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Military spending and stochastic growth

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Abstract

This study examines capital accumulation, military spending, arms accumulation, and output growth in a stochastic endogenous growth model. The analysis shows that higher (lower) growth in foreign military spending leads to faster (slower) economic growth in the home country if the home country's intertemporal substitution elasticity in consumption is smaller (larger); but more volatility in foreign military spending can lead to higher economic growth in the home country when its intertemporal substitution elasticity is large. In addition, shocks to output production may stimulate economic growth.

JEL classification: E20; E22; H56; O10; O40

Keywords: Military spending; Arms race; Capital accumulation; Stochastic growth

1. Introduction

The relationship between competitive arms accumulation and economic growth has been a major topic of dynamic economics and international relations in the political science. Earlier mathematical models of arms race presented by Richardson (1960) and Saaty (1968) have been extended and refined in numerous contributions in the 1970s–1990s, and notable ones are Brito (1972), Intriligator (1975), Simaan and Cruz (1975), Intriligator and Brito (1976), Deger and Sen (1984), van der Ploeg and de Zeeuw (1990), and Chang et al. (1996). At the same time, economic consequences of military spending and arms accumulation have also received enormous attention in both

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empirical studies and policy discussions. For example, Benoit (1973, 1978) and Deger and Sen (1983) have shown a positive, although sometimes not significant, effect of defense spending on economic growth, whereas Deger (1986) has found that defense spending definitely reduces national savings rates. The usual arguments for a positive effect of military spending on growth have been often put as follows: Rising military spending leads to increased security, demand creation, utilization of excess capacity, and various indirect spinoffs through education, employment, infrastructure, and other services that the military can provide (see Deger and Sen (1992) or Deger and Sen (1995) for a comprehensive survey). On the other hand, the direct resource allocation effect transfers potential investment resources to the military, reducing investment and growth. In a more sophisticated regression analysis, Landau (1993) has demonstrated that, for a sample of 71 countries over the time period 1969–1989, there exists a non-linear (quadratic) relationship between military spending and growth: At low levels of military expenditure, there will be a positive impact due to increased security and efficiency, while at higher levels of military expenditure, the negative resource-use effect will lead to lower growth.

As far as we know, all aforementioned empirical studies have approached their problems without explicit intertemporal optimization frameworks involving both capital and arms accumulation, although dynamic optimization is a routine component of theoretical studies on competitive arms accumulation. But theoretical studies on arms race have hardly taken into consideration of output production and capital accumulation. Instead, they have focused on the optimal allocation of consumption and military spending in an endowment economy without physical investment. When a formal dynamic model dealing with both arms and capital accumulation is presented by Zou (1995a), the finding is rather surprising: Competitive arms races among countries have no effect on capital accumulation in the long run, even though it can stimulate productive investment and output in the short run.

This paper intends to show that in a stochastic environment typical in a world of competitive arms races, military spending and arms accumulation affect long-run capital accumulation and growth rates in a rather complicated way. The introduction of stochastic elements to the model on arms races and capital accumulation seems to be a natural result of recent analytical advances in modeling government fiscal and monetary policies, public spending, private investment, and output growth in stochastic environments as in Eaton (1981), Gertler and Grinols (1982), Bertola and Drazen (1993), Pindyck and Solimano (1993), Turnovsky (1993, 2000), Grinols and Turnovsky (1993, 1994), Obstfeld (1994), and Ramey and Ramey (1995). Defense spending has been and may always be an important component of public policies and government expenditures, and a change in a country's military investment crucially depends on stochastic elements in domestic and international economic, political, and military factors. As this paper is going to show, a stochastic setting not only makes the model more realistic, it also produces rather different analytical results.

We organize the paper as follows. In Section 2, we present a simple stochastic growth model treating military spending as a pure consumption good and with some special function forms for preferences and production technology derive a closed-form solution linking the long-run economic growth rate to technology, preferences, and military instability (shocks). In Section 3, treating military spending as an investment good and allowing explicit dynamics of arms accumulation, we obtain another closed-form solution of the expected endogenous growth rate of capital and arms accumulation. We summarize our main findings in Section 4.

2. Model one: military spending as a consumption good

As in Brito (1972), Deger and Sen (1984), van der Ploeg and de Zeeuw (1990), and Zou (1995a), there are two countries in this model: the home country and the foreign country, and they are in a state of military confrontation. Suppose the preference of the home country is defined on its consumption, c, its military spending, $m(t)$, and foreign military spending, m^*: $u(c,m,m^*)$. Suppose the utility function $u(c,m,m^*)$ is twice differentiable and concave. Following Brito (1972), Deger and Sen (1983,1984), van der Ploeg and de Zeeuw (1990), and Zou (1995a), we assume that

$$u_1 > 0, \quad u_2 > 0, \quad u_3 < 0, \quad u_{11} < 0, \quad u_{22} < 0,$$
$$u_{12} = u_{21} > 0, \quad u_{13} = u_{31} < 0, \quad u_{23} = u_{32} > 0. \tag{1}$$

