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A dynamic model of capital and arms accumulation

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Abstract

How does competitive arms accumulation affect investment and capital accumulation? In a dynamic optimization framework including both investment and military spending, we find that, when the utility function is separable between consumption and the weapon stocks, an unanticipated rise in current military threat reduces current investment and an anticipated rise in future military threat stimulates current investment. But when the utility function is nonseparable between consumption and the weapon stocks, a current military threat may not decrease the short-run investment. In the long run, capital accumulation is independent of the military conflicts among countries regardless of the form of the utility function.

Key words: Capital accumulation; Military spending; Economic growth; Arms race
JEL classification: E20; E22; H56; O10; O40

1. Introduction

This paper examines both long-run and short-run responses of military spending and investment to competitive arms accumulation in a dynamic optimization model over an infinite horizon.

This approach is well justified for two reasons. First, the relation between military spending and capital accumulation has recently received considerable attention in policy discussions and empirical studies; see Deger and Sen (1983, 1992), Deger (1986), Hewitt (1991), McNamara (1992), Landau (1992), among

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others. Even though there are strong arguments for the existence of a negative impact of military tension on productive investment and output growth, empirical analysis often indicates some ambiguous effects or even a weak but positive effect; see Deger and Sen (1983) and Landau (1992). In addition, cross-country examination also shows that defense spending definitely reduces national saving ratios; see Deger (1986). Up till now, a well-grounded theoretical interpretation for these empirical findings is still lacking. Overall, empirical studies in this field have not explicitly modeled the dynamic relation among investment, consumption, and military spending. And very often some simple regression equations are estimated by putting military spending either as a dependent variable in Hewitt (1991) or as an explanatory variable in Landau (1992). If we derive the dynamics of investment and military spending from explicit dynamic optimization based on exogenously given preference, technology, military tension, and other factors, then not only can we verify whether empirical findings are consistent with theoretical predictions, but we can also provide insights on how to test the relation between military spending and investment in econometric studies.

Second, numerous theoretical studies on military spending often take output as given or ignore capital accumulation while focusing on the competitive arms accumulation in the dynamic games played by two countries in a state of confrontation. This long tradition begins with Richardson (1960) and continues with Saaty (1968), Brito (1972), Simaan and Cruz (1975), Intriligator (1975), and Intriligator and Brito (1976). For more recent studies along this line, see Deger and Sen (1984) and van der Ploeg and Zeeuw (1990).

In this paper we set up a dynamic optimization model including both capital and arms accumulation. We consider a typical country, say the home country, which is in a state of actual or potential military confrontation with a foreign country. The home country derives positive utility from its consumption and military defense services but disutility from the potential threat or invasion by the foreign country. When the foreign military threat rises, how should the home country respond? Intuitively, there exist two alternatives. One way is for the home country to cut both investment and consumption in the short run and devote more resource to arms accumulation; thus, investment is reduced as a result of rising military tension. But the home country can take another approach by reducing current consumption and increasing both investment and military spending in the short run. That will lead to a higher capital stock and a higher weapon stock in the home country. The expanding capital stock means a growing output, which in turn makes more consumption and military spending possible in the home country. In this way, the rising military tension accelerates the short-run investment and capital accumulation in the home country.

Which approach should the home country take? In this paper, we find that the answer crucially depends on the occurring time of the military threat and the assumption about the utility function. When the utility function is separable between consumption and the weapon stocks, we find that an unanticipated rise in current military threat reduces current investment and an anticipated rise in future military threat stimulates current investment. But when the utility function is nonseparable between consumption and the weapon stocks, a current military threat may not decrease the short-run investment. In the long run, no matter whether the utility function is separable or nonseparable, capital accumulation is determined by the famous modified golden rule, which is independent of the military conflicts among countries.

We organize this paper as follows. In Section 2, we set up the basic model with the utility function separable between consumption and the weapon stocks. In Section 3, we discuss the stability and the long-run equilibrium of the basic model. In Section 4, we demonstrate the short-run responses of both investment and military spending to different military shocks for the separable utility function. We consider the model with the nonseparable utility function in Section 5 and conclude this paper in Section 6.

