

Optimal monetary and tariff policy in open economies

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Abstract

This paper investigates the relationship between optimal monetary and tariff policy in open economies. In producer-currency pricing (PCP) case, as in Obstfeld and Rogoff (2002), optimal tariff policy rules are separable from optimal monetary policy rules. Except for PCP case, they are not separable from each other. The increase of tariffs will lead to a more insulated world economy in the sense that both home and foreign pay more attention to their domestic goals respectively. When tariffs are chosen optimally, except for PCP and reference-currency pricing (RCP) cases, optimal monetary policy is inward-looking. We also extend the model to consider gains from cooperation. Except for LCP case, there are gains from cooperation between Home and Foreign monetary policy makers. By comparison, there are gains from cooperation between Home and Foreign tariff policy makers in various cases.

Keywords: Open economies, Tariff policy, Monetary policy, Exchange rate pass-through elasticity, International cooperation

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1. Introduction

What's the relationship between optimal tariff and monetary policy rules in open economies? Though the problem is important for policy making, it attracts scant attention in new open economy macroeconomics literature. One exception is Obstfeld

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and Rogoff (2002). In a two-country sticky-wage model with incomplete international asset markets, they show that optimal tariff rules are separable from optimal monetary rules. The reason is, in their model, distortion caused by sticky-wage does not interact with the micro-level monopoly distortion. However, they draw their conclusion under the assumption that export firms set prices in producers' currency. In general, they do not answer whether optimal tariff policy rules are separable from optimal monetary policy rules or not. In this paper, we continue their research by introducing tariffs into Corsetti and Pesenti (2005), in which exchange rate pass-through elasticity plays a key role. The introduction of exchange rate pass-through elasticity allows us to consider three special cases: producer currency pricing or PCP, local currency pricing or LCP, and reference currency pricing or RCP.¹

When exchange rate pass-through elasticities are unity in both countries (corresponding to PCP case), our conclusion coincides with that in Obstfeld and Rogoff (2002), i.e. optimal tariff policy rules are separable from optimal monetary policy rules. However, except for PCP case, they are not separable from each other. The reason is that Home monetary policy maker stabilizes not only Home but also Foreign firm's markups in Home market. It seems weird that Home monetary policy maker, with an aim to minimize Home loss function, is required to stabilize Foreign firm's markup in Home market. What happens if Home monetary policy maker pursues an inward-looking policy? In order to reduce the sensitivity of profit to external shocks, Foreign firm will charge a higher price in Home market. Accordingly, Home household's purchasing power is reduced. Since both Home and Foreign tariffs influence Foreign firm's markup in Home market, optimal monetary policy rules are not separable from tariff policy rules. In the case of PCP, though Foreign tariff policy still affect Foreign firm's markup in Home market, Home monetary policy is independent of it. As a result, Home monetary policy's only objective is to stabilize Home firm's markup in Home market, in other words, it is inward-looking.

What's the influence of tariffs on Home optimal monetary policy rule? Except for the case of PCP, the imposition of tariffs in Home and Foreign can also reduce the sensitivity of Foreign firm's expected profit in Home market to external shocks. Accordingly, it is not necessary for Foreign firm to charge a higher price in Home to stabilize the sensitivity. It means that a more inward-looking monetary policy in Home will not bring about import price as high as that in Corsetti and Pesenti (2005). Thus, when Home and Foreign impose tariffs, optimal monetary policy rules

¹In PCP case, exporters in both countries set prices in producers' currency; In LCP case, exporters set prices in consumers' currency; In RCP case, exporters in both countries set prices in a common currency.

are more inward-looking than those in Corsetti and Pesenti (2005). Similar analysis implies that Home Nash monetary response to surprising increase of tariffs, both in Home and Foreign, is to ease its monetary stance. Therefore, the surprising increase of global tariffs will lead to a more insulated world economy in which each country pays more attention to its domestic goal and adopts a more aggressive monetary policy without exceedingly worrying about the retaliation from other country.

One merit of introducing exchange rate pass-through elasticity is that we can discuss some special cases. When both elasticities are nil (corresponding to LCP case), as in general case, optimal monetary policy rules are not separable from tariff policy rules. However, since Foreign tariff doesn't affect Foreign firm's markup in Home market in this case, Home monetary policy only responds to Home tariff policy. Similarly, Foreign monetary policy only responds to Foreign tariff policy. When one of elasticities is nil (corresponding to RCP case), without loss of generality, we consider the case in which there is no exchange rate pass-through to Home, but full exchange rate pass-through to Foreign. Under this circumstance, Home monetary policy responds to Home tariff, as a comparison, Foreign monetary policy is inward-looking.

A natural problem is what the optimal monetary policy rules will be if tariffs are set optimally? The answer depends on the exchange rate pass-through elasticities. In PCP case, since Home tariff policy doesn't influence Foreign firm's markup in Home market, thus, it doesn't influence Home policy loss function. By the same token, Foreign tariff policy doesn't influence Foreign policy loss function. Under this circumstance, there is no game to play between Home and Foreign. Similar analysis applies to RCP case. Except for these two cases, when tariffs are chosen optimally, optimal monetary policy rules are identical to their counterparts in PCP case, i.e. inward-looking. Intuitively, when Foreign firm's markup in Home market can be stabilized by Home tariff policy, Home monetary authority's attention will focus on Home firm's markup in Home market completely.

Thus, when tariffs are chosen optimally, optimal monetary policy rules are similar to those obtained in Obstfeld and Rogoff (2002), Devereux and Engel (2003). However, their conclusions are obtained when exchange rate pass-through is complete in both countries. As a comparison, our conclusion holds when exchange rate pass-through is not complete in both countries. After introducing optimal tariff policy rules, our conclusion contrasts sharply from Corsetti and Pesenti (2005), in which, when exchange rate pass-through is not complete in both countries, inward-looking policy rules are not optimal.

We also make an extension to discuss gains from cooperation between Home and Foreign policy makers. As far as monetary policy is concerned, except for LCP

case, there are gains from cooperation by signing a binding international monetary compact. The key to produce gains from cooperation is that Home monetary policy maker, whose policy can affect Home firm's markup in Foreign market, is required to internalize its externality to Foreign policy loss function. By comparison, in LCP case, Home monetary policy doesn't affect Home firm's markup in Foreign market. Consequently, there are no gains from cooperation. Our conclusion in PCP case contrasts with that in Corsetti and Pesenti (2005), since Home firm's markup in Foreign, after we introduce tariffs, is different from before.

What about tariff policy cooperation? In our model, cooperation between Home and Foreign policy makers produces gains for various cases. Our model also allows us to analyze the case in which Home and Foreign can cooperate in both monetary and tariff policy making. Our result shows that both countries will benefit from such kind of cooperation.

Related literature The literature on tariffs in new open economy macroeconomics is not much. Fender and Yip (2000) introduce tariffs in Obstfeld and Rogoff (1995). They show that tariff will decrease the tariff imposer's output in both the steady state and in the short run. In addition, a small tariff may make a country worse off. The outcome depends on the tradeoff between output and consumption through the change of terms of trade. Hwang and Turnovsky (2013) extend Obstfeld (2008) to consider the consequences of tariffs under PCP and LCP. The consequences depend on whether tariffs are anticipated or unanticipated. In the case that tariffs are fully anticipated, the effects of tariffs are independent of how exporters set prices. By comparison, when tariffs are unanticipated, the effects of tariffs depend on the pricing scheme adopted by exporters.

The detailed structure of our paper is as follows. In section 2, we introduce the model. Section 3 analyzes optimal monetary and tariff policy rules. In section 4, we make an extension to consider the cooperation between Home and Foreign policy makers. Section 5 concludes.

