**When Wealth Affects People's Impatience**

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**Very Preliminary Version**

**Abstract**: Zou [1994] has proved that the spirit of capitalism will cause endogenous growth using the AK model. We find that this conclusion needs to be reexamined under our endogenous time preference model. Contrastingly different patterns of growth will obtain under our framework.

# Introduction

Model [1] provides a case in which individual's marginal impatience is increasing along with the accumulation of wealth in the sense of Max Weber, finding that there is either a saddle-point stable steady state (with no boundary conditions to the time preference) or a balanced growth path (BGP) (if binding the time preference from the above). In Model [2], we consider the implications when people have decreasing marginal impatience in the sense of Becker and Mulligan [1997]. We reach the conclusion that the consumption-capital locus will follow a trajectory of an unbounded expanding cycle due to any disturbance, given that the intensity of capitalism is low enough, or follow the direction of the shock to infinite as time tends to $\infty$, otherwise. Finally, in Model [3], we treat an intermediate case where people's marginal impatience is an increasing function of capital over a certain range, but the relationship does not hold elsewhere. By a special form of preference function capturing our assumptions about people's variation of impatience, we show the existence of multi-equilibrium and analyze the local stability around the steady state under various conditions. By extend our model to Cobb-Douglas product function, we still prove the possibility of a cycle.

## The Model [1]

Suppose the existence of a representative in an economy, who accumulate wealth for its own sake; at the same time, as wealth level expands, his marginal impatience increases[1] modeled by an endogenous preference structure. Individual's problem is given by,

$$\max \quad \int_0^\infty [u(c_t)+v(k_t)]e^{-\int_0^t \rho(k_s)ds}dt$$

$$s.t. \quad \dot{k}_t = f(k_t)-c_t\,, \quad \rho'(k)>0\,.$$

To ensure the problem has an optimal solution, we adopt the standard assumption of a concave utility function as well as a neoclassical product function as in conventional Ramsey-Cass-Koopmans model.

The Euler equation: $-u_{cc}\dot{c} = v'(k)+u_c[f'(k)-\rho(k)]$

Steady State can be characterized by,

$$v'(\bar{k})+u'(\bar{c})[f'(\bar{k})-\rho(\bar{k})]=0$$

$$f(\bar{k})-\bar{c}=0\,,$$

which yields $f'(\bar{k})=\rho(\bar{k})-v'(\bar{k})/u'(\bar{c})<\rho(\bar{k})$.

1) Thus we have $\bar{k}>\bar{\bar{k}}$, where $\bar{\bar{k}}$ is defined by $f'(\bar{\bar{k}})=\rho(\bar{\bar{k}})$, i.e. the steady state level of capital when there is no spirit of capitalism in the economy.

2) The dynamics of the system is characterized by,

$\dot{c}=[\dfrac{v'(k)}{u'(c)}+f'(k)-\rho(k)]\sigma(c)^{-1}$, where $\sigma(c)=-u''(c)/u'(c)$, and

$\dot{k}=f(k)-c\,.$

If we assume the production function is of AK technology $f(k)=Ak$, and the utility

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[1] In the sense of Max Weber. [1958]

is given by $u = \log c + \beta \log k$, where $\beta$ represents the economy's intensity of the spirit of capitalism. We will have $\dot{c} = c \cdot [\frac{\beta c}{k} + A - \rho(k)]$.

Case 1: The steady state can be characterized by,

$\bar{c} = A\bar{k}$

$\rho(\bar{k}) = A(\beta + 1)$, given that the marginal impatience level $A(\beta+1)$ is attainable.

Linearize the dynamic system around steady state yields,

$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{bmatrix} A\beta & -A\bar{k}[\frac{A\beta}{\bar{k}} + \rho'(\bar{k})] \\ -1 & A \end{bmatrix} \begin{pmatrix} c - \bar{c} \\ k - \bar{k} \end{pmatrix}$, whose characteristic roots are given by $\mu_1$ and $\mu_2$.

The existence of **real roots** $\mu_1$ and $\mu_2$ is implied by,

$\mu_1 \cdot \mu_2 = A^2\beta - A\bar{k}(\frac{A\beta}{\bar{k}} + \rho'(\bar{k})) = -A\bar{k}\rho'(\bar{k}) < 0$[2], which also shows that the steady state is **saddle-point stable**.

**Remark:** The capital accumulation through spirit of capitalism will stop at some point in the future, given that individual's marginal impatience can grow to a sufficiently high level.

Case 2: If there is an upper bound for $\rho(\cdot)$, say, $\rho(\cdot) \le \bar{\rho} < A(\beta+1)$, then the economy will exhibit **endogenous growth** property. Suppose there exists a $k^*$ which satisfies $\rho(k) = \bar{\rho}$ for all $k \ge k^*$.

