

Optimal monetary policy in open economies: the role of reference currency in vertical production and trade

Chan Wang^a, Heng-fu Zou^{a,b,c}

^a*China Economics and Management Academy, Central University of Finance and Economics, Beijing, 100081, China*

^b*Institute for Advanced Study, Wuhan University, Wuhan, 430072, China*

^c*Institute for Advanced Study, Shenzhen University, Shenzhen, 518060, China*

Abstract

This paper examines optimal monetary policy rules in open economies with vertical production and trade in which we emphasize the role played by reference currency. As evidenced by empirical findings, we assume final goods prices are sticky, but intermediate goods prices are flexible. We find that the asymmetry of exporters' pricing behavior implies that the responses of monetary authorities to productivity shocks from the stage of final goods production are asymmetric but symmetric to productivity shocks from the stage of intermediate goods production. We also find that gains from cooperation are related to the covariance of productivity shocks in two stages. In addition, we give the conditions under which home and foreign are willing to take part in cooperation respectively. As for exchange rate policy, we find that the volatility of nominal exchange rate in RCP case is greater than that in LCP case, but smaller than that in PCP case. The volatility of real exchange rate in RCP case is, however, greater than those in PCP and LCP cases.

Keywords: Vertical production and trade, Reference-currency pricing, Optimal monetary policy, Monetary cooperation, Exchange rates

JEL classification: E5, F3, F4

1. Introduction

Currently, global economy is increasingly integrated by vertical production and trade processes. It means that the producer in a country not only uses home but also foreign intermediate goods to produce final goods, which are consumed by both home and foreign households. Using input-output tables from 10 OECD and four emerging market countries, Hummels et al. (2001) find that vertical specialization accounts for 21% of these countries's export, and grows almost 30% from 1970 to 1990. In addition, Feestra (1998), Hummels et al. (1998), Yi (2003), and Bridgman (2012) also emphasize the importance of vertical production and trade in the development of world economy.

Another important fact is that US dollar plays a dominant role in world economy, which is empirically relevant but has received limited attention in the literature. Using quarterly data of 23

*Corresponding author. Tel.: +86 1062288992; fax: + 86 1062288376.

Email addresses: wangchanist@gmail.com (Chan Wang), Hzoucema@gmail.com (Heng-fu Zou)

OECD countries from 1975 to 2003, Campa and Goldberg (2005) provide cross-country and time series evidence on pass-through into the import prices. They find that US has the lowest pass-through rates among the OECD countries. Gopinath, Itskhoki and Rigobon (2010) show that the exchange rate pass-through of the average good priced in dollars is much lower than that priced in nondollars. Goldberg and Tille (2008, 2009) provide the empirical evidence that clearly highlights the global role of the dollar. They find that dollar is used in the invoicing of trade not only between US and its trading partners but also trading partners having nothing to do with US. In addition, Shi and Xu (2010, hereafter SX) study the problem of twin dollarization which is motivated by the observation in East Asian economies, which not only borrow bonds denominated in US dollars but also price their export goods in US dollars. In terms of Devereux, Shi and Xu (2007, hereafter DSX), US dollar plays the role of reference currency with which both home and foreign exporters set prices.

How does the reference currency in the economy with vertical production and trade shape the optimal monetary policies? In order to answer this question, we introduce reference currency into a model with vertical production and trade similar to SX (2007). In addition, following Devereux and Engel (2007, hereafter DE), we assume that final goods prices are sticky, but intermediate goods prices are flexible, which is supported by vast of empirical literature¹.

Our research follows the tradition initiated by Friedman (1953), Mundell (1961) and Feldstein (1992) who believe that, when nominal goods prices are sticky, freely floating exchange rates can achieve relative price adjustment between countries. The reason is that flexible exchange rates can deal with real country-specific productivity or demand shocks. More recently, by combining intertemporal choice and nominal rigidities, Obstfeld and Rogoff (2000, hereafter OR) propose main framework to analyze optimal monetary policy rules in open economies, and conclude that a global monetary policy replicating the allocations under flexible wages is efficient. OR (2002) generalize OR(2000) to the case with incomplete international asset markets. If the shocks are global, optimal monetary policy rules involve replicating the allocations under flexible wages, furthermore, there are no gains from cooperation. By comparison, if shocks are country specific, optimal monetary policy rules cannot replicate the allocations under flexible wages, and gains from cooperation, though quantitatively small, can arise. However, OR's conclusions are based on the assumption that exporters set prices in the currency of the producers (PCP). DE (2003) not only consider the PCP case but also the situation in which exporters set prices in consumers' currency (LCP). In PCP case, optimal monetary policy rules can replicate flexible price allocations, and flexible exchange rate is optimal. Comparatively, in LCP case, optimal monetary policy rules cannot replicate flexible price allocations and fixed exchange rate is optimal. Building on DE (2003), DSX (2007) analyze the case in which both home and foreign exporters set prices in reference currency (denoted as reference currency pricing or RCP). When the coefficient of relative risk aversion is unity, foreign only responds to its own domestic shock, but home responds to both home and foreign shocks with response parameters being equal to their weights in world output.

The introduction of reference currency into the economy with vertical production and trade, together with the assumption that final goods prices are sticky, but intermediate goods prices are flexible, change the optimal monetary policy prescriptions given in the literature. When there is a positive productivity shock in the stage of intermediate goods production, no matter where it

¹A nonexhaustive list includes: Murphy et al. (1989), Clark (1999), Bils and Klenow (2004), Nakamura and Steinsson (2008)

comes from, both home and foreign monetary authorities expand money supply with the degree of expansion being equal to their weights in world output. Optimal monetary responses in our model are different from those in PCP case of SX (2007). In their model, due to transborder spillover effect, both home and foreign expand money supply as responses. However, with expenditure-switching effect being dominating the indirect positive transborder spillover effect, home monetary authority's response to productivity shock from home exceeds that of the foreign. When SX (2007) extend their model to consider the situation in which LCP is assumed in the stage of intermediate goods production but PCP in the final goods stage, optimal monetary responses are identical to ours. However, in their model, aiming at increasing aggregate demands, both home and foreign expand money supply as a response to a positive productivity shock. In our model, since intermediate goods prices are flexible, monetary expansion can not only increase aggregate demands but also influence relative prices.

When positive productivity shock occurs in the stage of final goods production, as DSX (2007), foreign only responds to its own domestic shock, but home responds to both home and foreign shocks with response parameters being equal to their weights in world output. Thus, our optimal monetary policy prescription is different from OR(2002), the PCP case of DE(2003) and Corsetti and Pesenti(2005, hereafter, CP). in which monetary policy involves that home responds only to its own domestic productivity shock. Our prescription also contrasts with SX (2007), in which home should respond to both home and foreign productivity shocks.

Therefore, in our model, the asymmetry of exporters' pricing behavior produces the asymmetric responses of home and foreign monetary authorities to the productivity shocks in the stage of final goods production. However, as for productivity shocks in the stage of intermediate goods production, home and foreign monetary authorities respond symmetrically.