2. The model

2.1. Households

The model is built on Corsetti and Pesenti (2005). The world is composed of two equally-sized countries, Home and Foreign. Both countries produce tradables by a continuum of firms, indexed by $h \in [0, 1]$ in Home country and $f \in [0, 1]$ in Foreign country. In addition, each country is inhabited by a continuum of households, indexed by $j \in [0, 1]$ in the Home country and $j^* \in [0, 1]$ in the Foreign country. Home representative household j 's expected utility is expressed by

$$U_t(j) = E_t \sum_{\tau=t}^{+\infty} \beta^{\tau-t} \left[\ln C_\tau(j) + \chi \ln \frac{M_\tau(j)}{P_\tau} - \kappa L_\tau(j) \right], \quad (1)$$

in which $0 < \beta < 1$ denotes the subjective discount factor and E_t is expectation conditional on information available at time t , $C_\tau(j)$ is the overall real consumption index, $\frac{M_\tau(j)}{P_\tau}$ real money balances, $L_\tau(j)$ total labor supply to Home firms. Foreign representative household's expected utility can be defined similarly, except that, as emphasized in Corsetti and Pesenti (2005), χ^* and κ^* need not be identical to χ and κ . Throughout the paper, we use asterisks to denote Foreign variables.

Overall real consumption index $C_t(j)$ is Cobb-Douglas function of the Home tradables index $C_{H,t}(j)$ and Foreign tradables index $C_{F,t}(j)$:

$$C_t(j) = C_{H,t}(j)^\gamma C_{F,t}(j)^{1-\gamma} \quad 0 < \gamma < 1, \quad (2)$$

in which Home and Foreign tradables indices $C_{H,t}(j)$ and $C_{F,t}(j)$ are defined respectively as

$$\begin{aligned} C_{H,t}(j) &= \left[\int_0^1 C_t(h, j)^{(\theta-1)/\theta} dh \right]^{\theta/(\theta-1)}, \\ C_{F,t}(j) &= \left[\int_0^1 C_t(f, j)^{(\theta^*-1)/\theta^*} df \right]^{\theta^*/(\theta^*-1)}. \end{aligned} \quad (3)$$

In equation (3), $C_t(h, j)$ and $C_t(f, j)$ are consumption of Home tradable good h and Foreign tradable good f at time t , $\theta > 1$ ² is the elasticity of substitution between Home tradables, which also describes Home firm's market power, θ^* is the Foreign counterpart of θ and has the same meaning.³

The price index for Home tradables, $P_{H,t}$, defined as the minimal expenditure required to buy a unit of composite good $C_{H,t}$, is

$$P_{H,t} = \left[\int_0^1 p_t(h)^{1-\theta} dh \right]^{1/(1-\theta)}, \quad (4)$$

²As explained in Obstfeld and Rogoff (1996), θ will be the price elasticity of demand faced by each Home firm. When the elasticity of demand is less than 1, firm's marginal revenue is negative, $\theta > 1$ is required to ensure an interior equilibrium in which the firms are willing to produce.

³From equation (2), we know the elasticity of substitution between Home and Foreign tradables is unity. The model follows Tille (2001) and assumes that within-country substitutability is greater than cross-country substitutability.

in which $p_t(h)$ is the price of Home product h . $P_{H,t}^*, P_{F,t}, P_{F,t}^*$ can be derived similarly.

We depart from Corsetti and Pesenti (2005) by assuming that Home and Foreign can impose tariffs on imports, as usual in the literature, tariff revenues in Home and Foreign are rebated to their public in lump-sum fashion. After tariffs are imposed, the prices paid by Home and Foreign households to buy imports are $(1 + t_{H,t}) p_t(f)$ and $(1 + t_{F,t}^*) p_t^*(h)$ respectively, in which $t_{H,t}$ and $t_{F,t}^*$ are ad valorem tariff rates set by Home and Foreign at time t respectively.

Under this circumstance, Home and Foreign CPIs can be derived as

$$P_t = \frac{P_{H,t}^\gamma [(1 + t_{H,t}) P_{F,t}]^{1-\gamma}}{\gamma_W}, \quad P_t^* = \frac{[(1 + t_{F,t}^*) P_{H,t}^*]^\gamma P_{F,t}^{*1-\gamma}}{\gamma_W}, \quad (5)$$

in which $\gamma_W = \gamma^\gamma (1 - \gamma)^{1-\gamma}$.

2.2. Technology and resource constraints

The introduction of tariffs doesn't influence firm's technology and resource constraints. Home product h is produced according to the following production function

$$Y_t(h) = \frac{L_t(h)}{\alpha_t}, \quad (6)$$

in which $L_t(h)$ is the labor demand by Home firm h , and α_t is Home productivity shock.

The resource constraint for Home product h is the same as that in Corsetti and Pesenti (2005) and is given by

$$Y_t(h) = \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^*. \quad (7)$$

Labor markets are perfect competitive and labors in Home and Foreign can not move freely across borders. Given the nominal wage W_t , the marginal cost of Home firm h is

$$MC_t(h) = MC_t = \alpha_t W_t, \quad (8)$$

where the first equality in equation (8) holds in symmetric equilibrium.

Home labor market clearing implies that

$$\int_0^1 L_t(j) dj = \int_0^1 L_t(h) dh \quad (9)$$

2.3. Budget constraints and household optimization

Home household j faces the following sequence of budget constraints

$$\begin{aligned} & \int_0^1 p_t(h) C_t(h, j) dh + \int_0^1 (1 + t_{H,t}) p_t(f) C_t(f, j) df + M_t(j) + B_{t+1}(j) \\ & + \epsilon_t B_{t+1}^*(j) \leq M_{t-1}(j) + (1 + i_t) B_t(j) + (1 + i_t^*) \epsilon_t B_t^*(j) + W_t L_t(j) \\ & + \int_0^1 \Pi_t(h) dh + T_t(j), \quad t = 0, 1, \dots, \end{aligned} \quad (10)$$

where B and B^* are bonds denominated in Home and Foreign currencies respectively, i_t and i_t^* denote the nominal interest rates on B and B^* respectively between $t - 1$ and t , ϵ_t is the nominal exchange rate which is defined as the home-currency price of foreign currency, $\Pi_t(h)$ is the profit received from firm h which distributes its profit among Home households equally, $T_t(j)$ is the lump-sum transfer from the Home government.

Home household j chooses consumption, money balances, bonds issued by Home and Foreign, labor supply to maximize his expected utility. The Euler equation is

$$1 = (1 + i_{t+1}) E_t Q_{t,t+1}(j), \quad (11)$$

in which $Q_{t,t+1}(j)$ is household j 's stochastic discount factor and has the expression

$$Q_{t,t+1}(j) = \beta \frac{P_t C_t(j)}{P_{t+1} C_{t+1}(j)}. \quad (12)$$

All Home households' stochastic discount factors are equal, since Home households choose the same consumption.

The first-order condition with respect to $M_t(j)$ is

$$M_t(j) = \frac{\chi P_t C_t(j)}{1 - \beta E_t \left[\frac{P_t C_t(j)}{P_{t+1} C_{t+1}(j)} \right]}. \quad (13)$$

Equation (13) is just a trade-off between the utility from consumption goods purchased by one unit of currency and derived transaction utility by keeping the money in hand and then converting the money back to consumption in the next period.

In the derivation of Home household's optimization conditions, we use the expressions for the Home demands for Home product j and Foreign product f , which

are

$$C_t(h, j) = \left(\frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}(j), \quad C_t(f, j) = \left(\frac{p_t(f)}{P_{F,t}} \right)^{-\theta^*} C_{F,t}(j), \quad (14)$$

and those for Home products index $C_{H,t}(j)$ and Foreign products index $C_{F,t}(j)$, which are given implicitly by

$$P_t C_t(j) = \frac{1}{\gamma} P_{H,t} C_{H,t}(j) = \frac{1}{1-\gamma} [(1+t_{H,t}) P_{F,t}] C_{F,t}(j). \quad (15)$$

Equation (15) is easy to understand, since overall real consumption index $C_t(j)$ is Cobb-Douglas function of the Home products index $C_{H,t}(j)$ and Foreign products index $C_{F,t}(j)$, the expenditures on Home and Foreign products are just constant fractions of total consumption expenditure.