The assumptions in (1) imply that the home country's marginal utility from its consumption and military spending is positive and diminishing, but the marginal utility from the foreign country's military spending (foreign military threat) is negative and diminishing. The assumption $u_{23} = u_{32} > 0$ implies that an increase in the foreign military threat will increase the marginal utility of the home country's defense, whereas $u_{12} = u_{21} > 0$ states that more security (military defense) for the home country raises its marginal utility of consumption and, similarly, $u_{13} = u_{31} < 0$ asserts that a rise in the foreign military threat reduces the home country's marginal utility of consumption. [1]

Following Eaton (1981) and Turnovsky (2000), output is produced by a stochastic technology,

$$\mathrm{d}Y = F(k)\,\mathrm{d}t + H(k)\,\mathrm{d}y, \quad F'(k) > 0, \; F''(k) < 0. \tag{2}$$

The equation above asserts that the flow of output over the period $(t, t+\mathrm{d}t)$ consists of two components. First, there is the deterministic component, described by the first term on the right-hand side, with $F(k)$ representing the mean rate of output production per unit time. In addition, there is a stochastic component, $H(k)\mathrm{d}y$, reflecting the various random elements affecting output production. The stochastic term $\mathrm{d}y$ is assumed to be temporally independent, normally distributed with mean zero and variance $\sigma_y^2\,\mathrm{d}t$. See more explanations and justifications in Turnovsky (2000).

[1] van der Ploeg and de Zeeuw (1990) have nicely justified these assumptions as follows: The welfare of one country depends on the level of security which is perceived to be an increasing function of its own weapons stock and a decreasing function of the foreign weapons stock. This may be because any imbalance in weapons stocks increases the likelihood of loosing a possible war and increases the likelihood that a war might in fact be initiated. Alternatively, a country may simply feel that it gains international prestige from having an army superior to its rivals. Both of these factors can in principle lead to a balance of terror.

We also assume that foreign military spending m^* follows a stochastic process,

$$\mathrm{d}m^* = \alpha m^* \,\mathrm{d}t + \sigma m^* \,\mathrm{d}z, \tag{3}$$

where $\alpha m^* \,\mathrm{d}t$ is the known mean level of foreign military spending, and $\sigma m^* \,\mathrm{d}z$ is the stochastic component. The stochastic term $\mathrm{d}z$ is assumed to be temporally independent, normally distributed with mean zero and variance $\sigma^2 \,\mathrm{d}t$. Eq. (3) is a special case of the more general form of government spending adopted by Bertola and Drazen (1993) in a continuous-time stochastic environment. In this paper, our model is not really a game, and some stochastic process is assumed on the foreign country.[2]

Suppose the covariance of $\mathrm{d}y$ and $\mathrm{d}z$ is

$$cov(\mathrm{d}y, \mathrm{d}z) = \sigma_{yz} \,\mathrm{d}t.$$

Since we take military spending as a consumption good, we can write the budget constraint for the home country as

$$\mathrm{d}k = \mathrm{d}Y - c\mathrm{d}t - m\mathrm{d}t. \tag{4}$$

The home country chooses its consumption path, $c(t)$, capital accumulation path, $k(t)$, and military spending path, $m(t)$, to maximize its discounted welfare with a constant time discount rate, ρ $(0 < \rho < 1)$, namely,

$$\max \mathrm{E}_0 \int_0^{\infty} u(c, m, m^*)\mathrm{e}^{-\rho t} \,\mathrm{d}t \tag{5}$$

subject to (4) with the technology and foreign military spending given in Eqs. (2) and (3), respectively, and with the initial capital stock in the home country given by $k(0)$.

2.1. Optimalities

To solve the problem, we introduce the value function, $V(k, m^*, t)$, and define the differential operator of it by

$$\begin{aligned} L(V(k, m^*, t)) &= \lim_{\mathrm{d}t \to 0} \mathrm{E}\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) \\ &= V_t + V_k(F(k) - c - m) + V_{m^*}\alpha m^* + \tfrac{1}{2} V_{km^*}\sigma_{yz}H(k)\sigma m^* \\ &\quad + \tfrac{1}{2} V_{kk}H(k)^2\sigma_y^2 + \tfrac{1}{2} V_{m^*m^*}\sigma^2 m^{*2}. \end{aligned}$$

Given the exponential time discounting, the value function can be assumed to be of the form

$$X(k, m^*)\mathrm{e}^{-\rho t} = V(k, m^*, t).$$

[2] We thank the referee for emphasizing this point.