How does reference currency in vertical trade and production influence international monetary policy cooperation? In traditional literature, the flexibility of exchange rate implies that gains from cooperation are likely to be small. In OR (2002), as mentioned above, when shocks are country specific, gains from cooperation exist, but they are quantitatively small. However, the research following OR (2002) finds that gains from cooperation depend on the following factors²: (1) the elasticity of exchange rate pass-through (CP 20005); (2) the intertemporal elasticity and the elasticity of substitution between goods produced by home and foreign (Clarida et al., 2002; Benigno and Benigno, 2003; Pappa, 2004); (3) imperfect correlation between domestic shocks across sectors (Canzoneri et al., 2005); (4) policymakers' imperfect information (Dellas 2006); (5) asymmetric trading structures (Liu and Pappa, 2008).

In our model, gains from cooperation depend on the covariance of productivity shocks in two stages. When the covariance is greater than zero, gains from cooperation arise. In addition, foreign is willing to cooperate unconditionally, but home is willing to only conditionally. If the covariance is less than zero, only when the variance of the shocks exceeds a critical value do gains from cooperation exist. In this circumstance, foreign, as before, is willing to cooperate, on the contrary, home refuses to cooperate. The key to produce gains from cooperation is that home monetary authority's Nash response to productivity shock in the stage of home final goods production will produce positive externality to foreign, by contrast, its Nash response to productivity shock in the stage of foreign final goods production can produce negative externality to foreign.

Choosing which currency to set prices, in the open economy macroeconomics models with nom-

²The list is within our knowledge.

inal rigidities, is a key element to determine optimal exchange rate policy. In PCP case of DE (2003), flexible exchange rate is optimal. By comparison, in LCP case, constant exchange rate is optimal. CP (2005) also reach the same conclusion. SX (2007) find, though optimal exchange rate being flexible, vertical production and trade can reduce its volatility. DE (2007) conclude that exchange rate policy involves a trade-off between smoothing fluctuations in real exchange rates to reduce distortions in consumption allocations and keeping nominal exchange rates flexible to adjust terms of trade. In our model, we find that the volatility of nominal exchange rate in RCP case is greater than that in LCP case, but smaller than that in PCP case. The volatility of real exchange rate in RCP is, however, greater than that in PCP and LCP cases. The reason is that the main welfare loss in RCP case is that monetary authorities cannot use expenditure-switching effect to change consumption demands of home households. As a result, they allow for some flexibility of real exchange rate to exchange for more powerful expenditure-switching effect in foreign. The comparisons between volatilities of nominal and real exchange rate in cooperative equilibrium and those in Nash equilibrium justify our claims.

The paper is organized as follows. Section 2 presents the basic model and solves for a flexible price equilibrium. Section 3 analyzes optimal monetary policy rules and their implications. Section 4 concludes.

2. The Model

We consider a static, two-country model with vertical production and trade. The model's structure is similar to that of SX (2007), but as DSX (2007), we introduce asymmetry in pricing behavior of exporters in home and foreign country in the sense that home exporters set prices in the currency of the producers (PCP), foreign exporters, however, set prices in the currency of the consumers (LCP). In terms of DSX (2007), home currency is the reference currency. In the following, we call the pricing scheme in which home and foreign exporters set prices in terms of reference currency as reference currency pricing (RCP).

The two countries, home and foreign, are both populated by a continuum of households of measure 1. The production sectors, both in home and foreign, are separated into two parts, one makes differentiated intermediate goods and the other uses home and foreign intermediate goods as inputs to make final goods which are consumed by the households worldwide.

Households enter bond market to trade in a full set of nominal state-contingent bonds before the period begins. By doing this, households can insure against risks incurred by productivity shocks worldwide, and also monetary policy uncertainty resulting from central banks' responses to productivity shocks worldwide. When the period begins, households' income is governed by an optimal risk-sharing rule. At this stage of the period, central banks announce optimal monetary policy rules, considering risk sharing between home and foreign, the way by which firms set prices, and the distribution of stochastic productivity shocks. Throughout the period, we assume that monetary authorities are able to commit to optimal monetary policy rules. Then firms set prices, consumption and production decisions are made, and the exchange rate is determined.

2.1. Households

The expected utility of representative home household i is

$$U(i) = E \left[\ln C(i) + \chi \ln \frac{M(i)}{P} - \eta L(i) \right],$$

in which $\frac{M(i)}{P}$ is the real money balances, $L(i)$ is the labor supply, and real consumption index $C(i)$ has an Armington form,

$$C(i) = 2C_H^{1/2}(i)C_F^{1/2}(i).$$

Consumption subindexes $C_H(i)$ and $C_F(i)$ are defined respectively by

$$C_H(i) = \left[\int_0^1 C_H(i, j_f)^{\frac{\lambda-1}{\lambda}} dj_f \right]^{\frac{\lambda}{\lambda-1}}, C_F(i) = \left[\int_0^1 C_F(i, j_f^*)^{\frac{\lambda-1}{\lambda}} dj_f^* \right]^{\frac{\lambda}{\lambda-1}},$$

where $\lambda > 1$ is the elasticity of substitution between final goods³, j_f denotes home final good j (thereafter, subscript f denotes final good, subscript i intermediate good), j_f^* denotes foreign final good j (throughout, asterisks denote foreign variables).

Home price index for overall real consumption $C(i)$ is

$$P = (P_{HHf})^{1/2} (P_{FHf})^{1/2}, \quad (1)$$

in which P_{HHf} (P_{FHf}) denotes price subindex for home (foreign) final goods sold in home. Home demand functions can be derived from cost minimization and are

$$C_H(i, j_f) = \frac{1}{2} \left[\frac{P_{HHf}(j_f)}{P_{HHf}} \right]^{-\lambda} \left[\frac{P_{HHf}}{P} \right]^{-1} C(i),$$

$$C_F(i, j_f^*) = \frac{1}{2} \left[\frac{P_{FHf}(j_f^*)}{P_{FHf}} \right]^{-\lambda} \left[\frac{P_{FHf}}{P} \right]^{-1} C(i).$$

Due to asymmetry in pricing behavior of exporters in home and foreign, foreign price index is

$$P^* = \left(\frac{P_{HFf}}{S} \right)^{1/2} (P_{FFf}^*)^{1/2}, \quad (2)$$

in which P_{HFf} (P_{FFf}^*) denotes price subindex for home (foreign) final goods sold in foreign, S is the home currency price of foreign currency.

The budget constraint for home representative household i can be written as

$$P(z)C(i, z) + M(i, z) + \sum_{z' \in Z} q(z')B(i, z') = W(z)L(i, z) + \Pi(i, z) + B(i, z) + M_0 + T(i, z),$$

where $z \in Z$ represents some particular state z , and Z is the set of all states. $B(i, z')$ is the amount of bonds held by household i which entitles household i to be paid B units of home currency when state z' occurs, and $q(z')$ is the home currency price of the state contingent bond; $W(z)L(i, z)$ is nominal wage income, $\Pi(i, z)$ represents profits from the ownership of home firms which distribute their profits among domestic households equally; $T(i, z)$ is lump-sum transfer

³Here we follow Tille (2001), cross-country substitutability is smaller than within-country substitutability.

from home government which rebates its seigniorage revenue to home households equally, it means $M(z) - M_0 = T(z)$; M_0 is initial holdings of nominal money balances.

