPP_t = \frac{IP_t}{POP_t} \times 1,000. \tag{21}$$

The population is expressed in thousands of inhabitants. Therefore, IPP_t is an index of industrial production per million people, and we found it to be nonstationary. It is stationary after detrending. We calculated the inflation rate (π_t) from the CPI, by using $\pi_t = \left(\frac{P_t}{P_{t-1}} - 1\right) \times 100$. Next, we adjusted the Bank Prime Loan Rate series (I_t) by the inflation rate (π_t), obtaining the real gross interest rate r_t, by using the formula $r_t = \frac{1+I_t}{1+\pi_t}$. We found the real gross interest rate, r_t, to be nonstationary. However, this estimation method requires stationary covariates, and we proceeded to find a stationary representation of this series. Thus, accordingly, we took the log difference of the real gross interest rate, $\Delta ln(r_t) = ln(r_t) - ln(r_{t-1})$ and found $\Delta ln(r_t)$ to be stationary.

3.3.2 Results of Model Specification Tests

For the model specification test, we use the methodology introduced in section 3.2.2. The null hypothesis of the GLR test is that the linear model, model 1, fits the data best. We use different types of nonlinear models as alternative hypothesis: model 2 and model 3.

In the Table 1 below, we present the bootstrap critical values in columns two through five. In the sixth column, we display the the *GLR* statistics, and column seven reports the *p*-values. The *p*-value for the linear model against *model 3* is less than 0.001, while the *p*-value for the linear model against *model 2* equals 0.003. The testing results indicate the existence of strong nonlinearities in the output, inflation and interest rate relationship.

Table 1: Results of Model Specification Tests

Models	Critical Values				GLR	Prob.
	1%	5%	10%	20%		
Models 1 v.s. 2	0.041	0.036	0.033	0.029	0.046	0.003
Models 1 v.s. 3	0.053	0.049	0.045	0.040	0.088	0.000

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Our comparisons of *model 2* and *model 3* each against the linear model have thus verified the presence of strong nonlinearity. However, we also need to take a step further: to treat *model 2* as the null model and test it against the more general *model 3*. To do so, we used a test statistic that was based on a similar *GLR* methodology (and a bootstrap procedure). The testing result shows that we cannot reject the null hypothesis that *model 2* is adequate against *model 3* at any conventional significant level. Therefore, our econometrics analysis will be based on the more parsimonious *model 2* in the remaining parts of this paper.

3.3.3 Estimation Results

In this section, we discuss the main traits of the estimated coefficient functions as well as the economic intuition behind them. We present empirical evidence on the scope for credit rationing in the U.S. credit market.

The Estimated Coefficient Functions

We present the estimation results of the partially linear varying coefficients model 2. Here, we focus on the nonlinearities displayed by the estimated coefficient functions with respect to the inflation rate. Figure 1 displays the estimated coefficient functions of model 2. Recall that model 2 is represented by (15). We will denote the corresponding estimated functions by $\hat{\gamma}$, $\hat{\beta}_1(\pi_t)$ and $\hat{\beta}_2(\pi_t)$. The estimated value for the constant parameter γ is $\hat{\gamma} = 0.0004$, with an associated standard error of 2.77×10^{-6}. So that $\hat{\gamma}$ is (highly) significantly different from zero. This is expected because there is an obvious trend in the output data.

The first panel in Figure 1 displays $\hat{\beta}_1(\pi_t)$. In this model, $\hat{\beta}_1(\pi_t)$ represents the varying intercept. In a standard linear regression, this coefficient would be constant and independent of the inflation rate: its diagram would take the form of a perfectly horizontal line for all values of π_t. However, we observe that the shape of $\hat{\beta}_1(\pi_t)$ is somewhat closer to a *V* shape with $\hat{\beta}_1(\pi_t)$ taking positive values between 0.095 and 0.118. Thus, $\beta_1(\pi_t)$ is a nonlinear function in π_t is supported by Figure 1.