The Home government returns seignorage and tariff revenues to domestic households in the form of lump-sum transfer, it means that Home government's budget constraint is

$$\int_0^1 [M_t(j) - M_{t-1}(j)] dj + \int_0^1 T_t(j) dj + \int_0^1 t_{H,t} P_{F,t} C_{F,t}(j) dj = 0. \quad (16)$$

In addition, market clearing in bonds markets implies

$$\int_0^1 B_t(j) dj + \int_0^1 B_t(j^*) dj^* = \int_0^1 B_t^*(j) dj + \int_0^1 B_t^*(j^*) dj^* = 0. \quad (17)$$

Finally, as in Corsetti and Pesenti (2005) and Devereux and Engel (2007), both Home and Foreign governments target nominal consumption, it means that Home monetary policy can be described as

$$P_t C_t = \mu_t. \quad (18)$$

Equations (11), (12), (18) and the condition $W_t = \kappa P_t C_t(j)$ imply

$$\frac{1}{\mu_t} = \beta (1 + i_{t+1}) E_t \left(\frac{1}{\mu_{t+1}} \right),$$

therefore, Home government can control nominal interest rate i by targeting nominal consumption. Home and Foreign nominal interest rates are linked by risk-adjusted uncovered interest parity, which is

$$\frac{1 + i_{t+1}}{1 + i_{t+1}^*} = \frac{E_t \left(\frac{\epsilon_{t+1}}{\mu_{t+1} \epsilon_t} \right)}{E_t \left(\frac{1}{\mu_{t+1}} \right)}. \quad (19)$$

2.4. Producer optimization and price setting

Following Obstfeld and Rogoff (1995, 2000, 2002), Betts and Devereux (2000), Devereux and Engel (2003, 2007), Corsetti and Pesenti (2001, 2005), we assume that price stickiness only last for one period. Thus, producers set prices at the beginning of the period and supply what is demanded within the period.⁴ The assumption is rationalized by the menu cost approach of Akerlof and Yellen (1985) and Mankiw (1985).

Home representative firm h sets prices in Home and Foreign markets at the beginning of the period to maximize its expected profits. In Home market, it chooses $p_t(h)$ at the end of period $t - 1$ to maximize

$$\max_{p_t(h)} E_{t-1} [Q_{t-1,t} \Pi_t(h)],$$

in which $\Pi_t(h) = (p_t(h) - MC_t) \int_0^1 C_t(h, j) dj$. The first-order condition of the above optimization problem is

$$p_t(h) = \frac{\theta}{\theta - 1} E_{t-1} (MC_t) = \frac{1}{\Phi} E_{t-1} (\alpha_t P_t C_t), \quad (20)$$

in which $\Phi \equiv \frac{\theta-1}{\theta\kappa}$. Equation (20) implies that nominal price charged by Home producer h in Home market is just a constant markup over expected marginal cost.

Now we turn to Home firm's price setting problem in its export market. In the open economy macroeconomics literature with nominal rigidities and monopolistic competition, which currency is used to price export goods is vital to determine exchange rate pass-through and therefore optimal monetary and exchange rate policy. When export prices are set in the producers' currency (denoted as producer-currency pricing or PCP), in the short run when prices are sticky, exchange rate pass-through is 100 percent. When export prices are set in consumers' currency (denoted as local-currency pricing or LCP), exchange rate pass-through is 0 percent.⁵ In this paper, we follow Corsetti and Pesenti (2005) and consider a range of possibilities for the

⁴Of course, firm is willing to supply only under the condition that the predetermined price exceeds marginal cost.

⁵In the recent open macroeconomics literature, many researchers examine PCP case. A nonexhaustive list of literature includes Obstfeld and Rogoff (1995, 2000, 2002), Corsetti and Pesenti (2001), Benigno and Benigno (2003). Nearly at the same time, Bacchetta and Wincoop (2000), Chari et al. (2002), and Duarte and Stockman (2005) study LCP case. Others, like Devereux and Engel (2003, 2007), Shi and Xu (2007), consider both cases simultaneously. Devereux et al. (2007) and Wang and Zou (2013) study a asymmetric case in which Home exports are priced in producer's currency, but Foreign exports are priced in consumers' currency.

price-setting behavior in export market. We assume that Foreign-currency price of home product h is

$$p_t^*(h) = \frac{\tilde{p}_t(h)}{\epsilon_t^{\eta^*}}, \quad 0 \leq \eta^* \leq 1, \quad (21)$$

in which $\tilde{p}_t(h)$ is the predetermined component of the Foreign-currency price of home product h , and $\eta^* \equiv \frac{\partial \ln p_t^*(h)}{\partial \ln(1/\epsilon_t)}$ is pass-through elasticity and measures the extent to which the Foreign-currency price is adjusted to the exchange rate. Furthermore, we assume that pass-through elasticity doesn't vary across producers and over time.⁶ With the device in hand, Home representative firm h can choose $\tilde{p}_t(h)$ to solve the following optimization problem

$$\max_{\tilde{p}_t(h)} E_{t-1} \left\{ Q_{t-1,t} \left[\left(\epsilon_t \frac{\tilde{p}_t(h)}{\epsilon_t^{\eta^*}} - MC_t^* \right) \int_0^1 C_t^*(h, j^*) dj^* \right] \right\}. \quad (22)$$

The solution to the problem (22) is

$$p_t^*(h) = \frac{\theta}{\theta - 1} \frac{1}{\epsilon_t^{\eta^*}} \frac{E_{t-1} \left[\frac{\gamma}{1-\gamma} \frac{P_t^* C_t^*}{P_t C_t} \frac{1}{1+t_{F,t}^*} \epsilon_t^{\eta^*} MC_t \right]}{E_{t-1} \left[\frac{\gamma}{1-\gamma} \frac{P_t^* C_t^*}{P_t C_t} \frac{1}{1+t_{F,t}^*} \epsilon_t \right]}. \quad (23)$$

In the derivation of equation (23), equations (15) and (12) and the condition that $W_t = \kappa P_t C_t(j)$ are used.

Furthermore, as emphasized in Corsetti and Pesenti (2001, 2005), in this paper, the range of shocks is restricted to ensure that firms' participation constraints are satisfied. For Home firms, they are

$$P_{H,t} \geq MC_t, \quad P_{H,t}^* \geq \frac{MC_t}{\epsilon_t}. \quad (24)$$

Equation (24) implies that, ex post, firms are willing to supply goods at the predetermined prices.

2.5. The closed-form solution of the model

In general, there doesn't exist closed-form solution for the symmetric equilibrium. However, considering the model's structure, we can derive one under the assumption that initial non-monetary wealth is zero, i.e. $B_{t_0} = B_{t_0}^* = 0$. The assumption,

⁶Obstfeld (2002) also adopts this device to determine the export prices. As emphasized by him, the assumption that pass-through elasticity doesn't vary across producers and over time is not innocuous.

together with logarithmic consumption preference and Cobb-Douglas overall real consumption index, imply that, in symmetric equilibrium, both Home and Foreign's current accounts are balanced. Consequently, from equations (15) and (18) and their Foreign analogs, we have

$$\epsilon_t = \frac{1 - \gamma}{\gamma} \frac{1 + t_{F,t}^* \mu_t}{1 + t_{H,t} \mu_t^*}. \quad (25)$$

Other things being unchanged, the imposition of Home tariff lowers demands of Home households for Foreign products. As a result, Home currency appreciates. The influence of Foreign tariff on exchange rate can be analyzed similarly.

From equations (8) and (18) and the condition $W_t = \kappa P_t C_t$, we obtain

$$MC_t = \kappa \alpha_t \mu_t. \quad (26)$$

If Home government eases its monetary stance (an increase in μ_t), Home households' utilities from working for an hour decrease. Therefore, in competitive labor market, Home households will lower labor supply and wage goes up. Thus, marginal cost faced by Home firms increases.

Equations (13) and (18) and the condition $W_t = \kappa P_t C_t$ imply

$$M_t = \frac{\chi \mu_t}{1 - \beta \mu_t E_t [\mu_{t+1}^{-1}]} \quad (27)$$

The above equation establishes the relationship between Home monetary stances and its money supply.