Now, the home country chooses consumption and military spending to maximize the expression

$$u(c,m,m^*) + L(X(k,m^*)\mathrm{e}^{-\rho t})$$
$$= u(c,m,m^*) - \rho X + X_k(F(k) - c - m) + X_{m^*}\alpha m^* + \tfrac{1}{2} X_{km^*}\sigma_{yz}H(k)m^*$$
$$+ \tfrac{1}{2} X_{kk}H(k)^2\sigma_y^2 + \tfrac{1}{2} X_{m^*m^*}\sigma^2 m^{*2}.$$

Taking partial derivatives with respect to c and m, respectively, of the above expression and canceling the term $\mathrm{e}^{-\rho t}$, we have

$$\frac{\partial u(c,m,m^*)}{\partial c} = X_k,$$
$$\frac{\partial u(c,m,m^*)}{\partial m} = X_k, \tag{6}$$

which asserts that the marginal values of consumption and military spending must be equal at an optimum. And from Eq. (6), we can determine the optimal values for c/k and m/k as the functions of X_k and X_{kk}. Furthermore, the value function must satisfy the Bellman equation

$$\max_{c,m}\{u(c,m,m^*) + L(X(k,m^*)\mathrm{e}^{-\rho t})\} = 0.$$

Substituting the optimal values from Eq. (6), we have

$$u(\tilde{c},\tilde{m},m^*) - \rho X + X_k(F(k) - \tilde{c} - \tilde{m}) + X_{m^*}\alpha m^* + \tfrac{1}{2}X_{km^*}\sigma_{yz}H(k)m^*$$
$$+ \tfrac{1}{2}X_{kk}H(k)^2\sigma_y^2 + \tfrac{1}{2}X_{m^*m^*}\sigma^2 m^{*2} = 0, \tag{7}$$

where ˜ denotes the optimal value.

2.2. Explicit solutions

Now from Eqs. (6) and (7), we get the optimal consumption, optimal military spending, and the value function. In order to find the explicit solutions, we specify the utility function and technology as follows:

$$u(c,m,m^*) = \frac{1}{1-\gamma}(c^\theta m^{1-\theta})^{1-\gamma}(m^*)^{-\lambda}, \tag{8}$$

$$F(k) = Ak, \quad H(k) = Ak \tag{9}$$

with parameters satisfying $0 < \theta < 1$; $\lambda > 0$ when $0 < \gamma < 1$; $\lambda < 0$ when $\gamma > 1$; and A is a positive constant. The restrictions on λ and γ are made to ensure that $\partial u/\partial m^* < 0$ for all possible cases. Furthermore, this utility function also can be regarded as being defined on the relative military status of the two countries since it can be rewritten as

$$c^{\theta(1-\gamma)}\left(\frac{1}{1-\gamma}\right)\left[\frac{m^{(1-\theta)(1-\gamma)}}{(m^*)^{\lambda}}\right],$$

where the term $(1/(1-\gamma))[m^{(1-\theta)(1-\gamma)}/(m^*)^{\lambda}]$ measures the relative military power of the home country versus the foreign country. It should be noticed that the production technology is the same as in Eaton (1981) and Turnovsky (1993, 2000), whereas the utility function is an extension of the ones in many continuous-time stochastic models on growth and asset pricing when the two military goods are included.

It is natural to conjecture that a solution for the value function takes the following form:

$$X(k,m^*)=\delta k^{1-\gamma}(m^*)^{-\lambda}, \tag{10}$$

where δ is to be determined. Therefore, we have

$$X_k=\delta(1-\gamma)k^{-\gamma}(m^*)^{-\lambda}; \quad X_{kk}=-\delta(1-\gamma)\gamma k^{-\gamma-1}(m^*)^{-\lambda}. \tag{11}$$

Substituting Eq. (11) into optimal condition (6) yields

$$\begin{aligned}&\theta(c^{\theta}m^{1-\theta})^{-\gamma}(m^*)^{-\lambda}c^{\theta-1}m^{1-\theta}=X_k,\\&(1-\theta)(c^{\theta}m^{1-\theta})^{-\gamma}(m^*)^{-\lambda}c^{\theta}m^{-\theta}=X_k.\end{aligned} \tag{12}$$

We define the total consumption as the sum of consumption and military spending

$$C=c+m.$$

From Eq. (12), we have

$$c=\theta C, \quad m=(1-\theta)C. \tag{13}$$

In addition, substituting Eq. (11) into Eq. (12), we obtain the total consumption-capital ratio as

$$\frac{C}{k}=[\delta(1-\gamma)(\theta^{\theta}(1-\theta)^{1-\theta})^{\gamma-1}]^{-1/\gamma}. \tag{14}$$

Substituting Eq. (14) into Eq. (7) leads to

$$\begin{aligned}&\frac{1}{1-\gamma}(\theta^{\theta}(1-\theta)^{1-\theta})^{1-\gamma}[\delta(1-\gamma)(\theta^{\theta}(1-\theta)^{1-\theta})^{\gamma-1}]^{(\gamma-1)/\gamma}k^{1-\gamma}(m^*)^{-\lambda}\\&\quad-\rho\delta k^{1-\gamma}(m^*)^{-\lambda}+\delta(1-\gamma)k^{-\gamma}(m^*)^{-\lambda}(Ak-C)-\lambda\delta k^{1-\gamma}(m^*)^{-\lambda-1}\alpha m^*\\&\quad-\frac{1}{2}\delta(1-\gamma)k^{1-\gamma}(m^*)^{-\lambda}A\sigma_{yz}+\frac{1}{2}\delta(1-\gamma)(-\gamma)k^{-\gamma-1}(m^*)^{-\lambda}A^2k^2\sigma_y^2\\&\quad+\frac{1}{2}\delta\lambda(\lambda+1)k^{1-\gamma}(m^*)^{-\lambda-2}\sigma^2m^{*2}=0.\end{aligned}$$