The second panel in Figure 1 displays the estimated coefficient function $\hat{\beta}_2(\pi_t)$ and it is of particular importance to our analysis. One of the main hypotheses from our simple theoretical
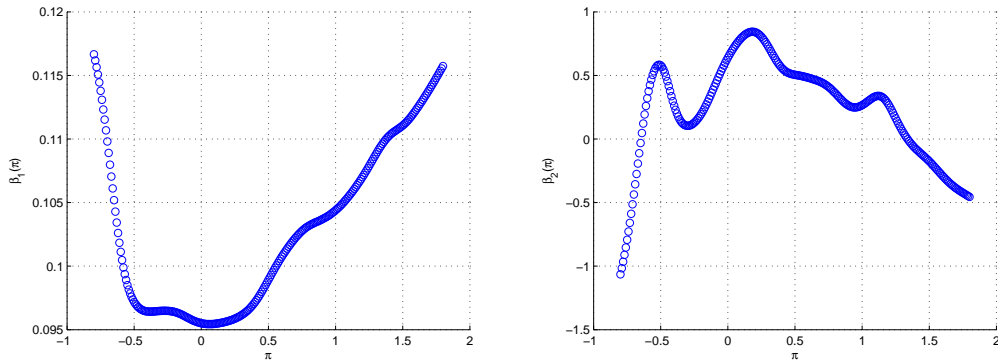
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Figure 1: Coefficients of Model 2, Local Linear Estimator

model is that the interest rate on loans having a nonlinear effect on output per capita and this effect depending on the level of the inflation rate. It is apparent that our hypothesis was verified and that the effect of r_t on Y_t varies significantly for different values of π_t, giving rise to threshold-effects. One can see that the second panel in Figure 1 looks (roughly) like an inverse U (or V) shape showing an obvious sign of nonlinearity.

Notice that the following transpires: for $\pi_t \in (-1, -0.6)$ and for $\pi_t \in (1.3, 1.8]$, the effect of the interest rate on output per capita is negative. The economy is in Walrasian regime. Credit is not rationed.

When $\pi_t \in (-0.6, 1.3)$, the effect of the interest rate on output per capita is positive. The economy is in Private Information regime. Credit is rationed.

Evidence on the Scope for Credit Rationing

Most of the previous research on credit rationing in the U.S. credit market has focused on the micro perspective. For example, Berger and Udell (1992) is based on the information of commercial bank loan contracts; Petrick (2005) is based on the household data; Duca and Rosenthal (1991) investigates credit rationing in the mortgage market.

In this paper, we supply a new perspective of how to look at the credit market at the aggregate level, one that allows for private information and expectations to effectively constrain this market. As we will show next, we find that the empirical evidence supports this opinion. In particular, we estimate the Walrasian region and Private Information region based on short-run macro data.

Our results from Figure 2 indicates that there exist two threshold, π_L and π_H, for inflation. The estimated values of π_L and π_H are -0.6% and 1.3%, respectively. Only when the monthly inflation rate is sufficiently low (i.e. $\pi_t < -0.6\%$) or high enough (i.e. $\pi_t > 1.3\%$,) and thus credit need not be rationed. However, for monthly inflation rates between -0.6% and 1.3%, the incentive compatibility constraint bind and reducing the amount of credit available in the market. Moreover, the severity of the adverse selection problem, seems to vary with the inflation rate as well, explaining why the peaks occur in the function $\hat{\beta}_2(\pi_t)$. As a final conclusion, we have that the *"indirect"* effect of π_t and Y_t is nonlinear and non-monotonic, and it varies significantly for different values of the monthly inflation rate indicating to some extent the information problem in the U.S. credit market.

The analysis of the effects of inflation on output per capita is also of the utmost importance in Macroeconomics (see Fisher (1993), Bullard and Keating (1995), Khan and Senhadji (2001), and Drukker et al (2008).) Our approach differs from the standard in the use of semiparametric estimation techniques, but our results are still comparable with the literature: we can also obtain functions that describe the magnitude of the impact that π_t has on Y_t, given the nonlinear effects of inflation.

Marginal Effects

We analyze the marginal effect of inflation as the partial derivative function of Y_t with respect to π_t keeping the interest rate r_t at a fixed value. When r_t is fixed we have $\Delta\, ln(r_t) = 0$. As a result, the marginal effects function for Model 2 is given by

$$\frac{\partial Y_t}{\partial \pi_t}|_{r_t \text{ fiexed}} = \frac{\partial \hat{\beta}_1(\pi_t)}{\partial \pi_t}. \tag{22}$$

One advantage of using the local linear estimation method is that, one also obtains the derivative estimates at the same time which we plot in Figure 2. From Figure 2, we can observe the marginal effects vary nonlinearly with the inflation rate. For example, when the initial inflation rate belongs to the interval $[-0.8\%, -0.5\%)$, an increase of inflation of one percent point reduces absolute output by 0.03 percentage points on average. However, as the initial inflation rate changes and it belongs, say, to the intervals $[0.0\%, 0.5\%)$ or $[0.5\%, 1.0\%)$, the effect on output is an increase of 0.0075 and 0.012 percentage points on average, respectively.