In symmetric equilibrium, the price of Home goods in Home market is given by equation (20) which is rewritten as

$$P_{H,t} = p_t(h) = \frac{\theta}{\theta - 1} E_{t-1} (MC_t). \quad (28)$$

Using equations (18) and (25), the price of Home goods in Foreign market given by equation (23) can be rewritten as

$$P_{H,t}^* = \frac{\theta}{\theta - 1} \frac{1}{\epsilon_t^{\eta^*}} \frac{E_{t-1} \left[\frac{1}{1+t_{H,t}} \epsilon_t^{\eta^*-1} MC_t \right]}{E_{t-1} \left[\frac{1}{1+t_{H,t}} \right]}. \quad (29)$$

Home equilibrium consumption is

$$C_t = \frac{\gamma_W \left(\frac{\theta-1}{\theta} \right)^\gamma \left(\frac{\theta^*-1}{\theta^*} \right)^{1-\gamma} \mu_t \epsilon_t^{-\eta(1-\gamma)} \left[E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \right) \right]^{1-\gamma}}{(1 + t_{H,t})^{1-\gamma} [E_{t-1} (MC_t)]^\gamma \left[E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \epsilon_t^{1-\eta} MC_t^* \right) \right]^{1-\gamma}}. \quad (30)$$

Noting that the imposition of Home tariff will lower Home consumption, unless in the extreme case in which Home economy keeps closed ($\gamma = 1$) or pass-through elasticity of exchange rate is unity ($\eta = 1$).

Finally, Home equilibrium employment is

$$L_t = \Phi \left[\gamma \frac{MC_t}{E_{t-1}(MC_t)} + (1 - \gamma) \frac{E_{t-1} \left(\frac{1}{1+t_{H,t}} \right) \epsilon_t^{\eta^*-1} MC_t}{(1 + t_{H,t}) E_{t-1} \left(\frac{1}{1+t_{H,t}} \epsilon_t^{\eta^*-1} MC_t \right)} \right]. \quad (31)$$

The imposition of Home tariff appreciates Home currency and lowers Foreign demands for Home products. As a result, Home employment decreases. The extent to which Home tariff influences Home employment depends on pass-through elasticity of exchange rate η^* .

Equations (26) – (31) and their Foreign counterparts, together with equation (25) constitute model's closed-form solution.

3. Optimal monetary and tariff policy rules

3.1. Policy objectives

Following the literature, we assume that derived utility from real money balances is small relative to that from consumption and labor supply. The assumption is supported empirically by Cooley and Hansen (1989). In the following, we only consider the non-monetary components of utility.

As in Corsetti and Pesenti (2005), both Home and Foreign authorities aim at bridging the gap between expected utility with flexible prices and that with sticky prices. However, when we introduce tariffs, flexible-prices allocations are no longer constrained efficient. The imposition of tariffs causes new distortions and thus drive a wedge between the employments without and with tariffs, when prices are flexible. Home flexible-price employment with tariffs is

$$L_t^{flex} = \Phi \left(\gamma + \frac{1 - \gamma}{1 + t_{H,t}} \right). \quad (32)$$

From the above equation, we know that the imposition of Home tariff will decrease Home flexible-price employment. The intuition is that the imposition of Home tariff will appreciate Home currency and thus decrease the demands for Home goods from Foreign households, consequently, Home output and employment goes down. It is noteworthy that the imposition of Foreign tariff does not influence Home flexible-price employment. As a matter of fact, Foreign tariff influences Home employment

by two channels. On the one hand, the imposition of Foreign tariff will increase prices of Home products in Foreign market, therefore, lowers the demands for Home products from Foreign households. On the other, the imposition of Foreign tariff depreciates Home currency, thus, increases Foreign households' purchasing power, as a result, the demands for Home products increase. The two effects of the imposition of Foreign tariff on Home flexible-price employment balance out each other in our model.

Another key variable under flexible prices is consumption which is given by

$$C_t^{flex} = \gamma \frac{\Phi_W}{\alpha_t^\gamma \alpha_t^{*1-\gamma} (1+t_{F,t}^*)^{1-\gamma}}, \quad (33)$$

in which $\Phi_W = \Phi^\gamma \Phi^{*1-\gamma}$. The analysis of the influence of imposition of tariffs on Home consumption under flexible prices is similar to that on Home employment. The imposition of Foreign tariff depreciates Home currency, consequently, the consumption demands of Home households for Foreign products decrease. Similarly, the effects of the imposition of Home tariff are from two channels: relative prices and exchange rate, and two effects offset against each other.

With both allocations under sticky prices and those under flexible prices in hand, we can calculate the gap between expected utility under flexible prices and that under sticky prices which Home government wishes to bridge, it is

$$E_{t-1} \left[\mathcal{W}_t^{flex} - \mathcal{W}_t \right] = E_{t-1} \left[\gamma \ln \frac{E_{t-1}(\alpha_t \mu_t)}{\alpha_t \mu_t} + (1-\gamma) \ln \left(\frac{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta \right)}{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta (1+t_{F,t}^*) E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \right)} \right) \right] \quad (34)$$

in which \mathcal{W}_t denotes the utility without monetary component. If governments impose no tariffs, by Jensen's inequality, the expected utility under flexible prices are never lower than that under sticky prices. After tariffs are introduced, we suppose the fluctuation of tariffs be slight such that the equation (34) is positive and can be used by Home government as policy loss function.

Following Corsetti and Pesenti (2005), we can equivalently express equation (34) in other three forms. The first is to express equation (34) as the expected value of

weighted average of logarithmic values of markups in Home market, plus a constant.

$$E_{t-1} \left[\mathcal{W}_t^{flex} - \mathcal{W}_t \right] = E_{t-1} \left(\gamma \ln \frac{P_{H,t}}{MC_t} + (1 - \gamma) \ln \frac{P_{F,t}}{\epsilon_t MC_t^*} \right) + \left(\gamma \ln \frac{\theta - 1}{\theta} + (1 - \gamma) \ln \frac{\theta^* - 1}{\theta^*} \right). \quad (35)$$

The second is the expected value of log of the ratio of price under sticky prices to that under flexible prices:

$$E_{t-1} \left[\mathcal{W}_t^{flex} - \mathcal{W}_t \right] = E_{t-1} \left(\ln \frac{P_t}{P_t^{flex}} \right). \quad (36)$$

With firms' participation constraints being considered, equation (36) implies that optima policy rules aim at bridging expected gap between log CPI under sticky prices and its analog under flexible prices.

The last is to express equation (34) as a function of output gaps and deviations from the law of one price in Home and Foreign.

$$\begin{aligned} E_{t-1} \left[\mathcal{W}_t^{flex} - \mathcal{W}_t \right] &= E_{t-1} \left\{ -\gamma \ln \frac{L_t}{\Phi \left(\gamma + \frac{1-\gamma}{1+t_{H,t}} \right)} - (1 - \gamma) \ln \frac{L_t^*}{\Phi^* \left(\frac{\gamma}{1+t_{F,t}^*} + (1 - \gamma) \right)} \right. \\ &\quad - \gamma \ln \left(\gamma + \frac{1 - \gamma}{1 + t_{H,t}} \right) + \gamma \ln \left(\gamma + \frac{1 - \gamma}{1 + t_{H,t}} \frac{P_{H,t}}{\epsilon_t P_{H,t}^*} \right) - (1 - \gamma) \ln \left(\frac{\gamma}{1 + t_{F,t}^*} + (1 - \gamma) \right) \\ &\quad \left. + (1 - \gamma) \ln \left(\frac{\gamma}{1 + t_{F,t}^*} + (1 - \gamma) \frac{P_{F,t}}{\epsilon_t P_{F,t}^*} \right) \right\}, \end{aligned} \quad (37)$$

in which $\frac{L_t}{\Phi \left(\gamma + \frac{1-\gamma}{1+t_{H,t}} \right)}$ and $\frac{L_t^*}{\Phi^* \left(\frac{\gamma}{1+t_{F,t}^*} + (1-\gamma) \right)}$ are output gaps in the Home and Foreign respectively. If the law of one price holds, then Home policies are separable from Foreign policies in the sense that Home output gap is independent of Foreign policies, vice versa. Once the law of one prices doesn't hold, optimal monetary and tariff policies in Home and Foreign are intertwined.