From which we can determine the coefficient δ from the following expression:

$$\frac{C}{k} = [\delta(1-\gamma)(\theta^{\theta}(1-\theta)^{1-\theta})^{\gamma-1}]^{-1/\gamma}$$
$$= \frac{\lambda\alpha + \frac{1}{2}(1-\gamma)\gamma A\sigma_y^2 - \frac{1}{2}\lambda(\lambda+1)\sigma^2 - (1-\gamma)A + \rho + \frac{1}{2}\lambda(1-\gamma)A\sigma_{yz}}{\gamma}.$$

Once δ is determined, the value function is in turn determined.

Now, we have the dynamic equation for the capital stock

$$\mathrm{d}k = (Ak - C)\mathrm{d}t + Ak\,\mathrm{d}y$$
$$= k\left[\left(A - \frac{C}{k}\right)\mathrm{d}t + A\,\mathrm{d}y\right]. \tag{15}$$

Hence, the expected growth rate of consumption and the capital stock, denoted as ϕ_1, is

$$\phi_1 = \mathrm{E}\frac{\mathrm{d}c/\mathrm{d}t}{c} = \mathrm{E}\frac{\mathrm{d}k/\mathrm{d}t}{k} = \left(A - \frac{C}{k}\right). \tag{16}$$

From which the solution of the capital stock starting from the initial capital $k(0)$ at time 0, is

$$k(t) = k(0)\mathrm{e}^{(\phi_1 - (\gamma/2)A^2\sigma^2)t + Ay(t) - Ay(0)}.$$

The stochastic path for the foreign military expenditure can be derived from Eq. (6) as

$$m^*(t) = m^*(0)\mathrm{e}^{(\alpha - [(\lambda+1)/2]\sigma^2) + \sigma z(t) - \sigma z(0)}.$$

The transversality condition

$$\lim_{t\to\infty} \mathrm{E}[\delta k^{1-\gamma}(m^*)^{-\lambda}\mathrm{e}^{-\rho t}] = 0$$

will be met if and only if

$$(1-\gamma)\left(A - \frac{C}{k} - \frac{\gamma}{2}A^2\sigma_y^2\right) - \lambda\left(\alpha - \frac{\lambda+1}{2}\sigma^2\right) - \rho < 0$$

which is equivalent to $C/k > 0$.

2.3. Comparative dynamics

Now, we focus on the effects of the growth and shocks in foreign military spending on the home country. For the mean growth in the foreign military threat, its effect on the home country's economic growth rate is given by

$$\frac{\partial\phi_1}{\partial\alpha} = -\frac{\lambda}{\gamma}.$$

Since $\lambda > 0$ when $0 < \gamma < 1$; and $\lambda < 0$ when $\gamma > 1$, we have $\frac{\partial\phi_1}{\partial\alpha} > 0$ when $\gamma > 1$; and $\frac{\partial\phi_1}{\partial\alpha} < 0$, when $0 < \gamma < 1$. That is to say, a higher (lower) growth in foreign military spending leads to faster (slower) economic growth in the home country if the home country's elasticity of intertemporal substitution in consumption, which is $1/\gamma$, is smaller (larger). These findings indicate that, when the foreign country raises its average level of military spending, the home country's marginal utility of military spending rises and it increases its current military spending and reduces its capital investment when its intertemporal substitution is relatively elastic, i.e., $0 < \gamma < 1$. Therefore the long-run growth rate is reduced. On the other hand, when $\gamma > 1$, the home country's elasticity of intertemporal substitution is small and it will cut consumption, raise investment, and produce more output.

As for the stochastic shocks to the foreign military threat, their effect on the home country's economic growth is given by

$$\frac{\partial\phi_1}{\partial\sigma^2} = \frac{\frac{1}{2}\lambda(\lambda+1)}{\gamma}.$$

Hence, $\partial\phi_1/\partial\sigma^2 > 0$ when $0 < \gamma < 1$ or when $\lambda < -1$ and $\gamma > 1$; $\partial\phi_1/\partial\sigma^2 < 0$ when $\gamma > 1$ and $0 > \lambda > -1$. These results seem to suggest that a higher elasticity of intertemporal substitution in consumption in the home country will result in higher economic growth in the home country when there is more volatility in the foreign military threat. But even in this case, caution should be exercised in order to avoid drawing a simple conclusion. It can also be the case that, when the foreign military threat causes larger disutility to the home country, i.e., a large absolute value of λ, a higher variance in the foreign military threat can lead to higher economic growth in the home country even for a small elasticity of intertemporal substitution (i.e., $\gamma > 1$). Furthermore, from our discussions above we notice that the mean and variance in foreign military growth tend to have opposite effects on the home country's economic growth.