We can solve home household i 's time-0 problem and obtain the following first-order conditions:

$$M(i, z) = \chi P(z) C(i, z), W(z) = \eta P(z) C(i, z).$$

The above two equations are intratemporal optimization conditions to ensure that marginal utilities from holding an extra unit of money or enjoying one additional unit of leisure are equal to marginal costs measured by forgone consumption respectively.

In addition, risk sharing between home and foreign households will equalize their marginal utilities from holding one unit of nominal state-contingent bond. It means that the following risk-sharing condition holds:

$$\Gamma P(z) C(i, z) = S(z) P^*(z) C^*(i^*, z) \quad \text{for any } i \text{ and } i^*,$$

in which Γ is the ratio of home and foreign households' Lagrange multipliers and is determined in an equilibrium of the market for state-contingent nominal bonds. And as shown in the appendix of DE(2003), $\Gamma = 1$ holds when log utility function of consumption is assumed. However, it is noteworthy that, in general, Γ differs from 1 to reflect the difference between home and foreign which is caused by asymmetry in pricing behavior of exporters in home and foreign.⁴

In symmetric equilibrium, optimization conditions and risk-sharing condition can be written as:

$$M = \chi PC \tag{3}$$

$$W = \eta PC \tag{4}$$

$$PC = SP^*C^* \tag{5}$$

2.2. Firm

Home final goods are produced by a continuum of firms indexed by $j_f \in [0, 1]$ with production function

$$Y_{Hf}(j_f) = 2\theta_f Y_{Hi}(j_f)^{1/2} Y_{Fi}(j_f)^{1/2}.$$

In which θ_f is home productivity shock in the stage of final goods production, $Y_{Hi}(j_f)$ ($Y_{Fi}(j_f)$) is an index which bundles differentiated home (foreign) intermediate goods together and is given by

$$\begin{aligned} Y_{Hi}(j_f) &= \left[\int_0^1 Y_{Hi}(j_f, j_i)^{\frac{\phi-1}{\phi}} dj_i \right]^{\frac{\phi}{\phi-1}}, \phi > 1 \\ \left(Y_{Fi}(j_f) &= \left[\int_0^1 Y_{Fi}(j_f, j_i^*)^{\frac{\phi-1}{\phi}} dj_i^* \right]^{\frac{\phi}{\phi-1}}, \phi > 1. \right) \end{aligned}$$

⁴DSX(2007) consider a more general case in which Γ differs from 1.

The unit cost function of home representative firm $j_f \in [0, 1]$ is $C(P_{HHi}, P_{FHi}) = \frac{P_{HHi}^{1/2} P_{FHi}^{1/2}}{\theta_f}$. The demands for home and foreign intermediate goods baskets by firm j_f are respectively:

$$Y_{Hi}(j_f) = \frac{1}{2} \left(\frac{P_{HHi}}{C(P_{HHi}, P_{FHi})} \right)^{-1} Y_{Hf}(j_f),$$

$$Y_{Fi}(j_f) = \frac{1}{2} \left(\frac{P_{FHi}}{C(P_{HHi}, P_{FHi})} \right)^{-1} Y_{Hf}(j_f).$$

The expressions for $Y_{Hi}(j_f)$ and $Y_{Fi}(j_f)$ imply that the demand for home intermediate good j_i by home final good firm j_f is

$$Y_{Hi}(j_f, j_i) = \frac{1}{2} \left(\frac{P_{HHi}(j_i)}{P_{HHi}} \right)^{-\phi} \left(\frac{P_{HHi}}{C(P_{HHi}, P_{FHi})} \right)^{-1} Y_{Hf}(j_f),$$

for foreign intermediate good j_i^* is

$$Y_{Fi}(j_f, j_i^*) = \frac{1}{2} \left(\frac{P_{FHi}(j_i^*)}{P_{FHi}} \right)^{-\phi} \left(\frac{P_{FHi}}{C(P_{HHi}, P_{FHi})} \right)^{-1} Y_{Hf}(j_f).$$

Similarly, differentiated intermediate goods are produced by a continuum of firms indexed by $j_i \in [0, 1]$ with production function

$$Y_{Hi}(j_i) = \theta_i L(j_i),$$

where θ_i is home productivity shock in the stage of final-goods production, $L(j_i)$ is firm j_i 's labor demand which is provided by home households.

Following SX(2007), we assume that $\theta_f = \exp(u)$ and $\theta_i = \exp(v)$, in which u and v follow normal distribution with zero mean and variance-covariance matrix given by

$$\Sigma = \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{pmatrix}.$$

Foreign productivity shocks are also assumed to have the above properties. To keep calculations as simply as possible, we assume $\sigma_u^2 = \sigma_v^2 = \sigma_{u^*}^2 = \sigma_{v^*}^2 = \sigma^2$ and $\sigma_{uv} = \sigma_{u^*v^*}$. Thus, $-\sigma^2 \leq \sigma_{uv} \leq \sigma^2$ follows.

2.3. Flexible price equilibrium

We first solve flexible price equilibrium as a benchmark. When prices in both stages of production are flexible, firms set prices after global shocks are realized. The solutions for optimal pricing rules are given in Table 1. The prices are all set on the basis of cost functions adjusted by markup to reflect firms' market power. Since foreign exporters set prices in the currency of the consumers, their prices are adjusted by nominal exchange rate.

Market clearing condition in the home country can be written as

Table 1: Optimal prices for flexible price case

$P_{HHf} = \frac{\lambda}{\lambda-1} \frac{P_{HHi}^{1/2} P_{FHi}^{1/2}}{\theta_f}$	$P_{HFf} = \frac{\lambda}{\lambda-1} \frac{P_{HHi}^{1/2} P_{FHi}^{1/2}}{\theta_f}$
$P_{FHf} = \frac{\lambda}{\lambda-1} \frac{S \left(\frac{P_{HFi}}{S} \right)^{1/2} (P_{FFi}^*)^{1/2}}{\theta_f^*}$	$P_{FFf}^* = \frac{\lambda}{\lambda-1} \frac{\left(\frac{P_{HFi}}{S} \right)^{1/2} (P_{FFi}^*)^{1/2}}{\theta_f^*}$
$P_{HHi} = \frac{\phi}{\phi-1} \frac{W}{\theta_i}$	$P_{HFi} = \frac{\phi}{\phi-1} \frac{W}{\theta_i}$
$P_{FHi} = \frac{\phi}{\phi-1} \frac{W^*}{\theta_i^*} S$	$P_{FFi}^* = \frac{\phi}{\phi-1} \frac{W^*}{\theta_i^*}$

$$\theta_i L = \frac{1}{4} \frac{C(P_{HHi}, P_{FHi})}{P_{HHi}} \left(\frac{PC}{P_{HHf}} + \frac{SP^*C^*}{P_{HFf}} \right) + \frac{1}{4} \frac{SC^* \left(\frac{P_{HFi}}{S}, P_{FFi}^* \right)}{P_{HFi}} \left(\frac{PC}{P_{FHf}} + \frac{P^*C^*}{P_{FFf}^*} \right). \quad (6)$$

The first term on the right-hand side of the previous equation is the demand for home intermediate goods from home final goods firms, the second term is from foreign final goods producers.