2. The model

There are two countries in this model: the home country and the foreign country, and they are in a state of military confrontation. The preference of the home country is defined on consumption c, the home country's weapon stock m, and the foreign weapon stock m^*: $U(c, m, m^*)$. Furthermore $U(c, m, m^*)$ is concave and continuously differentiable in its arguments. As in Brito (1972), Deger and Sen (1983, 1984), and van der Ploeg and Zeeuw (1990), the following assumptions are imposed on the preference of the home country:

$$U_1 > 0, \quad U_2 > 0, \quad U_3 < 0, \quad U_{11} < 0, \quad U_{22} < 0, \tag{1}$$

$$U_{12} = U_{21} \geq 0, \quad U_{13} = U_{31} \leq 0, \quad U_{23} = U_{32} > 0. \tag{2}$$

All assumptions in (1) are self-evident. The assumption that $U_{23} > 0$ in (2) implies that an increase in the foreign weapon stock will increase the marginal utility of the home weapon stock and defense; see Deger and Sen (1984) for this reasoning. But the other two assumptions in (2) might not be accepted without some doubts because they raise the question why the utility from consumption relates to the weapon stocks. People may argue that the utility from consumption is independent of the weapon stocks. To take this consideration also into our model, we use the signs '≥ 0' and '≤ 0' and make it possible that the utility from consumption is independent of the weapon stocks.

The weapon accumulation in the home country is

$$\dot{m} = g - \delta m, \tag{3}$$

where g is military spending and δ is the depreciation rate of the weapon stock.

At any time, the resource available to the home country is the output $f(k)$, which is increasing and concave in the capital input k. If capital also depreciates at the rate δ, then the equation of motion for capital formation is given by

$$\dot{k} = f(k) - c - g - \delta k. \tag{4}$$

The production function used in (4) does not depend on the security represented by m in our model. Some may say that military spending enhances security and hence makes capital more productive. In this case, the positive effect of the military spending on capital accumulation is so obvious that we do not need further proof. To us, the more interesting and, in some sense, the more difficult case, is the independence of the production function from security and defense.

To make our dynamic system more tractable, we assume that the weapon stock and capital are essentially the same good and they can be added. In addition, we have already assumed that they have the same depreciation rate δ as in Eqs. (3) and (4).

Due to this assumption, we can define the total asset in the home country as w:

$$w = k + m. \tag{5}$$

Differentiate (5) with respect to time and use (3) and (4):

$$w = f(k) - c - \delta w. \tag{6}$$

The home country's objective is to maximize a discounted stream of utility over an infinite horizon with a positive time discount rate ρ:

$$\max \int_0^\infty U(c, m, m^*) e^{-\rho t} dt,$$

subject to constraints (3) and (4) or, equivalently, constraints (5) and (6). The initial total asset is given: $w(0) = k(0) + m(0)$.

For most part of this paper, we follow van der Ploeg and Zeeuw (1990) and assume that the utility function is separable in consumption and the weapon stocks:

$$U(c, m, m^*) = u(c) + v(m, m^*). \tag{7}$$

Then, assumptions (1) and (2) are modified to be

$$u' > 0, \quad u'' < 0, \quad v_1 > 0, \quad v_{11} < 0, \quad v_{12} > 0. \tag{8}$$

That is to say, the utility from consumption does not depend on the weapon stocks. In assuming this separability between consumption and defense, we have the following advantages. First, we can avoid the problem of negative and positive effects of the weapon stocks on consumption just as we have done in the case of the production function. Second, this separability makes it very easy to compare our model to the standard neoclassical growth model without arms accumulation if we assume the same utility function of consumption, $u(c)$. Third, while the long-run analysis in our paper always holds with and without the separability in the utility function, a separable utility function allows us to obtain clear-cut results in our short-run analysis in Section 4. We will point out how some ambiguous results will appear in the case of the nonseparable utility function in Section 5.

To solve the optimization problem, we formulate the corresponding Hamiltonian:

$$H = u(c) + v(m, m^*) + \lambda(f(k) - c - \delta w) + \gamma(w - k - m),$$

where λ is the marginal utility of one extra unit of the asset, or the shadow price of the total asset, in the home country and γ is the multiplier for the add-up condition or the identity of the total asset.

The first-order conditions necessary for the optimization are

$$u'(c) = \lambda, \tag{9}$$

$$v_1(m, m^*) = \lambda f'(k), \tag{10}$$

$$w = k + m, \tag{11}$$

$$\dot{\lambda}/\lambda = \delta + \rho - f'(k), \tag{12}$$

$$\dot{w} = f(k) - c - \delta w, \tag{13}$$

$$\lim_{t \to \infty} \lambda w e^{-\rho t} = 0. \tag{14}$$

The explanations for these necessary conditions are straightforward. Eq. (9) implies the equality between the marginal utility of one extra unit of asset and the marginal utility of consumption. Eq. (10) says that the marginal rate of substitution between consumption and arms equals the opportunity cost of arms, namely, the marginal productivity of capital. Eq. (11) repeats the asset add-up condition (5). The familiar Euler condition is given by Eq. (12), which governs the optimal choice between consumption and capital accumulation. Again, Eq. (13) is the dynamic budget constraint (6). The usual transversality condition is given by (14).