In addition, the stochastic shocks to output production in the home country have the same qualitative effect on economic growth as the mean growth in foreign military spending:

$$\frac{\partial\phi_1}{\partial\sigma_y^2} = -\frac{\frac{1}{2}\gamma A(1-\gamma)}{\gamma}.$$

Therefore, $\partial\phi_1/\partial\sigma_y^2 > 0$ when $\gamma > 1$; and $\partial\phi_1/\partial\sigma_y^2 < 0$ when $0 < \gamma < 1$. The general lesson to be drawn here is that shocks to output production are not always harmful for output growth. In our model they can even stimulate output growth when the elasticity of intertemporal substitution is less than one (or $\gamma > 1$). This is a confirmation of the theoretical ambiguity between production risk and output growth pointed out earlier by

Devereux and Smith (1994) and Obstfeld (1994). Interested readers could see Ramey and Ramey (1995) for more contrasting views and further discussions.

3. Model two: military spending as an investment good

When treating military spending as an investment good, the model allows capital accumulation as well as arms accumulation. To economize notations, in this section we use $m(t)$ to denote the weapons stock of the home country and $m^*(t)$ the weapons stock of the foreign country. The home country's total wealth is the sum of its capital and weapons stocks:

$$w = k + m,$$

where w is the home country's total wealth.

Similarly, the total wealth of the foreign country, $w^*(t)$, is the sum of its capital and weapons stocks: $k^*(t)$; and $m^*(t)$:

$$w^* = k^* + m^*.$$

The preferences of the home country are defined on its consumption, c, its total wealth, $w(t)$, and the foreign country's total wealth, $w^*(t)$: $u(c, w, w^*)$. To us, the inclusion of both the capital stock and arms in the utility function provides a more realistic picture of a country's power and status in a competitive arms race over a longer time period because a higher capital stock always produces more output, which can lead to more military spending and more arms accumulation. Arms accumulation without substantial capital accumulation and output expansion is hardly sustainable in the long run as shown by the current situations in North Korea and Cuba and the recent history of the Soviet Union. Of course, defining the utility function on wealth is not new at all, and it has been adopted for various other applications: nationalism and mercantilism in the sense of Bardhan (1967) and Zou (1997); the psychological benefits (costs) of foreign asset holding (foreign borrowing) in Blanchard (1983); and wealth effects or social-status effects of wealth in Kurz (1968), Frank (1985), Cole et al. (1992), Zou (1994, 1995b), and Bakshi and Chen (1996). Our approach includes the existing utility functions in the arms-race literature as special cases when total wealth is just weapons. Of course, the general results derived from our extended model can apply to those special cases. In particular, our analysis will continue to hold if we assume that the utility function is defined only on weapons stocks such as $u(c, m, m^*)$. In our current context, we will continue to assume that the utility function $u(c, w, w^*)$ still has the following properties as in the last section:

$$u_1 > 0, \quad u_2 > 0, \quad u_3 < 0, \quad u_{11} < 0, \quad u_{22} < 0,$$
$$u_{12} = u_{21} > 0, \quad u_{13} = u_{31} < 0, \quad u_{23} = u_{32} > 0. \tag{17}$$

With slight modification, the foreign country's total wealth, w^*, is assumed to follow a Brownian motion,

$$\mathrm{d}w^* = \alpha_{w^*} w^* \,\mathrm{d}t + \sigma_{w^*} w^* \,\mathrm{d}z^*, \tag{18}$$

where the stochastic term $\mathrm{d}z^*$ is assumed to be a temporally independent, normally distributed with mean zero and variance $\mathrm{d}t$.

Now, the budget constraint for the home country is

$$\mathrm{d}w = \mathrm{d}k + \mathrm{d}m = \mathrm{d}Y - c\,\mathrm{d}t, \tag{19}$$

which states that the net increase in the home country's wealth (capital and weapons) is its net savings (output minus consumption).

The home country chooses its capital stock, weapon stock, and consumption to maximize its discounted utility, namely,

$$\max \mathrm{E}_0 \int_0^\infty u(c, w, w^*)\mathrm{e}^{-\rho t}\,\mathrm{d}t$$

subject to the budget constraint (19) and the initial stocks given by $k(0)$ and $m(0)$, respectively.