Since M and M^* are chosen by monetary authorities in home and foreign, $\theta_f, \theta_f^*, \theta_i, \theta_i^*$ are exogenous productivity shocks, flexible price equilibrium consists of 17 equations: 8 pricing equations in Table 1, home price index (1) and its foreign equivalent (2), money demand equation (3) and its foreign equivalent, labor supply equation (4) and its foreign equivalent, risk sharing condition (5), market clearing condition (6) and its foreign equivalent. From these 17 equations, 17 endogenous variables $P_{HHf}, P_{HFf}, P_{FHf}, P_{FFf}^*, P_{HHi}, P_{HFi}, P_{FHi}, P_{FFi}^*, S, W, W^*, P, P^*, C, C^*, L, L^*$ can be determined.

Solving flexible price equilibrium system yields

$$\tilde{\tau}_i = \frac{\theta_i}{\theta_i^*}, \quad (7)$$

in which $\tilde{\tau}_i$ is the terms of trade in the stage of intermediate goods production under flexible prices. Similarly, the terms of trade at the stage of final goods production is

$$\tilde{\tau}_f = \frac{\theta_f}{\theta_f^*}. \quad (8)$$

From equations (7) and (8), the terms of trade in two stages all depend on relative productivity shocks of their own stage.

Nominal exchange rate is given by

$$\tilde{S} = \frac{\tilde{M}}{\tilde{M}^*} = \frac{\tilde{W}}{\tilde{W}^*}. \quad (9)$$

Consumption is identical between home and foreign and is given by

$$\tilde{C} = \tilde{C}^* = \left(\eta \hat{\lambda} \hat{\phi} \right)^{-1} \left(\theta_f \theta_f^* \theta_i \theta_i^* \right)^{1/2}. \quad (10)$$

In which $\hat{\lambda} \equiv \frac{\lambda}{\lambda-1}$ is final goods producers' markup, $\hat{\phi}$ is defined similarly. From equation (10), consumption depends on a geometric weighted average of global productivity shocks.

Employment is also equated between home and foreign and has the form

$$\tilde{L} = \tilde{L}^* = \frac{1}{\eta \widehat{\lambda \hat{\phi}}}. \quad (11)$$

It is noteworthy that employment under flexible prices is not affected by global productivity shocks, and the conclusion doesn't hold when we consider a general CRRA utility function of consumption.

PPP holds and real exchange rate is

$$\tilde{R} = \frac{\tilde{S} \tilde{P}^*}{\tilde{P}} = 1. \quad (12)$$

Following the convention initiated by OR (1995, 2000, 2002), we focus on the "real" component of a representative household's utility and assume $\chi \rightarrow 0$. Thus, expected utility under flexible prices is given by

$$E\tilde{U} = E\tilde{U}^* = -\ln(\eta \widehat{\lambda \hat{\phi}}) - \frac{1}{\widehat{\lambda \hat{\phi}}}, \quad (13)$$

in which $-\ln(\eta \widehat{\lambda \hat{\phi}})$ is the expected utility from consumption and $-\frac{1}{\widehat{\lambda \hat{\phi}}}$ from labor.

Comparing the above flexible price allocations with those in SX (2007) when $\rho = 1$, then comparing flexible price allocations in DSX (2007) with those in DE(2003), we have the following

Proposition 1. *RCP in international trade can not change the global allocations and welfare when prices are flexible. It holds for both horizontal and vertical production and trade.*

3. Optimal monetary policy rules

Now, we consider optimal monetary policy rules when final goods prices are sticky, but intermediate goods prices are flexible. This type of price stickiness seems to be supported by US data. Murphy et al. (1989) and Clark (1999) find that final goods prices are significantly less volatile than intermediate goods; Bils and Klenow (2004) estimate that price flexibility of "raw goods" is about 3-4 times than that of processed goods; Nakamura and Steinsson (2008) also find that final goods prices are less volatile than that of intermediate goods.

We assume that monetary authorities can set monetary policy rules with commitment, in the sense explained in DE(2003) that the monetary authorities take account of the effect of their policy rules on households' expected consumptions, since their monetary decisions can influence the levels of pre-set prices. In line with OR(2002),DE(2003),DSX(2007), SX(2007), the optimal monetary policy rules are log-linear functions of global productivity shocks and given by

$$m = a_1 u + a_2 u^* + a_3 v + a_4 v^*, \quad (14)$$

$$m^* = b_1 u + b_2 u^* + b_3 v + b_4 v^*, \quad (15)$$

in which $m = \ln M$. From now on, lower-case letter denotes logarithmic value of a variable.

When final goods prices are sticky, but intermediate goods prices are flexible, clearly, intermediate-goods firms' pricing schemes remain unchanged, final goods firms, however, will choose prices before the realization of the productivity shocks, and their optimal prices are given by Table 2

Table 2: Optimal final goods prices when they are sticky

$P_{HHf} = \frac{\lambda}{\lambda-1} E \left(\frac{P_{HHi}^{1/2} P_{FHi}^{1/2}}{\theta_f} \right)$	$P_{HFf} = \frac{\lambda}{\lambda-1} E \left(\frac{P_{HHi}^{1/2} P_{FHi}^{1/2}}{\theta_f} \right)$
$P_{FHf} = \frac{\lambda}{\lambda-1} E \left(S \frac{\left(\frac{P_{HEi}}{S} \right)^{1/2} (P_{FFi}^*)^{1/2}}{\theta_f^*} \right)$	$P_{FFf}^* = \frac{\lambda}{\lambda-1} E \left(\frac{\left(\frac{P_{HEi}}{S} \right)^{1/2} (P_{FFi}^*)^{1/2}}{\theta_f^*} \right)$

The pricing functions in Table 2 are derived from the problem facing final goods firms who maximize expected discounted profits, using the nominal discount factor of their own country's households to discount. We use four pricing functions in Table 2 to replace their counterparts in Table 1. These four pricing equations, together with other 13 equations in flexible price equilibrium system, form a new equilibrium system.

Now we turn to solve the exchange rate and consumptions in the new equilibrium system. From equation (3) and its foreign equivalent, risk-sharing condition (5), we have

$$s - Es = m - m^*. \quad (16)$$

From equation (1), (3) and the fact that both P_{HHf} and P_{FHf} are predetermined, we have

$$c - Ec = m. \quad (17)$$

From equation (2), (3)'s foreign equivalent, (16) and that P_{HFf} and P_{FFf}^* are sticky, we have

$$c^* - Ec^* = \frac{1}{2}(m + m^*). \quad (18)$$

Observing equations (17) and (18) tells us that, unlike foreign consumption, home consumption is not affected by foreign monetary policy. The reason is that home CPI is predetermined. The same equations are also obtained in DSX (2007).

The objective of each monetary authority is to maximize expected utility of its own country's households. Since we follow the convention of the literature to ignore utility from real balances, it's enough to solve expected utilities from consumption and labor and express them as functions of parameters chosen by monetary authorities.