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We can make three points from these results. First, the partial marginal effects increase with the inflation rate. Second, negative partial marginal effects are associated with rates of inflation that are low enough. And, third, positive partial marginal effects are observed for rates of inflation that are sufficiently high.
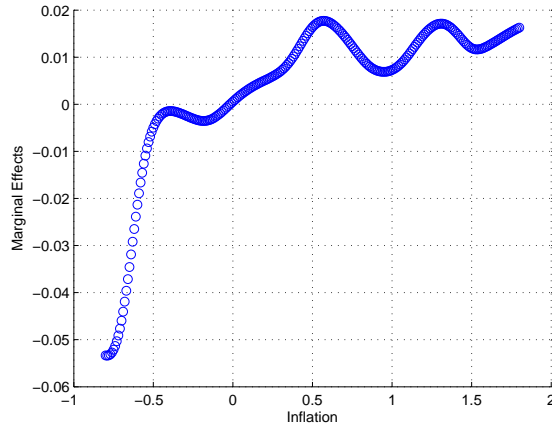

Figure 2: Marginal Effects with Fixed Interest Rate

4   Conclusions

In this paper, we extend the standard semiparametric smooth coefficients model to allow for nonstationary dependent variables by introducing a time trend among the regressors. We find the varying coefficient associated with the time trend t and other stationary regressors have different convergence rates. We establish the asymptotic properties of the new estimation.

We applied this new technique of estimation and inference to evaluate whether credit rationing is present in the U.S. credit market. We directly test the following hypotheses: 1) Inflation is a key state variable that has nonlinear effects on output per capita; 2) The real interest rate on loans has significant effects on output per capita that are nonlinear as well; 3) The nonlinear coefficient associated with the interest rate can help detect the presence of credit rationing in the U.S. market.

We found that the estimated smooth varying coefficients displayed strong nonlinearities with respect to the inflation rate, verifying the adequacy of having used a semiparametric smooth

coefficient model, and also confirming our hypotheses. We showed that, in general, the marginal effects of inflation on output per capita can be either positive or negative. Moreover, the marginal effect function is a monotonically increasing and concave function of π_t which display positive values when the monthly inflation rate is high enough, but negative values otherwise.

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Appendix A: Proof of Theorem 1

Proof of Theorem 1:

First, note that the right hand side of (5) has the form of $A_{1n}^{-1} A_{2n}$, where

$$A_{1n} = \left[\sum_{t=1}^{n} \begin{pmatrix} X_t \\ (Z_t - z)X_t \end{pmatrix}^{\otimes 2} K_h(Z_t - z) \right]$$

and

$$A_{2n} = \sum_{t=1}^{n} \begin{pmatrix} X_t \\ (Z_t - z)X_t \end{pmatrix} Y_t K_h(Z_t - z).$$

Define $H_n = \begin{pmatrix} 1 & 0 \\ 0 & h \end{pmatrix} \otimes D_n$. Then we can write $H_n A_{1n}^{-1} A_{2n} = H_n A_{1n}^{-1} H_n H_n^{-1} A_{2n} = [H_n^{-1} A_{1n} H_n^{-1}]^{-1} H_n^{-1} A_{2n}$.
Thus, $\widehat{\beta}(z)$ and $\widehat{\beta}'(z)$ can be re-expressed as follows:

$$H_n \begin{pmatrix} \widehat{\beta}(z) \\ \widehat{\beta}'(z) \end{pmatrix} = S_n(z)^{-1} n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) Y_t \begin{pmatrix} 1 \\ Z_{t,z,h} \end{pmatrix} \otimes \left(D_n^{-1} X_t \right), \qquad \text{(A.1)}$$

where $S_n = H_n^{-1} A_n H_n^{-1}$, $Z_{t,z,h} = (Z_t - z)/h$. By adding and subtracting terms we obtain

$$\begin{aligned} Y_t &= X_t^T \beta(Z_t) + u_t, \quad 1 \le t \le n, \\ &= X_t^T \left(\beta(z) + \beta'(z)(Z_t - z) + \beta(Z_t) - \beta(z) - \beta'(z)(Z_t - z) \right) + u_t. \end{aligned} \qquad \text{(A.2)}$$