3.1. Optimalities

We define the discounted value function $\bar{V}(w, w^*, t)$ to be

$$\bar{V}(w, w^*, t) = \bar{X}(w, w^*)\mathrm{e}^{-\rho t}.$$

Denote the share of the weapons stock in total wealth as

$$n = \frac{m}{k+m}.$$

The home country chooses the share of the weapons stock in total wealth, n, and consumption path, $c(t)$, to maximize the following expression:

$$\begin{aligned} &u(c, w, w^*) - \rho\bar{X} + \bar{X}_w(F((1-n)w) - c) + \bar{X}_{w^*}\alpha_{w^*}w^* \\ &\quad + \tfrac{1}{2}\bar{X}_{ww^*}\sigma_{yz^*}H((1-n)w)w^* + \tfrac{1}{2}\bar{X}_{ww}H((1-n)w)^2\sigma_y^2 + \tfrac{1}{2}\bar{X}_{w^*w^*}\sigma_{w^*}^2 w^{*2}. \end{aligned}$$

The conditions for the optimization problem are

$$\frac{\partial u(c, w, w^*)}{\partial c} = \bar{X}_w, \tag{20}$$

$$\begin{aligned} &-\bar{X}_w F'((1-n)w) - \tfrac{1}{2}\bar{X}_{ww^*}\sigma_{yz^*}H'((1-n)w)w^* \\ &-\bar{X}_{ww}H((1-n)w)H'((1-n)w)\sigma_y^2 = 0. \end{aligned} \tag{21}$$

From Eqs. (20) and (21), we can derive the optimal choices for the weapons share and the consumption path as the functions of $\bar{X}_w$, $\bar{X}_{ww^*}$, and $\bar{X}_{ww}$. With the substitution of the optimal values for the weapons share and consumption, the value function must satisfy the following Bellman equation:

$$u(c,w,w^*) - \rho\bar{X} + \bar{X}_w(F((1-n)w) - c) + \bar{X}_{w^*}\alpha_{w^*}w^* + \tfrac{1}{2}\bar{X}_{ww^*}\sigma_{yz^*}H((1-n)w)w^* + \tfrac{1}{2}\bar{X}_{ww}H((1-n)w)^2\sigma_y^2 + \tfrac{1}{2}\bar{X}_{w^*w^*}\sigma_{w^*}^2 w^{*2} = 0. \tag{22}$$

3.2. Explicit solutions

To derive the explicit solutions for the weapons share and consumption path, we specified the utility function as

$$u(c,w,w^*) = \frac{c^{1-\xi}}{1-\xi}\left(\frac{w}{w^*}\right)^{-\eta}, \tag{23}$$

where ξ and η satisfy the following conditions: if $0 < \xi < 1$, then $-1 < \eta < 0$; if $\xi > 1$, then $\eta > 0$. These conditions guarantee that the utility function is increasing and concave in the relative wealth ratio of the home country to over the foreign country, (w/w^*).

The home country's production technology is the same as in the last section:

$$F(k) = Ak, \quad H(k) = Ak$$

and

$$\mathrm{d}Y = Ak\,\mathrm{d}t + Ak\,\mathrm{d}y.$$

Given the specified utility function (23), the value function is conjectured as

$$\bar{X}(w,w^*) = \chi w^{1-\xi-\eta}(w^*)^{\eta}, \tag{24}$$

where the coefficient χ is to be determined.

Taking partial derivations, we have

$$\bar{X}_w = \chi(1-\xi-\eta)w^{-\xi-\eta}(w^*)^{-\eta}, \quad \bar{X}_{ww} = \chi(1-\xi-\eta)(-\xi-\eta)w^{-\xi-\eta-1}(w^*)^{\eta},$$

$$\bar{X}_{w^*} = \chi\eta w1^{-\xi-\eta}(w^*)^{\eta-1}, \quad \bar{X}_{ww^*} = \chi(1-\xi-\eta)\eta w^{-\xi-\eta}(w^*)^{\eta-1}.$$

Substituting the above expressions of partial derivatives into Eqs. (20) and (21), we have

$$c^{-\xi} = \chi(1-\xi-\eta)w^{-\xi}$$

or

$$\frac{c}{w} = (\chi(1-\xi-\eta))^{-1/\xi}, \tag{25}$$

$$-(1-\xi-\eta)A\chi w^{1-\xi-\eta}(w^*)^{\eta} - \tfrac{1}{2}(1-\xi-\eta)\eta\sigma_{yz^*}\chi w^{1-\xi-\eta}(w^*)^{\eta} + (1-\xi-\eta)(\xi+\eta)\chi w^{1-\xi-\eta}(w^*)^{\eta}(1-n)\sigma_y^2 = 0$$

or

$$-(1-\xi-\eta)A-\tfrac{1}{2}(1-\xi-\eta)\eta\sigma_{yz^*}+(1-\xi-\eta)(\xi+\eta)(1-n)\sigma_y^2=0. \tag{26}$$

Substituting Eqs. (25) and (26) into the Bellman equation yields

$$\begin{aligned}
&(\chi(1-\xi-\eta))^{-(1-\xi)/\xi}\frac{w^{1-\xi}}{1-\xi}\left(\frac{w}{w^*}\right)^{-\eta}-\rho\chi w^{1-\xi-\eta}(w^*)^{\eta}+\chi\eta w^{1-\xi-\eta}(w^*)^{\eta-1}\alpha_{w^*}w^*\\
&\quad+\chi w^{1-\xi}\left(\frac{w}{w^*}\right)^{-\eta}(1-\xi-\eta)(A(1-n)-(\chi(1-\xi-\eta))^{-1/\xi})\\
&\quad+\frac{1}{2}\chi(1-\xi-\eta)\eta(1-n)w^{1-\xi}\left(\frac{w}{w^*}\right)^{-\eta}\sigma_{yz^*}\\
&\quad-\frac{1}{2}(1-\xi-\eta)(\xi+\eta)\chi(1-n)^2w^{1-\xi}\left(\frac{w}{w^*}\right)^{-\eta}\sigma_y^2\\
&\quad+\frac{1}{2}\eta(\eta+1)\sigma_{w^*}^2\chi w^{1-\xi}\left(\frac{w}{w^*}\right)^{-\eta}=0.
\end{aligned} \tag{27}$$