Instead of working with three differential equations in c, m, and k, we can solve c, m, and k in terms of λ, w, and m^*, and substitute them into Eqs. (12) and (13). Denote the solutions as $c(\lambda, w, m^*)$, $m(\lambda, w, m^*)$, and $k(\lambda, w, m^*)$ (see Appendix 1 for the properties of these functions) and denote

$$h(\lambda, w, m^*) = \delta + \rho - f'(k(\lambda, w, m^*)),$$

$$g(\lambda, w, m^*) = f(k(\lambda, w, m^*)) - c(\lambda, w, m^*) - \delta w,$$

then, we have

$$\dot{\lambda} = \lambda h(\lambda, w, m^*), \tag{15a}$$

$$\dot{w} = g(\lambda, w, m^*). \tag{15b}$$

In Appendix 1, it is established that $h_\lambda > 0$, $h_w > 0$, $h_{m^*} < 0$, $g_\lambda > 0$, $g_{m^*} < 0$, and g_w does not possess a definite sign. We will focus on Eqs. (15a) and (15b) in Sections 3 and 4 of this paper.

3. The long-run effects of the foreign military threat

Let $\bar{\lambda}$, $\bar{w}$, $\bar{c}$, $\bar{m}$, and $\bar{k}$ be the long-run equilibrium values of the corresponding variables. Upon linearizing (15) around the steady state values $\bar{\lambda}$ and $\bar{w}$, we obtain

$$\dot{\lambda} = \bar{\lambda} h_\lambda(\lambda - \bar{\lambda}) + \bar{\lambda} h_w(w - \bar{w}), \tag{16a}$$

$$\dot{w} = g_\lambda(\lambda - \bar{\lambda}) + g_w(w - \bar{w}), \tag{16b}$$

here all the partial derivatives are evaluated at the steady state values $\bar{\lambda}$ and $\bar{w}$. In the steady state, $\dot{\lambda} = 0$ and $\dot{w} = 0$ in (15). The phase diagram is presented in Fig. 1. The $\dot{\lambda} = 0$ locus is downward-sloping because the slope $\mathrm{d}\lambda/\mathrm{d}w = -(h_w/h_\lambda)$ is less than zero. The $\dot{w} = 0$ locus has an ambiguous sign because g_λ is positive but g_w does not have a definite sign from Eq. (A.4e) of Appendix 1. In Fig. 1, we draw the $w = 0$ locus as a downward-sloping line; the dynamics are the same whether it is upward- or downward-sloping. For the existence of a perfect foresight equilibrium in our mode, it is required that

$$\Delta' \equiv \bar{\lambda}(h_\lambda g_w - h_w g_\lambda) < 0. \tag{17}$$

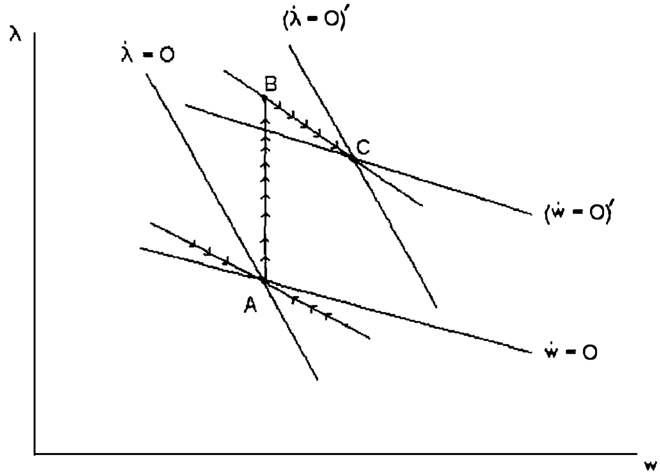

Fig. 1. The phase diagram and the effect of a permanent rise in the foreign military threat.