Following DSX (2007) and SX (2007), home expected employment is identical to its foreign counterpart and has the form

$$EL = EL^* = \frac{1}{\eta \widehat{\lambda} \widehat{\phi}}. \quad (19)$$

Home expected utility from consumption can be expressed as

$$\begin{aligned} Ec = & -\ln(\eta \widehat{\lambda} \widehat{\phi}) - \frac{\sigma_c^2}{2} - \frac{1}{4}(\sigma_u^2 + \sigma_{u^*}^2) - \frac{1}{8}(\sigma_v^2 + \sigma_{v^*}^2) \\ & + \frac{1}{2}(\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}) - \frac{1}{4}(\sigma_{uv} + \sigma_{u^*v^*}), \end{aligned} \quad (20)$$

and its foreign counterpart is

$$\begin{aligned}
Ec^* &= -\ln\left(\eta\widehat{\lambda\phi}\right) - \frac{\sigma_c^2}{2} - \frac{\sigma_s^2}{8} - \frac{1}{4}(\sigma_u^2 + \sigma_{u^*}^2) - \frac{1}{8}(\sigma_v^2 + \sigma_{v^*}^2) + \frac{1}{4}(\sigma_{su} - \sigma_{su^*}) \\
&+ \frac{1}{2}(\sigma_{c^*u} + \sigma_{c^*u^*} + \sigma_{c^*v} + \sigma_{c^*v^*}) - \frac{1}{4}(\sigma_{uv} + \sigma_{u^*v^*}).
\end{aligned} \tag{21}$$

Since home currency is designated as reference currency, home CPI index is insulated from the volatility of exchange rate. As a result, foreign monetary policy can not affect the mean and volatility of home consumption, which is verified by equations (17) and (20). Foreign CPI index, however, is influenced directly by the volatility of exchange rate. Therefore, both home and foreign monetary policies can affect the mean and volatility of foreign consumption, which is showed by equations (18) and (21).

Thus home household's expected utility is

$$\begin{aligned}
EU &= Ec - \frac{1}{\eta}EL = -\ln\left(\eta\widehat{\lambda\phi}\right) - \frac{1}{\eta\widehat{\lambda\phi}} - \frac{\sigma_c^2}{2} - \frac{1}{4}(\sigma_u^2 + \sigma_{u^*}^2) - \frac{1}{8}(\sigma_v^2 + \sigma_{v^*}^2) \\
&+ \frac{1}{2}(\sigma_{cu} + \sigma_{cu^*} + \sigma_{cv} + \sigma_{cv^*}) - \frac{1}{4}(\sigma_{uv} + \sigma_{u^*v^*}),
\end{aligned} \tag{22}$$

it's foreign counterpart is

$$\begin{aligned}
EU^* &= Ec^* - \frac{1}{\eta}EL^* = -\ln\left(\eta\widehat{\lambda\phi}\right) - \frac{1}{\eta\widehat{\lambda\phi}} - \frac{\sigma_c^2}{2} - \frac{\sigma_s^2}{8} - \frac{1}{4}(\sigma_u^2 + \sigma_{u^*}^2) - \frac{1}{8}(\sigma_v^2 + \sigma_{v^*}^2) \\
&+ \frac{1}{4}(\sigma_{su} - \sigma_{su^*}) + \frac{1}{2}(\sigma_{c^*u} + \sigma_{c^*u^*} + \sigma_{c^*v} + \sigma_{c^*v^*}) - \frac{1}{4}(\sigma_{uv} + \sigma_{u^*v^*}).
\end{aligned} \tag{23}$$

Using equations (14), (15), (16), (17), and (18), variance and covariance terms involving variables c, c^* and s can be expressed in terms of $a = \{a_1, a_2, a_3, a_4\}$ and $b = \{b_1, b_2, b_3, b_4\}$. Both home and foreign monetary authorities can choose the parameters of monetary policy rules to maximize the expected utility of their own country's households. In other words, home and foreign monetary policy makers play the following Nash game

$$\max_a EU(a, b^N) \quad \max_b EU^*(a^N, b) \tag{P1}$$

The solution to (P1) is given by proposition 2.

Proposition 2. *The solution to problem (P1) is*

$$\begin{aligned}
a_1^N &= \frac{1}{2}, \quad a_2^N = \frac{1}{2}, \quad a_3^N = \frac{1}{2}, \quad a_4^N = \frac{1}{2}, \\
b_1^N &= 0, \quad b_2^N = 1, \quad b_3^N = \frac{1}{2}, \quad b_4^N = \frac{1}{2}.
\end{aligned}$$

Proof. See *Technical Appendix*. ■

Comparing Nash solution in RCP case with those in PCP and LCP cases implies that the asymmetry of exporters' pricing behavior leads to monetary authorities' asymmetric responses. Home monetary authority's response is identical to that in LCP case, foreign monetary authority, however, responds as if it faces PCP case.

If there is a positive productivity shock in the intermediate goods stage, home and foreign monetary authorities all increase money supply, no matter where it originates from. Furthermore, home and foreign set response parameters to mirror their weights in world economy. Now we assume that there is a positive productivity shock in home intermediate goods stage. Since intermediate goods exporters set prices after the shock is realized, the prices of home intermediate goods go down in home and foreign. As a result, final goods producers in home and foreign will substitute foreign intermediate goods with home intermediate goods. It means that the shock is adverse to foreign intermediate goods producers. In order to boost the demand for its domestic intermediate goods, foreign monetary authority depreciates its currency to make its domestic intermediate goods more competitive. However, the depreciation will depress the demand for home intermediate goods. In addition, it also lowers foreign demand for home final goods. In order to counterbalance foreign monetary authority's adverse effects, home monetary authority expands money supply equally.

As far as productivity shocks in the stage of final goods production are concerned, home monetary authority responds to both home and foreign shocks, its foreign counterpart, however, responds only to its own domestic shock. Before we analyze the logic behind the different responses between home and foreign, it's instrumental to derive consumption of home and foreign households respectively. Home households' consumption of home and foreign final goods are

$$C_H = \frac{1}{2\widehat{\lambda}\widehat{\phi}\widehat{\eta}} \exp\left(-\frac{1}{4}\sigma^2\right) (\theta_f\theta_f^*\theta_i\theta_i^*)^{1/2}, \quad C_F = \frac{1}{2\widehat{\lambda}\widehat{\phi}\widehat{\eta}} \exp\left(-\frac{1}{4}\sigma^2\right) (\theta_f\theta_f^*\theta_i\theta_i^*)^{1/2}. \quad (24)$$

For foreign households, they are

$$C_H^* = \frac{1}{2\widehat{\lambda}\widehat{\phi}\widehat{\eta}} \exp\left(-\frac{1}{4}\sigma^2\right) (\theta_f\theta_f^*\theta_i\theta_i^*)^{1/2}, \quad C_F^* = \frac{1}{2\widehat{\lambda}\widehat{\phi}\widehat{\eta}} \theta_f^* (\theta_i\theta_i^*)^{1/2}. \quad (25)$$

If a positive productivity shock occurs in the home final goods stage, with sticky final goods prices, home monetary authority's optimal response should be expansionary to boost demand in home. In addition, optimal monetary policy rules also require a change of relative prices in foreign to switch foreign demand towards domestic final goods. As a result, demand of home households for domestic and foreign final goods increase, foreign demand for home final goods also increases. From equations (24) and (25), after a positive productivity shock in the home final goods stage, home households increase consumption of home and foreign final goods equally. However, foreign households, while increasing consumption of home final goods as home households, remain consumption of their own domestic consumption of final goods unaffected. For this to happen, there must be a level effect in home and expenditure-switching effect in foreign. Home monetary authority's unilateral expansion of money supply can achieve both desired effects.