Plug (A.2) into (A.1), and we have,

$$\begin{aligned} H_n \begin{pmatrix} \widehat{\beta}(z) \\ \widehat{\beta}'(z) \end{pmatrix} =\ & H_n \begin{pmatrix} \beta(z) \\ \beta'(z) \end{pmatrix} + S_n(z)^{-1} n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) \\ & \begin{pmatrix} 1 \\ Z_{t,z,h} \end{pmatrix} \otimes \left(D_n^{-1} X_t \right) \left[X_t^T \left(\beta(Z_t) - \beta(z) - \beta'(z)(Z_t - z) \right) + u_t \right] \\ =\ & H_n \begin{pmatrix} \beta(z) \\ \beta'(z) \end{pmatrix} + S_n(z)^{-1} n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) \\ & \begin{pmatrix} 1 \\ Z_{t,z,h} \end{pmatrix} \otimes \left(D_n^{-1} X_t \right) \left[X_t^T \left(\beta(Z_t) - \beta(z) - \beta'(z)(Z_t - z) \right) \right] \\ & + S_n(z)^{-1} n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) \begin{pmatrix} 1 \\ Z_{t,z,h} \end{pmatrix} \otimes \left(D_n^{-1} X_t \right) u_t, \end{aligned}$$

and

$$\begin{aligned} S_n(z) &= H_n^{-1} A_{1n} H_n^{-1} \\ &= n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) \begin{pmatrix} 1 \\ Z_{t,z,h} \end{pmatrix}^{\otimes 2} \otimes \left(D_n^{-1} X_t \right)^{\otimes 2} = \begin{pmatrix} S_{n,0}(z) & S_{n,1}(z) \\ S_{n,1}(z) & S_{n,2}(z) \end{pmatrix}, \end{aligned}$$

where for $j = 0, 1, 2$, we use the notation

$$S_{n,j}(z) = \frac{1}{n} \sum_{t=1}^{n} K_h(Z_t - z) Z_{t,z,h}^{j} \left(D_n^{-1} X_t \right)^{\otimes 2}.$$

Now, to facilitate the analysis of $S_{n,j}(z)$, we first express $S_n(z)$ as

$$S_{n,j}(z) = \begin{pmatrix} F_{n,j,0}(z) & F_{n,j,1}(z) \\ F_{n,j,1}(z)^T & F_{n,j,2}(z) \end{pmatrix}, \tag{A.3}$$

where (since $(D_n^{-1}X_t)^T = (X_{t1}^T, t/n)$)

$$F_{n,j,0}(z) = \frac{1}{n} \sum_{t=1}^{n} Z_{t,z,h}^j X_{t1} X_{t1}^T K_h(Z_t - z), \qquad F_{n,j,1}(z) = \frac{1}{n} \sum_{t=1}^{n} K_h(Z_t - z) Z_{t,z,h}^j X_{t1} (t/n),$$

and

$$F_{n,j,2}(z) = \frac{1}{n} \sum_{t=1}^{n} Z_{t,z,h}^j K_h(Z_t - z) \left(t^2/n^2 \right).$$

Define

$$M_0(Z_t) = f_z(Z_t)E(X_{t1}X_{t1}^T|Z_t), \quad M_1(Z_t) = (1/2)f_z(Z_t)E(X_{t1}|Z_t) \quad \text{and} \quad M_2(Z_t) = (1/3)f_z(Z_t).$$

By noting that X_{t1} and Z_t are stationary and using the standard change-of-variable and a Taylor's expansion argument, we know that $n^{-2}\sum_{t=1}^n t = (1/2) + O(n^{-1})$ and $n^{-3}\sum_{t=1}^n t^2 = (1/3) + O(n^{-1})$. By the law of iterative expectation, we have

$$\begin{aligned} E[F_{n,j,0}(z)] &= E\left[Z_{t,z,h}^j X_{t1} X_{t1}^T K_h(Z_t - z) \right] \\ &= E\left[Z_{t,z,h}^j E\left(X_{t1} X_{t1}^T|Z_t \right) K_h(Z_t - z) \right] \\ &= \frac{1}{h} \int \left(\frac{Z_t - z}{h} \right)^j f_z(Z_t) E\left(X_{t1} X_{t1}^T|Z_t \right) K_h(Z_t - z) dZ_t \\ &= \int v^j M_0(z)K(v)dv + O(h^2) \\ &= M_0(z)\mu_j(K) + O(h^2), \end{aligned}$$

where $\mu_j(K) = \int v^j K(v)dv$ as defined before.