The geometry of the intuition for (17) is that the $\dot{\lambda} = 0$ locus is steeper than the $\dot{w} = 0$ locus. With condition (17), it is easy to check that the positive eigenvalue of the dynamic system is given by

$$\mu_1 \equiv [(\bar{\lambda} h_\lambda + g_w) + \sqrt{(\bar{\lambda} h_\lambda + g_w)^2 - 4\Delta'}]/2 > 0, \tag{18a}$$

and the negative eigenvalue is

$$\mu_2 \equiv [(\bar{\lambda} h_\lambda + g_w) - \sqrt{(\bar{\lambda} h_\lambda + g_w)^2 - 4\Delta'}]/2 < 0. \tag{18b}$$

As there is one negative eigenvalue μ_2 corresponding to one state variable w and one positive eigenvalue μ_1 corresponding to one jumping variable λ, the dynamic system (15) has a unique perfect foresight path converging to the steady state. It is important to note that, without (17), we will have either a totally unstable dynamic system (i.e., two positive eigenvalues) or a totally stable dynamic system (i.e., two negative eigenvalues). In the former, we cannot do too much analysis on effects of exogenous shocks because any perturbation to the dynamic system will lead to either an explosive or a corner solution. In the latter, a unique perfect foresight equilibrium does not exist because any initial point in the neighborhood of the equilibrium will converge to the steady state and it does not matter how the dynamic paths are moving in the short run. In fact, the dynamic path can be increasing, or decreasing, or any continuous function of time and, therefore, in a totally stable dynamic system, it is meaningless to talk about the short-run effects of any shock. This is why we will only examine the dynamic system (15) when condition (17) holds.

Now suppose that there is a permanent increase in the foreign weapon stock. We want to know how the long-run equilibrium values of the endogenous variables are affected.

Proposition 1. In the long run, a permanent increase in the foreign threat leads to less consumption and more arms accumulation in the home country, but it does not alter the long-run capital stock in the home country.

To show this proposition, we differentiate the two steady state equations, $h(\bar{\lambda}, \bar{w}, m^*) = 0$ and $g(\bar{\lambda}, \bar{w}, m^*) = 0$, with respect to the foreign weapon stock m^*, and solve for $\mathrm{d}\lambda/\mathrm{d}m^*$ and $\mathrm{d}w/\mathrm{d}m^*$:

$$\mathrm{d}\lambda/\mathrm{d}m^* = \bar{\lambda}(g_{m^*}h_w - h_{m^*}g_w)/\Delta',$$

$$\mathrm{d}w/\mathrm{d}m^* = \bar{\lambda}(h_{m^*}g_\lambda - g_{m^*}h_\lambda)/\Delta'.$$

Upon substituting all these partial derivatives from Appendix 1:

$$\mathrm{d}\lambda/\mathrm{d}m^* = -\bar{\lambda}\delta f''(\bar{k})(\mathrm{d}k/\mathrm{d}m^*)/\Delta' > 0, \tag{19}$$

$$\mathrm{d}w/\mathrm{d}m^* = \bar{\lambda} f''(\bar{k})(\mathrm{d}k/\mathrm{d}m^*)(\mathrm{d}c/\mathrm{d}\lambda)/\Delta' > 0, \tag{20}$$

which are positive because $\mathrm{d}k/\mathrm{d}m^* < 0$ and $\mathrm{d}c/\mathrm{d}\lambda < 0$ from Appendix 1 and $\Delta' < 0$ from (17). Thus a permanent increase in the foreign weapon stock raises the total asset and the shadow price of the total asset in the home country.

As $u'(\bar{c}) = \bar{\lambda}$ and $\mathrm{d}c/\mathrm{d}\lambda = 1/u''(\bar{c}) < 0$, the foreign military threat reduces the long-run consumption: $\mathrm{d}c/\mathrm{d}m^* = (\mathrm{d}c/\mathrm{d}\lambda)(\mathrm{d}\lambda/\mathrm{d}m^*) < 0$.

To see that the long-run capital stock is not affected, just observe the steady state condition $\dot{\lambda} = 0$, which is the same as

$$f'(\bar{k}) = \delta + \rho.$$

Thus the long-run capital stock is determined by the equality of the marginal productivity of capital and the time discount rate plus the capital depreciation rate. It needs to be pointed out that, as shown in Section 5 later, this result always holds no matter whether the utility function is separable or nonseparable in consumption and the weapon stocks.

Since the long-run capital stock is not changed, the long-run equilibrium value of arms in the home country is higher as a result of a higher total asset in the home country [see expression (20)]. In fact, $\mathrm{d}w/\mathrm{d}m^* = \mathrm{d}m/\mathrm{d}m^*$: the long-run increase in the total asset due to a permanent shock of the foreign military threat only reflects the long-run increase in arms accumulation in the home country.