From the above equation, we have

$$\begin{aligned}
&(\chi(1-\xi-\eta))^{-1/\xi}\\
&\quad=\frac{(1-\xi-\eta)\frac{1}{2}[(\xi+\eta)(1-n)^2\sigma_y^2-\eta(1-n)\sigma_{yz^*}-2A(1-n)]}{(1-\xi-\eta)\xi/(1-\xi)}\\
&\qquad+\frac{\rho-\eta\alpha_{w^*}-\frac{1}{2}\eta(\eta+1)\sigma_{w^*}^2}{(1-\xi-\eta)\xi/(1-\xi)}.
\end{aligned} \tag{28}$$

Substitution Eq. (28) into Eq. (25), we have

$$\frac{c}{w}=\frac{\rho-\eta\alpha_{w^*}+(1-\xi-\eta)\frac{1}{2}[(\xi+\eta)(1-n)^2\sigma_y^2-\eta(1-n)\sigma_{yz^*}-2A(1-n)]-\frac{1}{2}\eta(\eta+1)\sigma_{w^*}^2}{(1-\xi-\eta)\xi/(1-\xi)}$$

and from Eq. (26) we can determine the optimal weapons share in total wealth n as

$$n=\frac{-A-\frac{1}{2}\eta\sigma_{yz^*}}{(\xi+\eta)\sigma_y^2}+1.$$

Similarly, we get the mean growth rate of the economy, denoted as ϕ_2,

$$\phi_2=\mathrm{E}\left(\frac{\mathrm{d}w/\mathrm{d}t}{w}\right)=\left(A(1-n)-\frac{c}{w}\right). \tag{29}$$

The transversality condition in this case is

$$\lim_{t\to\infty}\mathrm{E}[\delta w^{1-\xi-\eta}(w^*)^{-\eta}\mathrm{e}^{-\rho t}]=0,$$

which is also equivalent to the positivity of the consumption–wealth ratio.

3.3. Comparative dynamics

As in Section 2.3, we first examine how the change in the mean growth of the foreign wealth and weapons stock affects the economic growth of the home country. From (29) and the corresponding optimal conditions for c/w and n, we have

$$\frac{\partial \phi_2}{\partial \alpha_{w^*}} = \frac{(1-\xi)\eta}{\xi(1-\xi-\eta)}.$$

Because $\eta < 0$ if $0 < \xi < 1$; and $\eta > 0$ if $\xi > 1$, we obtain

$$\frac{\partial \phi_2}{\partial \alpha_{w^*}} > 0,$$

when $\xi > 1$, and $\eta > 0$;

$$\frac{\partial \phi_2}{\partial \alpha_{w^*}} < 0$$

when $0 < \xi < 1$ and $\eta < 0$.

This result is quite similar to the one in Section 2 when we treat military spending as a consumption good: a rise in the growth of foreign wealth and weapons stock raises the home country's economic growth, i.e., capital and weapon growth, if the home country has a lower elasticity of intertemporal substitution in consumption.

As for the stochastic shocks to the foreign wealth, their effect on the home country's economic growth is given by $\partial\phi_2/\partial\sigma_{w^*}^2$

$$\frac{\partial \phi_2}{\partial \sigma_{w^*}^2} = \frac{\frac{1}{2}(1-\xi)\eta(\eta-1)}{\xi(1-\xi-\eta)}.$$

Therefore,

$$\frac{\partial \phi_2}{\partial \sigma_{w^*}^2} < 0$$

when $\xi > 1$ and $0 < \eta < 1$;

$$\frac{\partial \phi_2}{\partial \sigma_{w^*}^2} > 0$$

when $0 < \xi < 1$ and $-1 < \eta < 0$; and when $\xi > 1$ and $\eta > 1$.

Even though the expressions seem more complicated than in the case of military spending as a consumption good, the economic reasoning and intuition are almost identical. If the home country's elasticity of intertemporal substitution in consumption is relatively larger, it reacts to rising volatility in the foreign military and capital stocks by cutting consumption and investing more in arms and capital accumulation, and a higher economic growth rate follows in the home country. On the other hand, with a lower elasticity of intertemporal substitution in consumption, the home country will raise spending on consumption and weapons and cut capital investment as a result of rising volatility in foreign military and capital stocks. Hence, the home country's economic growth suffers. Furthermore, if the home country derives a higher intensity of utility from security and relative economic and military power measured by η in the expression of $(1-\xi)^{-1}(w/w^*)^{-\eta}$ in our model, then a higher value of η $(\eta > 1)$ can lead to higher economic growth even with a relatively lower elasticity of intertemporal substitution, i.e., $\xi > 1$.