3.2. Nash monetary equilibrium

At the end of each period, governments announce to the public their monetary and tariff policy rules. Then, within the period, governments observe and respond to productivity shocks. In addition, we assume that governments can commit to their announced rules. In the following, we begin with analyzing Nash monetary equilibrium.

Home government chooses μ_t to minimize equation (34), taking $\{\alpha_t, \alpha_t^*, t_{H,t}, t_{F,t}^*, \mu_t^*\}$ as given. Foreign government's problem can be described similarly.

The relationship between optimal monetary policy rules and tariff policy rules has attracted scant academic attention in open economy macroeconomics literature. In Obstfeld and Rogoff (2002), since the sticky-wage distortion does not interact with the micro-level monopoly distortion, optimal monetary policy rules are separable from optimal tariff rules. However, They don't answer whether this conclusion is general or not. In this paper, we have

Proposition 1. *Except for PCP case ($\eta = \eta^* = 1$), optimal monetary policy rules are not separable from tariff policy rules.*

Proof. *The reaction function of Home government is*

$$\begin{aligned}
1 - \eta(1 - \gamma) &= \frac{\gamma \alpha_t \mu_t}{E_{t-1}(\alpha_t \mu_t)} + \frac{(1 - \gamma)(1 - \eta) \alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta}{E_{t-1}\left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta\right)} \\
&= \frac{\gamma \frac{L_t}{\Phi}}{\left(\gamma + \frac{1-\gamma}{1+t_{H,t}} \frac{P_{H,t}}{\epsilon_t P_{H,t}^*}\right)} + \frac{(1 - \gamma)(1 - \eta) \frac{1}{1+t_{F,t}^*} \frac{L_t^*}{\Phi^*}}{E_{t-1}\left(\frac{1}{1+t_{F,t}^*}\right) \left(\frac{\gamma}{1+t_{F,t}^*} + (1 - \gamma) \frac{P_{F,t}}{\epsilon_t P_{F,t}^*}\right)} \\
&= \gamma \frac{\theta}{\theta - 1} \frac{MC_t}{P_{H,t}} + (1 - \gamma)(1 - \eta) \frac{\theta^*}{\theta^* - 1} \frac{\frac{1}{1+t_{F,t}^*} \epsilon_t MC_t^*}{E_{t-1}\left(\frac{1}{1+t_{F,t}^*}\right) P_{F,t}}. \quad (38)
\end{aligned}$$

Similarly, the reaction function of Foreign government is

$$\begin{aligned}
1 - \eta^* \gamma &= \frac{(1 - \gamma) \alpha_t^* \mu_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)} + \frac{\gamma(1 - \eta^*) \alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}}{E_{t-1}\left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}\right)} \\
&= \frac{(1 - \gamma) \frac{L_t^*}{\Phi^*}}{(1 - \gamma) + \frac{\gamma}{1+t_{F,t}^*} \frac{\epsilon_t P_{F,t}^*}{P_{F,t}}} + \frac{\gamma(1 - \eta^*) \frac{1}{1+t_{H,t}} \frac{L_t}{\Phi}}{E_{t-1}\left(\frac{1}{1+t_{H,t}}\right) \left(\gamma \frac{\epsilon_t P_{H,t}}{P_{H,t}} + \frac{1-\gamma}{1+t_{H,t}}\right)} \\
&= (1 - \gamma) \frac{\theta^*}{\theta^* - 1} \frac{MC_t^*}{P_{F,t}^*} + \gamma(1 - \eta^*) \frac{\theta}{\theta - 1} \frac{\frac{1}{1+t_{H,t}} MC_t}{E_{t-1}\left(\frac{1}{1+t_{H,t}}\right) \epsilon_t P_{H,t}^*}. \quad (39)
\end{aligned}$$

The solution to equations (38) and (39) is the monetary Nash equilibrium. Notice that, except for $\eta = \eta^* = 1$, μ_t is a function of $t_{H,t}$ and $t_{F,t}^*$, so is μ_t^* , therefore, optimal monetary policy rules are not separable from tariff policy rules. ■

In the above proof, the reaction functions are expressed respectively as: (1). explicit functions of productivity shocks, monetary policies and tariff policies; (2). implicit functions of output gaps, deviations from the law of one price and tariff policies; (3). implicit functions of the markups and tariff policies.

How optimal monetary policy rules interact with tariff policies? Enlightened by Corsetti and Pesenti (2005), we consider Foreign representative firm's expected profit

$$\begin{aligned} E_{t-1} (Q_{t-1,t}^* \Pi_t^*(f)) &= E_{t-1} \left\{ Q_{t-1,t}^* \left[(p_t^*(f) - MC_t^*) C_{F,t}^* + \left(\frac{p_t(f)}{\epsilon_t} - MC_t^* \right) C_{F,t} \right] \right\} \\ &= E_{t-1} \left\{ \Omega - \Lambda \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta}{\tilde{p}_t^*(f)} \right\}, \end{aligned} \quad (40)$$

in which $\Omega > 0$ and $\Lambda > 0$ are expressions that are independent of α_t, μ_t and $\tilde{p}_t^*(f)$.⁷ If Home government pursues the inward-looking policy to stabilize domestic markups, i.e. $\mu_t = 1/\alpha_t$,⁸ in light of the concavity of expected profit accruing to Foreign representative firm to α_t , when $\eta \neq 1$, then Home stabilization policy will increase uncertainty to Foreign firm's expected profit from the Home market.

To buffer the above-mentioned risk, Foreign representative firm will charge a higher export price in Home market. Foreign representative firm's reaction can be justified by considering the elasticity of it's expected profit to the Home government's monetary stance μ_t

$$-\frac{\partial Q_{t-1,t}^* \Pi_t^*(f)}{\partial \mu_t} \frac{\mu_t}{Q_{t-1,t}^* \Pi_t^*(f)} = (1-\eta) \frac{\Lambda \alpha_t^* \alpha_t^{\eta-1} (1+t_{H,t})^{\eta-1} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta}{\tilde{p}_t^*(f) \Omega - \Lambda \alpha_t^* \alpha_t^{\eta-1} (1+t_{H,t})^{\eta-1} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta}, \quad (41)$$

in which the elasticity is valued at $\mu_t = 1/\alpha_t$. From equation (41), when $\eta < 1$, a higher price $\tilde{p}_t^*(f)$ will, to some degree, insulate Foreign firm's expected profit from external shocks.

Since a higher price $\tilde{p}_t^*(f)$ will reduce Home households' purchasing power, Home government has to take a more open view and take Foreign firm's markup in Home market into account.

However, after we introduce tariffs, when pass-through elasticities are less than unity, it is not necessary for Foreign firm to charge export price as high as that in

⁷It can be verified that $\Omega = Q_{t-1,t}^* (p_t^*(f) - MC_t^*) C_{F,t}^* + \frac{\beta \gamma \mu_{t-1}^*}{1+t_{F,t}^*}$ and $\Lambda = \beta \gamma^\eta (1-\gamma)^{1-\eta} \kappa^* \mu_{t-1}^*$

⁸When $\mu_t = 1/\alpha_t$, we have $\frac{P_{H,t}}{MC_t} = \frac{\theta}{\theta-1}$.

Corsetti and Pesenti (2005) to reduce risk. In our setting, as showed in equation (41), the imposition of tariffs in Home and Foreign can also reduce the sensitivity of Foreign firm's expected profit in Home market to external shocks. Accordingly, the introduction of tariffs leads to a lower export price $\widehat{p}_t^*(f)$ charged by Foreign firm. Therefore, comparing with what is implied in Corsetti and Pesenti (2005), a more inward-looking Home monetary policy will not substantially reduce Home households' purchasing power.