The driving force for Proposition 1 is the modified golden rule of the long-run capital accumulation. Since the optimal capital stock in the long run is determined by the time preference and the depreciation rate of capital in our model, the total resource available in the home country is fixed in the long run if the time preference and the capital depreciation rate remain the same. Facing more foreign military threat, the home country has to choose 'less butter and more guns'.

This analytical result is also depicted in Fig. 1. As a result of a permanent increase in the foreign weapon stock, both the $\dot{w} = 0$ locus and the $\dot{\lambda} = 0$ locus shift upward to the right. The unique perfect foresight path is from the initial equilibrium point A to point B and, then, from point B to point C – the new equilibrium. At the new equilibrium point C, the home country has more weapon accumulation and less consumption (note that consumption is a decreasing function of the shadow price λ).

4. The short-run effects of the foreign military threat

While the long-run 'superneutrality' of the foreign military threat holds, an equally, if not more, interesting question is how the short-run investment and military spending are affected by the military threat. It is natural to ask whether the foreign military threat accelerates capital formation in the home country or decelerates it. The scenario here resembles the classical case of inflation and growth. As shown by Sidrauski (1967), inflation is superneutral because the long-run capital stock is independent from inflation. But in the short run, Fischer (1979) demonstrates that inflation often stimulates investment along the transitional path towards the long-run equilibrium. If we follow Fischer's approach here, we need to examine the impact of the foreign military threat on the negative eigenvalue μ_2 given in Eq. (18b). It is obvious that the foreign threat does affect the negative eigenvalue in (18b), but the differentiation of μ_2 with respect to the military threat parameter m^* does not yield a definite sign. However there exists another approach developed by Kenneth Judd (1983, 1985, 1987) which is especially helpful in tracing the short-run impacts of exogenous shocks on endogenous variables. See also Dixit (1990) for a lucid presentation of the Judd approach.

Following Judd (1987) and Dixit (1990), we suppose that initially, i.e., at time $t = 0$, the home country is in the steady state corresponding to the foreign military threat m^*. Now let the foreign military threat change as follows:

$$x^*(t) = m^* + \varepsilon z(t), \tag{21}$$

where $z(t)$ is the intertemporal change in the foreign weapon stock and ε is a small perturbation of the military threat. In this paper, we might take $z(t)$ as a step function of time and then a *temporary* change in the foreign weapon stock during time $t \subseteq [t_1, t_2]$ can be represented by $z(t) = 1$ for $t \subseteq [t_1, t_2]$ and $z(t) = 0$ otherwise. From this example, we can see that the temporary shocks can be easily handled with this technique. Of course, $z(t)$ can take other function forms such a ramp function and an impulse function. Eventually $z(t)$ is assumed to be constant.

Substitute $x^*(t)$ for m^* into (15):

$$\dot{\lambda} = \lambda h(\lambda, w, m^* + \varepsilon z(t)), \tag{22a}$$

$$\dot{w} = g(\lambda, w, m^* + \varepsilon z(t)). \tag{22b}$$

The solution to the dynamic system (22) will be smooth in both t and ε as the preference and technology are continuously differentiable. We write the solution as $\lambda(t, \varepsilon)$ and $w(t, \varepsilon)$. Differentiating (22) with respect to ε and linearizing:

$$\begin{bmatrix} \dot{\lambda}_\varepsilon \\ \dot{w}_\varepsilon \end{bmatrix} = \begin{bmatrix} \bar{\lambda} h_\lambda & \bar{\lambda} h_w \\ g_\lambda & g_w \end{bmatrix} \begin{bmatrix} \lambda_\varepsilon \\ w_\varepsilon \end{bmatrix} + \begin{bmatrix} \bar{\lambda} h_{m^*} z(t) \\ g_{m^*} z(t) \end{bmatrix}, \tag{23}$$

where all partial derivatives of h and g are evaluated at the initial steady state values $\bar{\lambda}$, $\bar{w}$, and $\varepsilon = 0$. In the last section we already studied the Jacobian matrix in (23) and found its two eigenvalues [see Eqs. (18a) and (18b)]. Next, to solve (23), we take the Laplace transforms of $\lambda(t)$, $w(t)$, and $z(t)$ and denote them $\Lambda(s)$, $W(s)$, and $Z(s)$ for $s > 0$, respectively,