According to the same step as above, we have

$$E[F_{n,j,1}(z)] = M_1(z)\mu_j(K) + O(h^2), \tag{A.4}$$

$$E[F_{n,j,2}(z)] = M_2(z)\mu_j(K) + O(h^2). \tag{A.5}$$

By the kernel theory for the stationary mixing case (see Theorem 1 of Cai, Fan and Yao (2000) for details) one can easily show that for $l = 0, 1, 2$ and $j = 0, 1, 2$,

$$\text{Var}\left[F_{n,j,l}(z) \right] = O((nh)^{-1}).$$

Therefore,

$$F_{n,j,l}(z) = M_l(z)\mu_j(K) + O_p(h^2 + (nh)^{-1/2}). \tag{A.6}$$

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We have defined $S(z)$ earlier. Recall equation (6):

$$S(z) = \begin{pmatrix} M_0(z) & M_1(z) \\ M_1(z)^T & M_2(z) \end{pmatrix}.$$

By definition of $S(z)$ above, together with equation (A.6), (A.4), (A.5) and (A.3), we have

$$S_{n,j}(z) = \mu_j(K) S(z) + o_p(\delta_n), \tag{A.7}$$

where $\delta_n = h^2 + (nh)^{-1/2}$. By noting that $\mu_0(K) = 1$ and $\mu_1(K) = 0$, we immediately obtain from the definition of $S_n(z)$ (A.3), (A.6) and (A.7) that

$$S_n(z) = \begin{pmatrix} 1 & 0 \\ 0 & \mu_2(K) \end{pmatrix} \otimes S(z) + O_p(\delta_n). \tag{A.8}$$

From (A.8), we immediately obtain that

$$S_{n,0}(z)^{-1} = S(z)^{-1} + o_p(\delta_n). \tag{A.9}$$

$S_{n,0}(z)$ is the upper-left corner $d \times d$ matrix of $S_n(z)$. From (A.1), we have

$$D_n\left[\widehat{\beta}(z) - \beta(z)\right] \equiv L_{1n} + L_{2n}, \tag{A.10}$$

where

$$L_{1n} = S_{n,0}(z)^{-1} B_n(z), \tag{A.11}$$

with

$$B_n(z) = n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) D_n^{-1} X_t X_t^T \{\beta(Z_t) - \beta(z) - (Z_t - z)\beta'(z)\},$$

and

$$L_{2n} = S_{n,0}(z)^{-1} n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) u_t D_n^{-1} X_t.$$

Define,

$$G_{n,0}(z) = n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) X_{t1} X_{t1}^T \{\beta_{(1)}(Z_t) - \beta_{(1)}(z) - (Z_t - z)\beta'_{(1)}(z)\},$$

$$G_{n,1}(z) = \sum_{t=1}^{n} K_h(Z_t - z) X_{t1} (t/n) \{\beta_{(2)}(Z_t) - \beta_{(2)}(z) - (Z_t - z)\beta'_{(2)}(z)\},$$

$$G_{n,2}(z) = n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) (t/n) X_{t1}^T n\{\beta_{(1)}(Z_t) - \beta_{(1)}(z) - (Z_t - z)\beta'_{(1)}(z)\},$$

$$G_{n,3}(z) = n \, n^{-1} \sum_{t=1}^{n} K_h(Z_t - z) (t^2/n^2) \, n\{\beta_{(2)}(Z_t) - \beta_{(2)}(z) - (Z_t - z)\beta'_{(2)}(z)\},$$

so that

$$B_n(z) = \begin{pmatrix} G_{n,0}(z) + G_{n,1}(z) \\ G_{n,2}(z) + G_{n,3}(z) \end{pmatrix}. \tag{A.12}$$