Once more, we notice that the mean growth and stochastic shocks in foreign capital and weapons stocks have opposite effects on the home country's economic growth.

Our findings in this section and the last section regarding the effects of a foreign military threat on domestic economic growth stand in sharp contrast to Zou's (1995a). In a first deterministic dynamic optimization framework with both investment and military spending, Zou finds that, when the utility function is separable between consumption and the weapon stocks, an unanticipated rise in current military threat reduces current investment and an anticipated rise in future military threat stimulates current investment. But in the long run capital accumulation and output production is independent of the military conflicts among nations regardless of the forms of the utility function. The introduction of stochastic elements to the model has offered new and deep insights on the interaction between military spending and capital accumulation.

For stochastic shocks to domestic output production, σ_y^2, their effects on economic growth are not clear-cut because they may interact with the foreign capital and weapons stocks. But, when $\sigma_{yz^*} = 0$, the effects of σ_y^2 on output growth are given by

$$\frac{\partial \phi_2}{\partial \sigma_y^2} = -\frac{A^2}{(\xi+\eta)(\sigma_y^2)^2}\,\frac{\xi+1}{2\xi}.$$

Hence

$$\frac{\partial \phi_2}{\partial \sigma_y^2} > 0$$

when $0 < \xi < 1$ and $-1 < \eta < -\xi$. And

$$\frac{\partial \phi_2}{\partial \sigma_y^2} < 0$$

when $0 < \xi < 1$ and $0 > \eta > -\xi$; or when $\xi > 1$ and $\eta > 0$. The general lesson is again quite clear: stochastic shocks to output production may raise precautionary savings and investment and accelerate output growth under certain circumstances. Our analysis involving both capital and arms accumulation offers further insights on the ongoing theoretical and empirical inquiry into volatility and output growth with an additional perspective on national security and output production by modeling the interaction between military spending and productive investment.

4. Concluding remarks

This study has examined capital accumulation, military spending, arms accumulation, and output growth in a stochastic endogenous growth model. The analysis shows that higher (lower) growth in foreign military spending leads to faster (slower) economic growth in the home country if the home country's intertemporal substitution elasticity in consumption is smaller (larger); but more volatility in foreign military spending can lead to higher economic growth in the home country when its intertemporal substitution elasticity is large. In addition, shocks to output production may stimulate economic growth.

Given these highly complicated theoretical relationships between military spending and output growth, it is not strange at all that we can see both negative and positive associations between economic growth and military spending in many cross-country, time-series studies in 1970s–1990s; see Landau (1993) and Ram (1995) for a review on the empirics on military expenditures and economic growth. To single out a few recent cases, a positive association between military spending and growth is found by Stewart (1991), Mueller and Atesoglu (1993), Macnair et al. (1995); a negative association is found by Mintz and Huang (1990), and Ward and Davis (1992); a nonlinear relationship is found by Landau (1993); and no significant association is found by Mintz and Stevenson (1995).

Our theoretical relationships are obtained without taking into consideration of the often-mentioned military spinoffs of positive externalities on output production though education, research and development, and technological innovations. A straightforward extension of our model is to introduce military spending into the production. It is expected that the positive effect of military sector on economic growth is stronger, but analytical (explicit) solutions in the stochastic setting are more difficult to obtain when the production function has two inputs: capital and weapons stocks.

To have a more realistic picture of the power struggle in international politics, it also seems desirable to extend our model to consider the two-country dynamic equilibrium in arms race and capital accumulation without assuming the foreign country's action as an exogenous stochastic process. The analytical tool for a much simpler environment of financial market equilibrium with two-person dynamics is provided by Dumas (1989), but the military threat to each other in our two-country case will be an analytical challenge if we want to have explicit solutions as in Dumas.

While the Cold War has ended, ethnic conflicts have erupted into civil wars in eastern Europe, central Asia, and, especially, Africa. The economic causes and consequences of civil conflicts and violence have received considerable attention in recent years, and, of the 27 major armed conflicts that occurred in 1999, all but two took place within national boundaries (Collier, 1999, 2000). Our model can be viewed as an initial approach to exploring how ethnic and civil conflicts affect military spending and economic growth within a nation.

Finally, the rise and decline of nations and great powers in the last five hundred years have been extensively studied by various authors, and two notable cases are Olson (1982) and Kennedy (1987). To us, a serious attempt to the rise and decline of nations and powers should necessarily incorporate production technology and military technology in dynamic models. This is a topic we should pursue in the near future. To be honest, as long as military conflicts among nations and within nations are a constant companion of human beings, the study on arms and economic growth will have its permanent value in our effort to understand the economic, political, and social behavior of the human race.

5. For further reading

The following references may also be of interest to the reader: Brito and Intriligator (1995); Cole et al., 1995; Fershtman and Weiss 1993; Fershtman et al., 1996; Merton,1971.

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