$$\Lambda(s) = \int_0^\infty \lambda(t) \mathrm{e}^{-st} \mathrm{d}t,$$

$$W(s) = \int_0^\infty w(t) \mathrm{e}^{-st} \mathrm{d}t,$$

$$Z(s) = \int_0^\infty z(t) \mathrm{e}^{-st} \mathrm{d}t,$$

With these Laplace transforms, (23) is converted to

$$s \begin{bmatrix} \Lambda_\varepsilon(s) \\ W_\varepsilon(s) \end{bmatrix} = \begin{bmatrix} \bar{\lambda} h_\lambda & \bar{\lambda} h_w \\ g_\lambda & g_w \end{bmatrix} \begin{bmatrix} \Lambda_\varepsilon(s) \\ W_\varepsilon(s) \end{bmatrix} + \begin{bmatrix} \bar{\lambda} h_{m^*} Z(s) + \lambda_\varepsilon(0) \\ g_{m^*} Z(s) + w_\varepsilon(0) \end{bmatrix}$$

or

$$\begin{bmatrix} \Lambda_\varepsilon(s) \\ W_\varepsilon(s) \end{bmatrix} = \begin{bmatrix} s - \bar{\lambda} h_\lambda & -\bar{\lambda} h_w \\ -g_\lambda & s - g_w \end{bmatrix}^{-1} + \begin{bmatrix} \bar{\lambda} h_{m^*} Z(s) + \lambda_\varepsilon(0) \\ g_{m^*} Z(s) \end{bmatrix}. \tag{24}$$

In deriving (24), we have used the fact that the initial total asset $w(0)$ is given and cannot change, i.e., $w_\varepsilon(0) = 0$, but the shadow price of the total asset λ can jump. Thus we have dropped $w_\varepsilon(0)$ and retained $\lambda_\varepsilon(0)$ in (24).

To determine $\lambda_\varepsilon(0)$ in (24), we note that the existence of a saddle-point equilibrium in our model implies a finite total asset in the home country: a bounded capital stock and a bounded weapon stock. In addition, $z(t)$ is constant for sufficiently large time t. Therefore $W_\varepsilon(s)$ is finite for all $s > 0$. In particular, $W_\varepsilon(s)$ is finite when s equals the positive eigenvalue μ_1. But when $s = \mu_1$, the inverse matrix in (24) is singular. To remove this singularity, the only possibility is to set implicitly the numerators in (24) to zero:

$$(\mu_1 - g_w)[\bar{\lambda} h_{m^*} Z(\mu_1) + \lambda_\varepsilon(0)] + \bar{\lambda} h_w g_{m^*} Z(\mu_1) = 0 \tag{25a}$$

and

$$g_\lambda[\bar{\lambda} h_{m^*} Z(\mu_1) + \lambda_\varepsilon(0)] + (\mu_1 - \bar{\lambda} h_\lambda) g_{m^*} Z(\mu_1) = 0. \tag{25b}$$

Solving (25a) for $\lambda_\varepsilon(0)$:

$$\lambda_\varepsilon(0) = \{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda} \mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} Z(\mu_1). \tag{26}$$

In (26), the coefficient for $Z(\mu_1)$ is positive because $[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}]$ is positive upon substituting the relevant terms from Appendix 1 [which is equal to $\bar{\lambda}\delta f''(\bar{k}) \mathrm{d}k/\mathrm{d}m^* > 0$], $-\bar{\lambda}\mu_1 h_{m^*}$ is also positive for $h_{m^*} < 0$ from Appendix 1, and $(\mu_1 - g_w)$ is positive for μ_1 is larger than $(\bar{\lambda} h_\lambda + g_w)$ and $h_\lambda > 0$ from both Eq. (18a) and Appendix 1. Therefore we have established:

Proposition 2. When the utility function is separable between consumption and the weapon stocks, any perfectly anticipated increase in the future military threat from the foreign country raises the shadow price of the total asset today in the home country.

This interpretation of expression (26) is true because $Z(\mu_1)$ can be regarded as the present value of future military threat from the foreign country discounted at the positive eigenvalue:

$$Z(\mu_1) = \int_0^\infty z(t) \mathrm{e}^{-\mu_1 t} \mathrm{d}t.$$

For example, if the foreign weapon stock rises by one unit from time $t = T$ on, then, $z(t) = 0$ for $t \subseteq [0, T)$ and $z(t) = 1$ for $t \subseteq [T, \infty)$, and

$$Z(\mu_1) = \int_0^{\infty} \mathrm{e}^{-\mu_1 t} \mathrm{d}t = \mathrm{e}^{-\mu_1 T}/\mu_1.$$

In this case, the initial shadow price of the total asset will jump up by

$$\lambda_\varepsilon(0) = \{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda}\mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} \mathrm{e}^{-\mu_1 T}/\mu_1.$$

Since $\lambda = u'(c)$, we can derive the initial response of consumption from the initial response of the shadow price. In fact,

$$\begin{aligned} c_\varepsilon(0) &= \lambda_\varepsilon(0)/u''(c^*) \\ &= \{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda}\mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} Z(\mu_1)/u''(c^*). \end{aligned}$$

Hence, when the utility function is separable, any perfectly anticipated future military threat reduces current consumption in the home country.