When there is a positive productivity shock in the foreign final goods stage, foreign monetary authority can't look for expenditure-switching effect to work in home. Therefore, it will expand money supply more aggressively than home monetary authority when facing the same situation. Foreign monetary authority aims at using level effect to boost its domestic households' demand, at the same time, divert their demand to their own domestic final goods. However, foreign authority's aggressive money supply will impair home export. In order to offset this adverse effect to some

degree, home monetary authority follows its foreign counterpart and expands money supply less aggressively. Consequently, as showed by equations (24) and (25), home households increase consumption for home and foreign final goods equally. Foreign households, by comparison, consume more of their own domestic final goods than imported final goods.

As emphasized in SX(2007), it seems that the introduction of vertical production and trade implies that there are gains from cooperation between global monetary authorities. However, their assumption of PCP in the final goods stages implies that there are no cooperation gains. After we introduce RCP, their conclusion does not hold any more. Now we discuss cooperative choices of monetary policy rules.

Proposition 3. (1). *If policymakers could cooperate equally in choosing their monetary policy rules, then the cooperative solution is*

$$\begin{aligned} a_1^C &= \frac{2}{3}, \quad a_2^C = \frac{1}{3}, \quad a_3^C = \frac{1}{2}, \quad a_4^C = \frac{1}{2}, \\ b_1^C &= 0, \quad b_2^C = 1, \quad b_3^C = \frac{1}{2}, \quad b_4^C = \frac{1}{2}. \end{aligned}$$

(2). *If $\sigma_{uv} > 0$, then there are gains from cooperation. Moreover, foreign is willing to cooperate unconditionally, but home is willing to cooperate only under the condition that $\sigma_{uv} > \frac{1}{36}\sigma^2$.*

(3). *If $\sigma_{uv} < 0$, then there are gains from cooperation only when $\sigma^2 > -48\sigma_{uv}$. But, in this case, home is not willing to cooperate any more. Foreign, as before, is still willing to cooperate.*

Proof. (1). Global monetary authorities take part in cooperation and maximize

$$EV = \frac{1}{2}EU + \frac{1}{2}EU^*.$$

The rest of the proof is similar to that of Proposition 2, therefore, it is omitted.

(2). When monetary policy rules are chosen by Nash players, the welfare of home and foreign households are respectively

$$EU = -\ln(\eta\widehat{\lambda\phi}) - \frac{1}{\widehat{\lambda\phi}} - \frac{1}{4}\sigma^2, \quad (26)$$

$$EU^* = -\ln(\eta\widehat{\lambda\phi}) - \frac{1}{\widehat{\lambda\phi}} - \frac{1}{8}\sigma^2. \quad (27)$$

When monetary policy rules are set cooperatively, the counterparts of equations (26) and (27) are respectively

$$EU = -\ln(\eta\widehat{\lambda\phi}) - \frac{1}{\widehat{\lambda\phi}} - \frac{5}{18}\sigma^2 + \sigma_{uv}, \quad (28)$$

$$EU^* = -\ln(\eta\widehat{\lambda\phi}) - \frac{1}{\widehat{\lambda\phi}} - \frac{1}{18}\sigma^2 + \sigma_{uv}. \quad (29)$$

If $\sigma_{uv} > 0$, (28) + (29) > (26) + (27) holds. Therefore, there are gains from cooperation. Moreover, (29) > (27) holds, it means that foreign households are better off if their monetary policy rules are set cooperatively. Similarly, if $\sigma_{uv} > \frac{1}{36}\sigma^2$, then (28) > (26) holds, home prefers cooperation.

(3). If $\sigma_{uv} < 0$, (28) + (29) > (26) + (27) holds only when $\sigma^2 > -48\sigma_{uv}$. Furthermore, (28) < (26) holds, it means that home households are worse off if their monetary policy are set cooperatively.

Therefore, home refuses cooperation even if there are gains from cooperation. But under the condition $\sigma^2 > -48\sigma_{uv}$, we have (29) > (27), thus, foreign is still willing to cooperate. ■

When a productivity shock comes from the stage of intermediate goods production, both home and foreign's Nash responses are identical to cooperative responses, no matter which country the shock stems from. If a productivity shock comes from the stage of final goods production, the same is still true for foreign, but home's cooperative response to a domestic productivity shock is greater than its Nash response, by comparison, its cooperative response to a foreign productivity shock is smaller than its Nash response.

To facilitate the analysis of the difference between Nash and cooperative monetary policy rules, it's helpful to derive the consumption of home and foreign final goods in cooperative equilibrium. For home households, they are

$$C_H = \frac{1}{2\lambda\widehat{\phi}\eta} \exp\left(-\frac{1}{9}\sigma^2 + \sigma_{uv}\right) \left(\theta_f^{2/3}\theta_f^{*1/3}\theta_i^{1/2}\theta_i^{*1/2}\right),$$

$$C_F = \frac{1}{2\lambda\widehat{\phi}\eta} \exp\left(-\frac{4}{9}\sigma^2 + \sigma_{uv}\right) \left(\theta_f^{2/3}\theta_f^{*1/3}\theta_i^{1/2}\theta_i^{*1/2}\right).$$

For foreign household, they are

$$C_H^* = \frac{1}{2\lambda\widehat{\phi}\eta} \exp\left(-\frac{1}{9}\sigma^2 + \sigma_{uv}\right) \left(\theta_f^{2/3}\theta_f^{*1/3}\theta_i^{1/2}\theta_i^{*1/2}\right),$$

$$C_F^* = \frac{1}{2\lambda\widehat{\phi}\eta} \exp(\sigma_{uv}) \theta_f^* (\theta_i\theta_i^*)^{1/2}.$$

When a productivity shock is from the stage of home final goods production, by the previous analysis, home monetary authority's expansion of money supply will induce home households to import more foreign final goods by level effect. At the same time, by expenditure-switching effect, foreign households can consume more final goods imported from home without lowering the consumption of their own domestic final goods. This means that home monetary authority's response produces a positive externality to foreign which is not internalized by home when playing Nash game. Consequently, its Nash response is smaller than cooperative one which is made after taking total benefits and costs to global households into account.