Proposition 2. *.When the governments can impose tariffs, though Home government's optimal monetary policy, as that in Corsetti and Pesenti (2005), still stabilizes a CPI-weighted average of the markups of all firms selling in Home market, a more greater weight is given to Home firm's markup. That is, in our setting, optimal monetary policy rules are more inward-looking than those in Corsetti and Pesenti (2005). The conclusion holds for the case in which pass-through elasticities are less than unity.*

What are the Home Nash monetary responses to tariffs? When pass-through elasticities are less than unity, Home monetary stance eases, i.e. an increase in μ_t , to accommodate to a surprising increase of Home tariff $t_{H,t}$. What's the intuition? As showed in equation (41), a surprising increase of Home tariff $t_{H,t}$ will reduce the sensitivity of Foreign firm's expected profit to external shocks. Though a more aggressive and inward-looking monetary policy to close domestic output gap will lower Foreign firm's expected profit, it will not incur great loss from the decline of domestic households' purchasing power. As a result, Home government eases its monetary stance. Similarly, when Foreign government increases tariff unexpectedly, Home Nash monetary response is also to ease its monetary stance. In other words, the surprising increase of tariffs, both in Home and Foreign, results in a more insulated world economy in the sense that each country pays more attention to its domestic goal and adopts a more aggressive monetary policy without exceedingly worrying about the retaliation from other country's firms.

3.3. Some special cases of monetary Nash equilibrium

3.3.1. PCP ($\eta = \eta^* = 1$) and LCP ($\eta = \eta^* = 0$)

When pass-through of exchange rate is complete in both countries, the Nash monetary equilibrium can be described as the solution to the following equations

$$1 = \frac{\alpha_t \mu_t}{E_{t-1}(\alpha_t \mu_t)} \quad 1 = \frac{\alpha_t^* \mu_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)}. \quad (42)$$

Nash equilibrium monetary policies, in this circumstance, are identical to those given in Corsetti and Pesenti (2005). Home government uses Nash monetary policy to stabilize domestic firm's markup, so does Foreign government. However, contrary to the conclusion in the literature, Nash monetary equilibrium does not support the flexible-price allocations⁹ Thus, the classical conclusion given by Friedman (1953) calling for flexible exchange rate to bring the economy back to flexible-price level in the presence of price rigidity, after we consider the interaction between tariff and monetary policies, needs to be reconsidered.

It's noteworthy that, in PCP case, as what is obtained in Obstfeld and Rogoff (2002), optimal monetary policy rules are separable from tariff policies. The reason is that, in PCP case, Home monetary policy can not influence Foreign firm's markup in Home market.¹⁰ Though Foreign tariff policy still influences the markup, Home policymaker disregards it and pays all its attention to stabilize Home firm's markup in Home market. The Foreign monetary policy maker's response can be analyzed similarly.

What will happen when pass-through of exchange rate is nil in both countries? The situation is, by Devereux and Engel (2003), more in accord with the empirical findings. the Nash monetary equilibrium, in this setting, is given by the solution to the following equations

$$\mu_t = \left[\gamma \frac{\alpha_t}{E_{t-1}(\alpha_t \mu_t)} + (1 - \gamma) \frac{\frac{1}{1+t_{H,t}} \alpha_t^*}{E_{t-1} \left(\frac{1}{1+t_{H,t}} \alpha_t^* \mu_t \right)} \right]^{-1}, \quad (43)$$

and

$$\mu_t^* = \left[(1 - \gamma) \frac{\alpha_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)} + \gamma \frac{\frac{1}{1+t_{F,t}^*} \alpha_t}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \alpha_t \mu_t^* \right)} \right]^{-1}. \quad (44)$$

⁹In the case of PCP, in general, Nash monetary policies support flexible-price allocations. The conclusion holds in the following literature: Obstfeld and Rogoff (2000), Obstfeld and Rogoff (2002) for the case of unitary relative risk aversion, Devereux and Engel (2003), Corsetti and Pesenti (2005), Shi and Xu (2007) for the case of perfectly correlated productivity shocks in the two stages, Wang and Zou (2013). The list is nonexhaustive.

¹⁰Notice that the log of Foreign firm's markup in Home market is $\ln \frac{P_{F,t}}{\epsilon_t M C_t^*}$, which equals to $\ln \frac{E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \epsilon_t^{1-\eta} M C_t^* \right)}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \right) \epsilon_t^{1-\eta} M C_t^*} + \ln \frac{\theta^*}{\theta^*-1}$. Obviously, when $\eta = 1$, the markup is independent of Home monetary policy.

In comparison with Devereux and Engel (2003), Corsetti and Pesenti (2005), optimal Nash monetary policy, in our model, doesn't involve fixed exchange rate. Imposition of tariffs induces policymakers to make light of the repercussions of exchange rate volatility on import prices, and make more inward-looking monetary policy to stabilize domestic output and prices. Therefore, to some degree, they ignore the impact of their monetary policies on exchange rate volatility. Accordingly, in both PCP and LCP cases, exchange rate is volatile. In addition, when $\eta = 0$, only Home tariff policy influences the elasticity of Foreign firm's expected profit to the Home monetary stance. Consequently, Home monetary policymaker only responds to Home tariff and eases its monetary stance after Home tariff increases.

3.3.2. RCP case ($\eta = 0$ and $\eta^* = 1$, or $\eta = 1$ and $\eta^* = 0$)

Following Devereux et al. (2007), reference currency is one with which all export goods are preset. In real world economy, US dollar, to some degree, plays the role of reference currency and is used for most of trans-border trade involving the US. We analyze the case $\eta = 0$ and $\eta^* = 1$ as an example, in which there is no exchange rate pass-through to Home, however, full exchange rate pass-through to Foreign. Therefore, in this case, Home is much like US in reality.

The Nash monetary equilibrium, in RCP case, is the solution to the following equations

$$\mu_t = \left[\gamma \frac{\alpha_t}{E_{t-1}(\alpha_t \mu_t)} + (1 - \gamma) \frac{\frac{1}{1+t_{H,t}} \alpha_t^*}{E_{t-1}\left(\frac{1}{1+t_{H,t}} \alpha_t^* \mu_t\right)} \right]^{-1}, \quad (45)$$

and

$$1 = \frac{\alpha_t^* \mu_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)}. \quad (46)$$

From equations (45) and (46), we conclude that Home adjusts its monetary stance to accommodate both Home and Foreign productivity shocks, Foreign, however, focuses only on its own domestic shock.¹¹ In this case, since Foreign monetary stance doesn't influence Home firm's markup in Foreign market¹², Foreign monetary policy

¹¹The same conclusion is obtained in Devereux et al. (2007) for the case of unitary relative risk aversion. In addition, Wang and Zou (2013) also get a similar result, when final goods prices are sticky but intermediate goods prices are flexible, and productivity shocks are from final goods production.

¹²The log of Home firms' markup in Foreign market is $\ln \frac{P_{H,t}^* \epsilon_t}{MC_t} = \ln \frac{E_{t-1}\left(\frac{1}{1+t_{H,t}} \epsilon_t^{\eta^*-1} MC_t\right)}{E_{t-1}\left(\frac{1}{1+t_{H,t}}\right) \epsilon_t^{\eta^*-1} MC_t} + \ln \frac{\theta}{\theta-1}$,

maker takes a totally inward-looking monetary policy and disregards tariff policy. As a comparison, Home monetary policy maker will take Foreign firm's markup in Home market into account. With the increase of home tariff, as explained in the case of LCP, Home eases its monetary stance.