With the knowledge of $\lambda_\varepsilon(0)$ and $w_\varepsilon(0)$ (the latter always equals zero), we can follow Judd (1987) and substitute these two values back into (23) while setting $t = 0$:

$$\begin{aligned} \dot{\lambda}_\varepsilon(0) &= \bar{\lambda} h_\lambda \lambda_\varepsilon(0) + \bar{\lambda} h_{m^*} z(0) \\ &= \bar{\lambda} h_\lambda \{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda}\mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} Z(\mu_1) + \bar{\lambda} h_{m^*} z(0), \end{aligned} \tag{27}$$

$$\begin{aligned} \dot{w}_\varepsilon(0) &= g_\lambda \lambda_\varepsilon(0) + g_{m^*} z(0) \\ &= g_\lambda \{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda}\mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} Z(\mu_1) + g_{m^*} z(0). \end{aligned} \tag{28}$$

In (27), $h_\lambda > 0$ and $h_{m^*} < 0$ from Appendix 1. Therefore, when the utility function is separable, a perfectly anticipated rise in the foreign arms accumulation speeds up the change in the initial shadow price while an unanticipated rise in the current foreign weapon stock lowers the speed of the initial change in the shadow price.

Eq. (28) is the most important equation we have tried to derive so far. It tells us how the short-run or the current asset accumulation, i.e., the sum of current investment and military spending, responds to military shocks. As $g_\lambda > 0$ and $g_{m^*} < 0$ from Appendix 1, we have:

Proposition 3. When the utility function is separable, an unanticipated rise in current foreign military threat reduces current asset accumulation, and a perfectly anticipated rise in future foreign military threat accelerates the current asset accumulation.

With Proposition 3 and the optimal condition between capital and arms accumulation for the case of a separable utility function in consumption and the weapon stocks, namely, $v_1(m, m^*) = \lambda f'(k)$, we can show how current investment and military spending are affected by the foreign military shocks.

Proposition 4. When the utility function is separable, an unanticipated rise in current foreign military threat reduces current investment, and a perfectly anticipated rise in future foreign military threat stimulates current investment.

To prove this proposition, we first note from Proposition 3 that, with an unanticipated rise in current foreign weapon stock, there will be an asset decumulation in the home country; since

$$\dot{w}_\varepsilon(0) = \dot{k}_\varepsilon(0) + \dot{m}_\varepsilon(0),$$

either current investment or current military spending or both will be reduced. In addition, from (27), the current shadow price of the asset is likely to be reduced, which is to say, for $u'(c) = \lambda$ with a separable utility function, current consumption is not going to be reduced as a result of an unanticipated rise in the current foreign military shock. Then go back to the optimal condition for the case of a separable utility function:

$$v_1(m, m^*) = \lambda f'(k). \tag{10}$$

Suppose that military spending remains the same or is reduced. The left-hand side is larger because m^* is higher and m is lower or remains the same [note $v_{12}(m, m^*) > 0$]. On the right-hand side, λ is likely to be lower from (27). To restore the equilibrium condition, current investment needs to be cut.

When there is an anticipated future increase in the foreign military threat, the current asset accumulation is going to accelerate from (28). That is to say, either current investment or current military spending or both will increase. It is very easy to see that current investment is going to increase. Just look at the Euler equation,

$$\dot{\lambda}/\lambda = \delta + \rho - f'(k). \tag{12}$$

As shown in Eq. (26), the current shadow price λ rises when more foreign military threat emerges in the future. Therefore, the optimal condition (12) calls for more current investment and more capital formation to reduce the marginal productivity of capital.

We use Fig. 2 to illustrate the effects of an anticipated temporary rise in future military threat. In Fig. 2, anticipating a temporary rise of military threat in the future, the home country will cut consumption in the short run and invest more in both capital and arms stocks. This is depicted in the dynamic path from the initial equilibrium A to point B. Since the threat is anticipated to be temporary in the future, with the accumulation of more capital and arms in the short run, the home country will gradually reduce its asset accumulation and increase its consumption; eventually the economy will restore its initial equilibrium A. The stage of asset decumulation is depicted on the dynamic path from point B to point A in Fig. 2.