When a productivity shock is from the stage of foreign final goods production, RCP implies that foreign monetary authority needs an aggressive expansion to take advantage of its domestic level and expenditure-switching effect. But substantial depreciation of foreign currency reduces home export. In order to counteract, home monetary authority expands less aggressively than its foreign counterpart. To some extent, home monetary authority's response causes foreign central bank's effort to be less effective. Home monetary authority's Nash response, which aims to maximize home representative household's welfare, unlike cooperative one, doesn't consider its adverse effect on foreign. Accordingly, home Nash response is greater than cooperative one.

The conclusion of proposition 3 contrasts with other contributions in the literature. As showed in DE(2003), SX(2007), there are no gains from cooperation between global central banks. But our analysis implies that gains from cooperation depend on covariance between productivity shocks in two stages. If it is greater than zero, gains from cooperation can arise. In this circumstance, both countries are willing to cooperate when the covariance is greater than a proportion of the variance ($\sigma_{uv} > \frac{1}{36}\sigma^2$). Global gain from cooperation is $\frac{1}{24}\sigma^2 + 2\sigma_{uv}$ in which $\sigma_{uv} - \frac{1}{36}\sigma^2$ goes to home and

$\frac{5}{72}\sigma^2 + \sigma_{uv}$ is gained by foreign. Thus, our conclusion contrasts with CP (2005) in which gains from cooperation exist except for two polar cases (PCP and LCP), but the size of gains can not be determined. If the covariance between productivity shocks in two stages is less than zero, only when the variance is large enough are there gains from cooperation. But gains from cooperation can not be achieved since home can be better off by playing Nash game.

Though Nash equilibrium can replicate flexible price allocations in OR(2000, 2002), PCP case of DE (2003) and CP (2005), for less restrictive preference specifications of Benigno and Benigno (2003), the conclusion does not hold. In addition, the presence of LCP causes it impossible to replicate flexible price allocations, as documented in DSX(2007) and LCP case of DE (2003) and CP(2005). When dual price stickiness are introduced in SX(2007), optimal monetary policy can not replicate flexible price allocations, unless the productivity shocks in two stages are perfectly correlated. In our model, the asymmetry of exporters' pricing behavior implies that optimal monetary policy rules do not support flexible price allocations either.

DSX (2007) draw a conclusion that in the Nash equilibrium, home representative household's expected utility is always lower than that of the foreign. A main reason to reach such a surprising conclusion is, as mentioned by them, they don't consider the role of offshore reference currency holding which is a main benefit to the home. When we consider the same question in a model with vertical production and trade structure, we have

Proposition 4. (1) *In PCP case, the expected utility of home household is equal to that of his foreign counterpart, so is in LCP case. The expected utility in LCP case is lower than that in the case of PCP.*

(2) *In RCP case, the expected utility of home household is lower than that of his foreign counterpart.*

(3) *If pricing structure is changed from PCP case to RCP case, both home and foreign households will become worse off; but if pricing structure is changed from LCP case to RCP case, then home household is indifferent, however, foreign household is better off.*

Proof. (1) *The expected utility in PCP case is*

$$EU(a^n, b^n) = EU^*(a^n, b^n) = -\ln(\widehat{\lambda\phi\eta}) - \frac{1}{\widehat{\lambda\phi}}. \quad (30)$$

In LCP case, it is

$$EU(a^n, b^n) = EU^*(a^n, b^n) = -\ln(\widehat{\lambda\phi\eta}) - \frac{1}{4}\sigma^2 - \frac{1}{\widehat{\lambda\phi}}. \quad (31)$$

Comparison of equation (30) with (31) verifies our conclusion.

(2) *In RCP case, the expected utility of home household is given by equation (26), its foreign equivalent is given by equation (27). Since (26) < (27), the conclusion follows.*

(3) *Comparing equation (30) with (26), then equation (30) with (27), the first part of the conclusion follows. Similarly, comparing equation (31) with (26), then equation (31) with (27), the second part of the conclusion follows. ■*

As emphasized in DSX (2007), in PCP case, monetary authorities can achieve both level and expenditure-switching effects. As a result, Nash monetary policy rules can replicate flexible price allocations and both home and foreign households can achieve utility level higher than those in LCP and RCP cases. In LCP case, however, expenditure-switching effect doesn't work for both

home and foreign monetary authorities, accordingly, home and foreign households' welfare is the lowest among three pricing structures. In RCP case, as explained by DSX (2007), foreign monetary authority can achieve both effects, but home can not. Therefore, comparing with LCP case, foreign is better off, but home's situation is not improved. It is interesting that both home and foreign households become worse off, when pricing structure is changed from PCP to RCP case. It is easy to understand that home becomes worse off, why is foreign worse off? Since, in PCP case, foreign monetary policy can influence both export and import, but, in RCP case, it can only influence import.

In DE(2003), optimal monetary policy rules in PCP case require nominal exchange rate to be flexible, but constant in LCP case. What should it be in RCP case? What about real exchange rate?

Proposition 5. (1). *The volatility of nominal exchange rate in RCP case is greater than that in LCP case, but smaller than that in PCP case.*

(2). *The volatility of real exchange rate in RCP case is greater than those in PCP and LCP cases.*

Proof. (1). *In PCP case, nominal exchange rate can be expressed as*

$$S = \frac{M}{M^*} = \frac{\exp(a_1u + a_2u^* + a_3v + a_4v^*)}{\exp(b_1u + b_2u^* + b_3v + b_4v^*)} = \frac{\exp(u + \frac{1}{2}v + \frac{1}{2}v^*)}{\exp(u^* + \frac{1}{2}v + \frac{1}{2}v^*)} = \frac{\theta_f}{\theta_f^*}. \quad (32)$$

Similarly, in LCP case,

$$S = \frac{\exp(\frac{1}{2}u + \frac{1}{2}u^* + \frac{1}{2}v + \frac{1}{2}v^*)}{\exp(\frac{1}{2}u + \frac{1}{2}u^* + \frac{1}{2}v + \frac{1}{2}v^*)} = 1. \quad (33)$$

In RCP case,

$$S = \frac{\exp(\frac{1}{2}u + \frac{1}{2}u^* + \frac{1}{2}v + \frac{1}{2}v^*)}{\exp(u^* + \frac{1}{2}v + \frac{1}{2}v^*)} = \left(\frac{\theta_f}{\theta_f^*}\right)^{1/2}. \quad (34)$$

Taking logarithms to both sides of equation (32), we have $s = u - u^$. The variance of s is, therefore, $2\sigma^2$. By the same procedures, we have the variance of s in LCP case is 0, in RCP case, $\frac{1}{2}\sigma^2$. Thus we complete the proof of the first part of the proposition 5.*