3.4. Nash tariff equilibrium

Home tariff policy maker aims to minimize $E_{t-1} [\mathcal{W}_t^{flex} - \mathcal{W}_t]$ by choosing $t_{H,t}$, taking $\{\alpha_t, \alpha_t^*, t_{F,t}^*, \mu_t, \mu_t^*\}$ as given. Foreign tariff policy maker's problem can be described similarly. Nash tariff equilibrium consists of the solution to the following reaction functions, in which equation (47) follows from Home tariff policy maker's optimization problem, and equation (48) is its Foreign analog.

$$\begin{aligned}
1 &= \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta}{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta\right)} = \frac{\frac{1}{1+t_{F,t}^*} \frac{L_t^*}{\Phi^*}}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*}\right) \left(\frac{\gamma}{1+t_{F,t}^*} + (1-\gamma) \frac{P_{F,t}}{\epsilon_t P_{F,t}^*}\right)} \\
&= \frac{\theta^* \frac{1}{1+t_{F,t}^*} \epsilon_t MC_t^*}{\theta^* - 1 E_{t-1} \left(\frac{1}{1+t_{F,t}^*}\right) P_{F,t}}, \tag{47}
\end{aligned}$$

and

$$\begin{aligned}
1 &= \frac{\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}}{E_{t-1} \left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}\right)} = \frac{\frac{1}{1+t_{H,t}} \frac{L_t}{\Phi}}{E_{t-1} \left(\frac{1}{1+t_{H,t}}\right) \left(\gamma \frac{\epsilon_t P_{H,t}^*}{P_{H,t}} + \frac{1-\gamma}{1+t_{H,t}}\right)} \\
&= \frac{\theta \frac{1}{1+t_{H,t}} MC_t}{\theta - 1 E_{t-1} \left(\frac{1}{1+t_{H,t}}\right) \epsilon_t P_{H,t}^*}. \tag{48}
\end{aligned}$$

As before, reaction functions are expressed in three equivalent ways. Home Nash tariff policy aims to stabilize Foreign representative firm's markup in Home market. Why? Observing equation (40) reveals that, for $\eta \neq 1$, the profit that will be accrued to Foreign firm, is a concave function of Home tariff policy. It means that the variation of Home tariff policy will add uncertainty to Foreign firm's revenue from Home market. As before, Foreign firm has incentive to charge a higher export

which is independent of Foreign monetary policy when $\eta^* = 1$.

price in Home market to reduce the sensitivity of its expected profit to Home tariff policy. It's verified by the following

$$\frac{\partial Q_{t-1,t}^* \Pi_t^*(f)}{\partial t_{H,t}} \frac{t_{H,t}}{Q_{t-1,t}^* \Pi_t^*(f)} = -(\eta - 1) \frac{\Lambda t_{H,t} (1 + t_{H,t})^{\eta-2} \alpha_t^* \mu_t^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta}{\tilde{p}_t^*(f) \Omega - \Lambda \alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta} \quad (49)$$

When pass-through elasticities do not take boundary values, a negative Home productivity shock (a rise in α_t) induces Foreign tariff policy maker to impose a higher tariff. Why? Since a surprising rise of α_t lowers Home firm's markup in Foreign market, Foreign tariff policy maker, with an aim to stabilize it, chooses a tariff that, at the margin, will equate its marginal influence on exchange rate pass-through with its marginal influence on expected production cost. As a result, Foreign tariff policy maker increases tariff to appreciate Foreign currency, thus, Home firm's markup in Foreign market is stabilized. Similarly, a negative Foreign productivity shock (a rise in α_t^*) induces Home tariff maker to increase tariff. What is the relationship between monetary and tariff policies? When Home monetary stance is eased, Home currency depreciates. By the channel of exchange rate pass-through, the markup of Foreign firm in Home market deteriorates. In order to stabilize it, Home tariff maker increases tariff to appreciate Home currency. Similarly, the ease of Home monetary stance will lead to Foreign tariff maker to impose a higher tariff, but the analysis is somewhat different. The ease of Home monetary stance influences Home firm's markup in Foreign market by two channels. On the one hand, the ease of Home monetary stance appreciates Foreign currency, and improves the markup by exchange rate pass-through. On the other, the ease of Home monetary stance raises Home firm's marginal cost, thus, deteriorates the markup. The net influence amounts to a fall of the markup.¹³ In order to stabilize it, Foreign tariff maker raises its tariff to appreciate Foreign currency.

When the exchange rate pass-through is complete in both Home and Foreign, the markup of Foreign firm in Home market is independent of Home tariff. Accordingly, Home tariff policy doesn't affect Home household's welfare. Similarly, Foreign tariff

¹³The reasoning can be verified by the following: $\frac{P_{H,t}^* \epsilon_t}{MC_t} = \frac{\theta}{\theta-1} \frac{\epsilon_t^{1-\eta^*} E_{t-1} \left(\frac{1}{1+t_{H,t}} \epsilon_t^{\eta^*-1} MC_t \right)}{E_{t-1} \left(\frac{1}{1+t_{H,t}} \right) MC_t} =$

$$\frac{\theta}{\theta-1} \frac{E_{t-1} \left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}} \right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^{1-\eta^*} \right)}{E_{t-1} \left(\frac{1}{1+t_{H,t}} \right) (1+t_{H,t}) \alpha_t \left(\frac{\mu_t}{1+t_{H,t}} \right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^{1-\eta^*}}$$

policy doesn't affect Foreign household's welfare. Therefore, in PCP case, there is no game to play for tariff policy makers. Similar reasoning implies that there is no game to play in RCP case.

In LCP case, the Nash tariff equilibrium is the solution to the following equations

$$1 = \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)}{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right) \right)} \quad 1 = \frac{\alpha_t \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)}{E_{t-1} \left(\alpha_t \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right) \right)}. \quad (50)$$

Unlike the general case, the Nash tariff equilibrium in LCP case has a dominant strategy equilibrium. The explanation of the responses of tariff policy to productivity shock and monetary stance is similar to that in general case.

3.5. Nash monetary equilibrium when tariffs are chosen optimally

In general case, we know that optimal monetary policy rules are not separable from tariff policies and the surprising increase of tariffs, both in Home and Foreign, results in a more insulated world economy. What are the optimal monetary policy rules when tariffs are also chosen optimally?

Proposition 3. *Except for PCP and RCP cases, when tariffs are chosen optimally, Nash monetary equilibrium is the solution to equations (42). It implies that optimal monetary policies, in this circumstance, are identical to those in PCP case, thus, totally inward-looking.*

Proof. *Substituting equations (47) and (48) into equations (38) and (39), we have equation (42). ■*

Intuitively, when Foreign firm's markup in Home market can be stabilized by Home tariff policy, Home monetary authority's attention will be diverted away from it to Home firm's markup in Home market completely. Thus, when tariffs are chosen optimally, optimal monetary policy rules are similar to those obtained in Obstfeld and Rogoff (2002), Devereux and Engel (2003), where optimal monetary policy requires that Home monetary policy responds only to Home productivity shock. However, a key difference between our result and theirs is that their results are obtained under the assumption that exchange rate pass-through is complete in both countries, by comparison, our result follows from incomplete exchange rate pass-through in both countries. In addition, when tariffs are chosen optimally, our conclusion contrasts sharply from Corsetti and Pesenti (2005), in which, when exchange rate pass-through is incomplete in both countries, inward-looking policy is not optimal.

4. An extension

Now we consider Home and Foreign cooperate in setting optimal monetary and tariff rules. The objective of Home and Foreign policy makers is to minimize a weighted average of Home and Foreign policy loss functions, i.e.