On the balanced growth path,

$\dot{c}/c = \frac{\beta c}{k} + A - \bar{\rho} = \gamma$, from which we will have,

$c/k = (\gamma + \bar{\rho} - A)/\beta = const.$. Further, we can see that,

$\dot{c}/c = \dot{k}/k = \gamma = A - \frac{c}{k}$.

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[2] Note that we also have $\mu_1 + \mu_2 = A + \beta / \bar{k} > 0$

The combination of the above two equations yields, $\gamma = A - \overline{\rho}/(1+\beta)$, which is the balanced growth rate of consumption and capital in the long run.

**Remark:** The balanced growth rate is positively related to technological coefficient and the intensity of capitalism (the wealth effect in utility), but negatively related to the long run impatience level.

## The Model [2]

Under the same framework as in Model [1], we assume individual's marginal impatience is decreasing as a result of capital accumulation[3] --- when people are getting wealthier, they become more patient.

Suppose the instantaneous preference takes the form, $\rho(k) = B/k$[4], where $B$ is a positive constant, i.e. $B > 0$.

Given the AK product function, the dynamic of this system is given by,

$$\dot{c} = c \cdot [\frac{\beta c}{k} + A - B/k],$$

$$\dot{k} = Ak - c.$$

The steady state is, $\overline{k} = B/A(\beta+1)$ and $\overline{c} = A\overline{k}$.

Linearize the dynamic system around steady state yields,

$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = \begin{bmatrix} A\beta & A^2 \\ -1 & A \end{bmatrix} \begin{pmatrix} c - \overline{c} \\ k - \overline{k} \end{pmatrix}$, whose characteristic roots are given by two complex numbers $\mu_1$ and $\mu_2$.

We easily see that $\mu_1 + \mu_2 = A(\beta+1) > 0$, and $\mu_1 \cdot \mu_2 = AB/\overline{k} = A^2(\beta+1) > 0$, which also shows that the dynamic system produces a **cycle**, given $\beta < 3$ [5]. More

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3 In the sense of Becker and Mulligan [1997].

4 This kind of preference also captures the idea that the long-run discount rate should be a constant, whatever the patterns of growth that the economy takes. For example, when there is endogenous growth, the discount rate would be asymptotic zero.

5 Note that $\beta$ represents the intensity of capitalism. When $\beta = 0$, there is no capitalism at all.

specifically, the complex roots are given by,

$\mu_1 = A(\beta+1)/2 + iA(\beta+1)^{1/2}(3-\beta)^{1/2}/2$,

and $\mu_2 = A(\beta+1)/2 - iA(\beta+1)^{1/2}(3-\beta)^{1/2}/2$, whose real part is strictly positive, which shows that the steady state is not stable as it will keep expanding following the trajectory of an unbounded cycle forever. In this case, **there is neither a balanced growth path nor stationary steady state equilibrium**.

Given $\beta \geq 3$, we will have two positive real characteristic roots. The steady state is a source and **unstable** for every slight disturbance[6].

# The Model [3]

We further assume the preference takes the form,

$$\rho(k) = 1/[(M-k)^2 + N],$$

where $M > 0, N > 0$, and thus we have,

$$\rho'(k) = 2(M-k)/[(M-k)^2+N]^2 \begin{cases} > 0, if & k < M \\ = 0, if & k = M \\ < 0, if & k > M \end{cases}.$$

This type of preference has the property that individual's marginal impatience goes up along with capital first, and then drops gradually to a constant as capital grows to infinity. Still we assume the production function is of AK technology $f(k) = Ak$, and the utility is given by $u = \log c + \beta \log k$, and the dynamics is given by,

$$\begin{aligned} \dot{c} &= c \cdot [\frac{\beta c}{k} + A - \rho(k)] \\ \dot{k} &= Ak - c \end{aligned}.$$

When $1/[A(\beta+1)] - M^2 \leq N < 1/[A(\beta+1)]$, the system will produce **multi-equilibrium** in which the steady state locus of consumption and capital is,

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6 The optimal consumption and capital will follow the direction of the disturbance to infinite as time tends to $\infty$.

$$(\bar{c},\bar{k})=(A\bar{k},M\pm\{\frac{1}{A(\beta+1)}-N\}^{1/2}).$$

Firstly, we linearize the system around steady state locus,

$\begin{pmatrix}\dot{c}\\ \dot{k}\end{pmatrix}=H\cdot\begin{pmatrix}c-\bar{c}\\ k-\bar{k}\end{pmatrix}$, where H is given by,

$$H=\begin{bmatrix}A\beta & -A^2\beta-2A\bar{k}(M-\bar{k})/[(M-\bar{k})^2+N]^2\\ -1 & A\end{bmatrix}.$$

Compute the trace and determinant of coefficient matrix H, and we have,

$Tr(H)=A(\beta+1)$,

$Det(H)=2A\bar{k}(M-\bar{k})/[(M-\bar{k})^2+N]^2$.