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Similar to (A.6), (A.4) and (A.5), by the kernel theory and an application of Taylor's expansion, it is easy to show that

$$E[G_{n,0}(z)] = h^2 M_0(z) \left[\frac{\mu_2(K)}{2} \beta''_{(1)}(z) \right] \{1 + O(h)\}$$

and $\mathrm{Var}[G_{n,0}(z)] = O((nh^2)^{-1})$, so that

$$G_{n,0}(z) = h^2 M_0(z) \left[\frac{\mu_2(K)}{2} \beta''_{(1)}(z) \right] \{1 + O_p(\gamma_n)\},$$

where $\gamma_n = h + (nh^2)^{-1/2}$. Further, following the proof above, we can easily show that

$$G_{n,1}(z) = nh^2 M_1(z) \left[\frac{\mu_2(K)}{2} \beta''_{(2)}(z) \right] \{1 + O_p(\gamma_n)\},$$

$$G_{n,2}(z) = h^2 M_1(z) \left[\frac{\mu_2(K)}{2} \beta''_{(1)}(z) \right] \{1 + O_p(\gamma_n)\},$$

and

$$G_{n,3}(z) = nh^2 M_2(z) \left[\frac{\mu_2(K)}{2} \beta''_{(2)}(z) \right] \{1 + O_p(\gamma_n)\}.$$

Plugging the above results into (A.12), we obtain

$$B_n(z) = h^2 S(z) D_n \left[\frac{\mu_2(K)}{2} \beta''(z) \right] \{1 + O_p(\gamma_n)\}. \tag{A.13}$$

Substituting (A.13) into (A.11) and using (A.9) lead to

$$L_{1n} = D_n h^2 \mu_2(K) \beta''(z) \{1 + O_p(\gamma_n)\},$$

Therefore,

$$D_n^{-1} L_{1n} = h^2 \mu_2(K) \beta''(z) + O_p(h^2 \gamma_n). \tag{A.14}$$

Finally, we consider L_{2n}. Define

$$T_n(z) = \sqrt{\frac{h}{n}} \sum_{t=1}^{n} K_h(Z_t - z) u_t D_n^{-1} X_t = \begin{pmatrix} T_{n,1}(z) \\ T_{n,2}(z) \end{pmatrix}$$

with

$$T_{n,1}(z) = \sqrt{\frac{h}{n}} \sum_{t=1}^{n} K_h(Z_t - z) u_t X_{t1}$$

and

$$T_{n,2}(z) = \sqrt{\frac{h}{n}} \sum_{t=1}^{n} (t/n) K_h(Z_t - z) u_t.$$

By combining the above expressions with (A.10) and (A.14), we obtain

$$\sqrt{nh} D_n \left[\widehat{\beta}(z) - \beta(z) - h^2 \mu_2(K) \beta''(z) + O_p(h^3) \right] = S_{n,0}(z)^{-1} T_n(z). \tag{A.15}$$

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To prove the asymptotic normality of the left hand side of (A.15), it suffices to establish the asymptotic normality of $T_n(z)$. Note that $T_{n,1}$ only involves stationary variables. Hence, by the kernel estimation theory for stationary mixing data (see Theorem 2 of Cai, Fan and Yao (2000) for details) we have

$$T_{n,1}(z) \xrightarrow{d} N(0, \sigma_u^2 v_0(K) M_0(z)). \qquad (A.16)$$

where $v_0(K) = \int K^2(v)v^2 dv$. Also, we have

$$T_{n,2}(z) \xrightarrow{d} N(0, \sigma_u^2 v_0(K) M_2(z)) = N(0, v_0(K)M_2(z)). \qquad (A.17)$$

The covariance matrix is given by

$$Cov(T_{n,1}, T_{n,2}) = \sigma_u^2 h^{-1} E[K_h(Z_t - z)X_{t1}(t/n)] = \sigma_u^2 v_0(K)M_1(z) + O(h).$$

Therefore, a combination of (A.16) and (A.17) leads to

$$T_n(z) \xrightarrow{d} N(0, V),$$

where

$$V = v_0(K)\begin{pmatrix} M_0(z) & M_1(z) \\ M_1(z)^T & M_2(z) \end{pmatrix} = v_0(K)S(z).$$

Therefore, by Slusky's theorem, we have

$$\sqrt{nh}\, D_n \left[\widehat{\beta}(z) - \beta(z) - h^2 \mu_2(K)\beta''(z)\right] \xrightarrow{d} N(0, v_0(K)S(z)^{-1}).$$

Q.E.D.