$$E_{t-1} \left[\xi \left(\mathcal{W}_t^{flex} - \mathcal{W}_t \right) + (1 - \xi) \left(\mathcal{W}_t^{*flex} - \mathcal{W}_t^* \right) \right], \quad (51)$$

in which $\xi \in (0, 1)$ represents the bargaining power of the Home. Cooperative equilibrium is the solution of the following equations

$$\begin{aligned} & \xi [1 - \eta(1 - \gamma)] + (1 - \xi) \gamma \eta^* = \xi \gamma \frac{\alpha_t \mu_t}{E_{t-1}(\alpha_t \mu_t)} + \\ & \xi (1 - \gamma) (1 - \eta) \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta}{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta \right)} + (1 - \xi) \gamma \eta^* \frac{\alpha_t \left(\frac{\mu_t}{1+t_{H,t}} \right)^\eta \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^{1-\eta^*}}{E_{t-1} \left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}} \right)^\eta \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^{1-\eta^*} \right)} \\ & = \xi \gamma \frac{\theta}{\theta - 1} \frac{MC_t}{P_{H,t}} + \xi (1 - \gamma) (1 - \eta) \frac{\theta^*}{\theta^* - 1} \frac{\frac{1}{1+t_{F,t}^*} \epsilon_t MC_t^*}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \right) P_{F,t}} \\ & + (1 - \xi) \gamma \eta^* \frac{\theta}{\theta - 1} \frac{\frac{1}{1+t_{H,t}} MC_t}{E_{t-1} \left(\frac{1}{1+t_{H,t}} \right) \epsilon_t P_{H,t}^*} \end{aligned} \quad (52)$$

and

$$\begin{aligned} (1 - \xi) (1 - \gamma \eta^*) + \xi \eta (1 - \gamma) & = \xi \eta (1 - \gamma) \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta}{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}} \right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^\eta \right)} \\ & + (1 - \xi) \gamma (1 - \eta^*) \frac{\alpha_t \left(\frac{\mu_t}{1+t_{H,t}} \right)^\eta \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^{1-\eta^*}}{E_{t-1} \left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}} \right)^\eta \left(\frac{\mu_t^*}{1+t_{F,t}^*} \right)^{1-\eta^*} \right)} + (1 - \xi) (1 - \gamma) \frac{\alpha_t^* \mu_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)} \\ & = \xi \eta (1 - \gamma) \frac{\theta^*}{\theta^* - 1} \frac{\frac{1}{1+t_{F,t}^*} \epsilon_t MC_t^*}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*} \right) P_{F,t}} + (1 - \xi) \gamma (1 - \eta^*) \frac{\theta}{\theta - 1} \frac{\frac{1}{1+t_{H,t}} MC_t}{E_{t-1} \left(\frac{1}{1+t_{H,t}} \right) \epsilon_t P_{H,t}^*} \\ & + (1 - \xi) (1 - \gamma) \frac{\theta^*}{\theta^* - 1} \frac{MC_t^*}{P_{F,t}^*}. \end{aligned} \quad (53)$$

Comparison of cooperative monetary policy rules given in equations (52) and (53) with their Nash counterparts given in equations (38) and (39) implies that, when exchange rate pass-through elasticities are between 0 and 1, there are gains from cooperation. The reason is that cooperative monetary policy in Home will internalize its effect on Foreign, which means that, besides Home and Foreign firm's markups in Home, Home monetary policy maker is also bound by international monetary compact to take Home firm's markup in Foreign into account.

In Corsetti and Pesenti (2005), there are not gains from cooperation in PCP and LCP cases. In our setting, it can be verified that, except for LCP case, gains from cooperation arise in all other cases. In LCP case, Home monetary policy doesn't affect Home firm's markup in Foreign, accordingly, doesn't produce externality. It is true for Foreign monetary policy. Thus, unlike Corsetti and Pesenti (2005), in which gains from cooperation are related to exchange rate pass-through elasticities in a non-monotonic way, our conclusion implies that

Proposition 4. *Except for a special case—LCP, gains from cooperation exist for all other cases.*

It's noteworthy that the consideration of tariff is vital to produce gains from cooperation in PCP case. Without tariff, Home firm's domestic markup is identical to its foreign analog, Once tariff is taken into account, its domestic markup is as before, however, its foreign analog is affected by Home tariff.

What is the cooperative tariff equilibrium? As their monetary counterparts, we consider the situation in which tariff policy makers in Home and Foreign can choose tariff rules cooperatively to minimize global loss function $E_{t-1}[\xi(\mathcal{W}_t^{flex} - \mathcal{W}_t) + (1 - \xi)(\mathcal{W}_t^{*flex} - \mathcal{W}_t^*)]$. The cooperative tariff equilibrium is the solution to the following equations

$$\begin{aligned} \xi(1 - \gamma)(1 - \eta) + (1 - \xi)\gamma(\eta^* - 1) &= \xi(1 - \gamma)(1 - \eta) \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta}{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta\right)} \\ + (1 - \xi)\gamma\eta^* \frac{\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}}{E_{t-1} \left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}\right)} &- (1 - \xi)\gamma \frac{\frac{1}{1+t_{H,t}}}{E_{t-1} \left(\frac{1}{1+t_{H,t}}\right)} \end{aligned} \quad (54)$$

and

$$\begin{aligned} \xi(1-\gamma)(1-\eta) + (1-\xi)\gamma(\eta^* - 1) &= (1-\xi)\gamma(\eta^* - 1) \frac{\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}}{E_{t-1} \left(\alpha_t \left(\frac{\mu_t}{1+t_{H,t}}\right)^{\eta^*} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^{1-\eta^*}\right)} \\ - \xi(1-\gamma)\eta \frac{\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta}{E_{t-1} \left(\alpha_t^* \left(\frac{\mu_t}{1+t_{H,t}}\right)^{1-\eta} \left(\frac{\mu_t^*}{1+t_{F,t}^*}\right)^\eta\right)} &+ \xi(1-\gamma) \frac{\frac{1}{1+t_{F,t}^*}}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*}\right)}. \end{aligned} \quad (55)$$

Comparison of Nash tariff equilibrium given in equations (47) and (48) with cooperative tariff equilibrium in (54) and (55) implies that there are gains from cooperation between Home and Foreign tariff policy makers. The conclusion also holds for various special cases. Thus, in our model, an international tariff compact is welcomed by two countries unconditionally. However, further research is needed to determine the size of the gains from cooperation and the way to divide the gains between two countries.

When tariff rules are set cooperatively, cooperative monetary equilibrium is the solution to the following equations

$$1 = \xi \frac{\alpha_t \mu_t}{E_{t-1}(\alpha_t \mu_t)} + (1-\xi) \frac{\frac{1}{1+t_{H,t}}}{E_{t-1} \left(\frac{1}{1+t_{H,t}}\right)} \quad (56)$$

and

$$1 = \xi \frac{\frac{1}{1+t_{F,t}^*}}{E_{t-1} \left(\frac{1}{1+t_{F,t}^*}\right)} + (1-\xi) \frac{\alpha_t^* \mu_t^*}{E_{t-1}(\alpha_t^* \mu_t^*)}. \quad (57)$$

From equations (54) and (55), both Home and Foreign tariffs are implicit functions of productivity shocks and monetary stances in Home and Foreign. Therefore, cooperative monetary equilibrium given in equations (56) and (57) implies that Home monetary stance should respond to productivity shocks in both countries. Comparison of equations (56) and (57) with the previous conclusions implies that there are gains from cooperation, when Home and Foreign sign a comprehensively cooperative compact in tariff and monetary policy making.

5. Conclusion

The paper analyzes the relationship between optimal tariff and monetary policy rules by introducing tariffs in Corsetti and Pesenti (2005). Our conclusion depends on

exchange rate pass-through elasticities. When exchange rate pass-through elasticities are unity in both countries (corresponding PCP case), our conclusion coincides with that in Obstfeld and Rogoff (2002), i.e. optimal tariff policy rules are separable from optimal monetary policy rules. However, except for PCP case, optimal tariff policy rules are not separable from optimal monetary policy rules.

After Home and Foreign impose tariffs, except for the case of PCP, optimal monetary policy rules are more inward-looking than those in Corsetti and Pesenti (2005). In addition, we also find that, except for PCP and RCP cases, when tariffs are chosen optimally, optimal monetary policy rules are identical to those in PCP case, i.e. inward-looking. Thus, when tariffs are chosen optimally, our conclusion is similar to Obstfeld and Rogoff (2002), Devereux and Engel (2003). The difference is that their conclusions hold in PCP case, however, our conclusion holds except for PCP and RCP cases. After introducing optimal tariff policy rules, our conclusion also contrasts sharply from Corsetti and Pesenti (2005), in which, except for PCP case, inward-looking policy rules are not optimal.

We also extend the model to consider gains from cooperation between Home and Foreign policy makers. Except for LCP case, there are gains from cooperation between Home and Foreign monetary policy makers. By comparison, there are gains from cooperation between Home and Foreign tariff makers in various cases. Our model also shows that, if Home and Foreign cooperate comprehensively in monetary and tariff policy making, both countries will benefit.

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