(2). *In PCP case, real exchange rate is*

$$R = \frac{SP^*}{P} = 1. \quad (35)$$

In LCP case, real exchange rate can be expressed as

$$R = \frac{S \left[E \left(\frac{(\hat{\phi}_{\frac{W}{\theta_i}})^{1/2} (\hat{\phi}_{\frac{SW^*}{\theta_i^*}})^{1/2}}{S\theta_f} \right) \right]^{1/2} \left[E \left(\frac{(\hat{\phi}_{\frac{W}{S\theta_i}})^{1/2} (\hat{\phi}_{\frac{W^*}{\theta_i^*}})^{1/2}}{\theta_f^*} \right) \right]^{1/2}}{\left[E \left(\frac{(\hat{\phi}_{\frac{W}{\theta_i}})^{1/2} (\hat{\phi}_{\frac{W^*}{S\theta_i^*}})^{1/2}}{\theta_f} \right) \right]^{1/2} \left[E \left(S \frac{(\hat{\phi}_{\frac{W}{S\theta_i}})^{1/2} (\hat{\phi}_{\frac{W^*}{\theta_i^*}})^{1/2}}{\theta_f^*} \right) \right]^{1/2}},$$

in which

$$W = \frac{\eta}{\chi} (\theta_f \theta_f^* \theta_i \theta_i^*)^{1/2}, W^* = \frac{\eta}{\chi} (\theta_f \theta_f^* \theta_i \theta_i^*)^{1/2}$$

In addition, from equation (33), $S = 1$. Using expressions for W, W^*, S , real exchange rate can be written as

$$R = 1 \tag{36}$$

In RCP case, real exchange rate can be expressed as

$$R = \frac{S^{1/2} \left[E \left(\frac{S^{-1/2} \left(\frac{W}{\theta_i} \right)^{1/2} \left(\frac{W^*}{\theta_i^*} \right)^{1/2}}{\theta_f^*} \right) \right]^{1/2}}{\left[E \left(\frac{S^{1/2} \left(\frac{W}{\theta_i} \right)^{1/2} \left(\frac{W^*}{\theta_i^*} \right)^{1/2}}{\theta_f^*} \right) \right]^{1/2}},$$

in which

$$W = \frac{\eta}{\chi} (\theta_f \theta_f^* \theta_i \theta_i^*)^{1/2}, W^* = \frac{\eta}{\chi} \theta_f^* \theta_i^{1/2} \theta_i^{*1/2},$$

In addition, by equation (34), $S = \left(\frac{\theta_f}{\theta_f^*} \right)^{1/2}$. Using expressions for W, W^*, S , the real exchange rate can be written as

$$R = \left[\exp \left(-\frac{1}{8} \sigma^2 \right) \right] \theta_f^{1/4} \theta_f^{*-1/4}. \tag{37}$$

Taking logarithms to both sides of equation (35), we have $r = 0$. The variance of r is, obviously, 0. By the same procedures, the variance of real exchange rate, in LCP case, is 0, in RCP case, $\frac{1}{8} \sigma^2$. Thus, we complete the proof of the second part of the proposition 5. ■

In DE(2007), optimal exchange rate policy is a trade-off between competing objectives, i.e. the desire to smooth fluctuations in real exchange rate and the need to allow flexibility in the nominal exchange rate. For one thing, optimal exchange rate policy requires smooth fluctuation in nominal exchange rate, since resulting smooth fluctuation in real exchange rate can reduce distortions in consumption allocations. For another, optimal exchange rate policy also requires flexibility of nominal exchange rate to facilitate adjustment of terms of trade. Optimal exchange rate policy involve a trade-off between these two objectives.

In our model, PCP case provides a benchmark. In PCP case, optimal exchange rate policy ignores the adjustment of terms of trade in the stage of intermediate goods production, since flexible prices can adjust terms of trade of the stage efficiently. Optimal fluctuation of nominal exchange rate coincides with that of terms of trade in the stage of the final goods production. As a result, both real exchange rate and terms of trade in the stage of final goods production are efficient.

As in PCP case, optimal exchange rate policy in RCP case also ignores the adjustment of terms of trade in the stage of intermediate goods production. However, optimal exchange rate can not adjust terms of trade in the stage of the final goods production, since it is predetermined. The only trade-off facing central bank is between the desire to smooth fluctuation of real exchange rate to ensure efficient consumption allocations and the need to allow flexibility in the nominal exchange rate to adjust the foreign prices of home final goods. Since the main welfare loss in our model is from central banks' inability to use expenditure-switching effect to influence home households' demand,

it's expected that central banks are willing to give up some benefits from smooth fluctuation in real exchange rate to exchange for more powerful expenditure-switching effect in foreign by allowing greater fluctuation in nominal exchange rate. It explains why the volatility of real exchange rate is higher in RCP case than in PCP and LCP cases. To justify our explanations, we solve the volatilities of nominal and real exchange rate in cooperative equilibrium.

Proposition 6. *In RCP case, nominal and real exchange rate in cooperative equilibrium are more volatile than those in Nash equilibrium.*

Proof. *The proof is similar to that of Proposition 5. The variance of nominal exchange rate in cooperative equilibrium is $\frac{8}{9}\sigma^2$, however, in Nash equilibrium, it is $\frac{1}{2}\sigma^2$. Similarly, the variance of real exchange rate in cooperative equilibrium is $\frac{2}{9}\sigma^2$, in Nash equilibrium, it is $\frac{1}{8}\sigma^2$. Simple comparisons prove the proposition. ■*

The conclusion in proposition 6 implies that, in RCP case, from a global view, it is desirable to increase simultaneously the volatility of nominal and real exchange rate up to a point at which marginal benefit from more powerful expenditure-switching effect in foreign equals marginal cost incurred from greater distortions in consumption allocations. But from individual country's perspective, the marginal benefit from a more flexible nominal exchange rate will be reaped partly by other country. As a result, in Nash equilibrium, less volatile nominal and real exchange rate are accepted by home and foreign.

4. Conclusion

In this paper, we examine optimal monetary policy rules in a model with vertical production and trade in which we emphasize the role played by reference currency. As evidenced by empirical findings, we assume final goods prices are sticky, but intermediate goods prices are flexible.

We find that when there is a positive productivity shock in the stage of intermediate goods production, no matter where it comes from, both home and foreign monetary authorities expand money supply with the degree of expansion being equal to their weights in world output. However, when positive productivity shocks occur in the stage of final goods production, foreign only responds to its own domestic shock, but home responds to both home and foreign shocks with response parameters being equal to their weights in world output. Thus, the asymmetry of exporters' pricing behavior in our model implies that the responses of monetary authorities to productivity shocks are different. If productivity shocks are from the stage of final goods production, home and foreign monetary authorities respond asymmetrically. By comparison, if productivity shocks are from the stage of intermediate goods production, home and foreign monetary authorities respond symmetrically.

Unlike what is found in the literature, in our model, gains from cooperation are related to the covariance of productivity shocks in two stages. When the covariance is greater than zero, there are gains from cooperation. Furthermore, foreign is willing to cooperate unconditionally, but home is willing to only conditionally. If the covariance is less than zero, only when the variance of the shocks is strictly greater than a critical value do gains from cooperation arise. In this circumstance, home, unlike foreign which is willing to cooperate as before, refuses to cooperate.

In addition, we also find that the volatility of nominal exchange rate in RCP case is greater than that in LCP case, but smaller than that in PCP case. The volatility of real exchange rate in RCP is, however, greater than those in PCP and LCP cases. The reason is that the main welfare loss in RCP case is that monetary authorities cannot use expenditure-switching effect to change

consumption demand of home households. Consequently, they allow for some flexibility of real exchange rate to exchange for more powerful expenditure-switching effect in foreign.

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