Thus when $\bar{k}=M-\{\frac{1}{A(\beta+1)}-N\}^{1/2}<M$, i.e. in the region of increasing marginal impatience, the steady state will be **unstable**. Here we claim that this system will **never produce a cycle**. If the system has a cycle, then we have $\Delta=[Tr(H)]^2-4Det(H)<0$, which simplifies to,

$$\frac{9}{8A(\beta+1)}-N<M\cdot\{\frac{1}{A(\beta+1)}-N\}^{1/2}.$$

Let $x=\frac{1}{A(\beta+1)}-N$, and the above is equivalent to,

$0<x<M\cdot\{M^2-\frac{1}{2A(\beta+1)}\}^{1/2}-\frac{1}{4A(\beta+1)}$. But the right side will never be positive, i.e. the inequality $M\cdot\{M^2-\frac{1}{2A(\beta+1)}\}^{1/2}-\frac{1}{4A(\beta+1)}>0$ has no solution.

From the above deduction, we easily exclude the case of a cycle.

Further, when $\bar{k}=M+\{\frac{1}{A(\beta+1)}-N\}^{1/2}>M$, i.e. in the region of decreasing marginal impatience, the steady state will be a **saddle**.

## Extensions

Consider the neo-classical technology, i.e. Cobb-Douglas product function, $y = Ak^{\alpha}$, with $0 < \alpha < 1$ and $A > 0$. Other assumptions remain as in Model [1] or Model [2]. The optimal conditions regarding the time path of consumption and capital accumulation can be written as,

$$\dot{c} = c \cdot [\frac{\beta c}{k} + \alpha A k^{\alpha-1} - \rho(k)],$$

$$\dot{k} = Ak^{\alpha} - c.$$

The steady state, if exists, can be given by the combination of the following two equations,

$$\overline{c} = A\overline{k}^{\alpha},$$

$$\beta A\overline{k}^{\alpha-1} + \alpha A\overline{k}^{\alpha-1} = \rho(\overline{k}).$$

Linearizing the dynamic system around $(\overline{c}, \overline{k})$, we have,

$$\begin{pmatrix} \dot{c} \\ \dot{k} \end{pmatrix} = J \cdot \begin{pmatrix} c - \overline{c} \\ k - \overline{k} \end{pmatrix}, \text{ where } J = \begin{bmatrix} A\beta\overline{k}^{\alpha-1} & A\overline{k}^{\alpha}[-A\beta\overline{k}^{\alpha-2} + A\alpha(\alpha-1)\overline{k}^{\alpha-2} - \rho'(\overline{k})] \\ -1 & \alpha A\overline{k}^{\alpha-1} \end{bmatrix}.$$

Denoting the characteristic roots of $J$ as $\mu_1$ and $\mu_2$ yields,

$$\mu_1 + \mu_2 = A(\alpha + \beta)\overline{k}^{\alpha-1} > 0,$$

$$\mu_1 \cdot \mu_2 = A\overline{k}^{\alpha}[A\overline{k}^{\alpha-2}(\alpha + \beta)(\alpha - 1) - \rho'(\overline{k})].$$

The discriminant of the characteristic polynomial of J is defined to be,

$$\Delta = [tr(J)]^2 - 4\det(A).$$

For simplicity, we adopt special forms of preference function, say, $\rho(k) = B/k$. And thus we have,

$$\overline{k}^{\alpha} = B/[(\alpha + \beta)A],$$

$$\mu_1 \cdot \mu_2 = A^2\overline{k}^{2\alpha-2}\alpha(\alpha + \beta), \text{ and finally}$$

$$\Delta = A^2\overline{k}^{2\alpha-2}(\alpha + \beta)(\beta - 3\alpha).$$

We claim that when $\beta < 3\alpha$, any disturbance to the steady state will generate a **unbounded expanding cycle**; when $\beta \geq 3\alpha$, we still have two positive roots and thus the steady state is a **source** and unstable for any disturbance as in the last case of Model [2].

**Remark:** Even under the standard assumption of a constant-to-scale technology, there is still possibility for unbounded growth, when the intensity of capitalism is high enough, or unbounded expanding cycle when $\beta$ is low[7].

## Income Tax and Consumption tax

This paper provides a simple framework for analyzing individual's behavior when capital affects people's impatience. It can be easily used to analyze the long run or short run effects of government policies, which is the next step of our work.

## References

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[7] Note that when there is no spirit of capitalism, i.e. $\beta = 0$, there is always a cycle due to any shocks.

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