As a corollary of Proposition 4, when the utility function is separable, more foreign military threat happening both today and in the future brings about an ambiguous impact on current investment in the home country.

We provide some economic intuitions for our propositions here.[1] In this model, since the utility from consumption is independent of the weapon stocks, an *unanticipated* rise in the *current* foreign military threat does not change the marginal utility of consumption and the steady state level of consumption is not affected by a momentary change in the military threat. Hence,

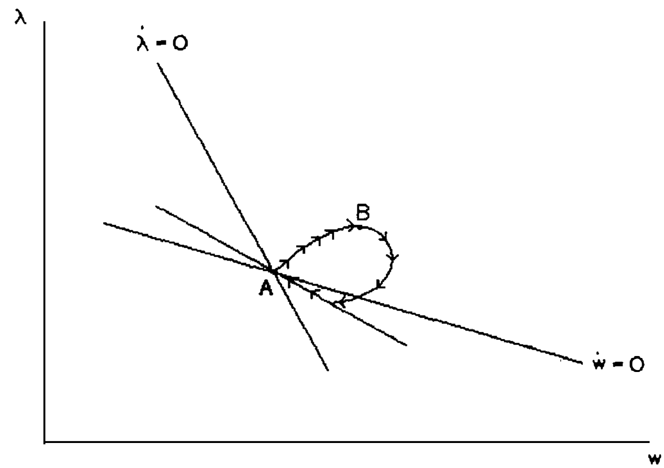

Fig. 2. An anticipated temporary rise in the future military threat.

[1] I thank two anonymous referees for providing further economic reasoning of these results and for applying these results to explain the stylized facts from empirical studies on military spending, savings and growth.

an unanticipated increase in current military threat, requiring increased armaments, can only be obtained by redirecting investment from capital formation to arms accumulation (see Judd, 1985, for a similar result about the effects of current government spending on current consumption and current investment); therefore, current investment is reduced. On the other hand, anticipating more foreign threat in the future, the home country can build up its defense by consuming less and investing more in both capital and arms today. More capital formation today means more output in the near future. With more output, more resource is available for more military spending, which in turn leads to a larger weapon stock. As a larger stock of weapon in the home country improves its position in the confrontation with the foreign country, the home country can afford to gradually slow down the rate of capital formation and channel more resource to consumption. In the long run, with a lower investment rate, the capital stock returns to its equilibrium level which is determined by the modified golden rule and is not affected by any military shock.

These propositions have strong empirical implications. As summarized in Deger (1986), econometric studies have found two interesting stylized facts regarding the impact of military expenditure on growth. The first says that the direct effect of defense spending on growth rates across countries seems to give an ambiguous relationship. These studies show that there is no impact or even, contrary to expectation, a somewhat weakly positive impact. The second empirically validated observation is that defense spending quite definitely reduces national saving–income ratios. Our results derived in this paper can explain quite well these two stylized facts. Superneutrality (Proposition 1) implies that the direct impact of defense spending and military shocks have little or no impact on the steady state capital stock. Thus the standard neoclassical result holds that the steady state growth rate is exogenously determined. Therefore, empirical studies of the effect of defense on growth would generally be inconclusive since they generally use cross-section data and therefore reflect long-run or steady state parameters. On the other hand, Proposition 4 indicates that current increases in the military threat reduce current investment. This result seems to explain well why saving–income ratios are often significantly negatively related to military spending. Further, as an anticipated future increase in the military threat stimulates current investment, countries planning well for future threats would increase current investment to get the rewards of both higher defense and consumption in the future. So we have the theoretical results validating cases like South Korea and Taiwan because they have always been anticipating and preparing for the break out of war with North Korea and mainland China, respectively, by accelerating their capital formation and arms accumulation.

These propositions also offer insights on testing the relation between military spending and investment. As we said in the introduction, both military spending and investment are endogenous variables in our model. A system of two simultaneous equations in terms of military spending and investment can be constructed to test how these two variables respond to current foreign military threat and future military threat. For example, we can propose the following form of regression equations:

$$\dot{k}(t) = \phi(\Delta m^*(t), \Delta m^*(t+1), \theta),$$

$$\dot{m}(t) = \kappa(\Delta m^*(t), \Delta m^*(t+1), \theta),$$

where $\Delta m^*(t)$ represents the change in the current military threat and $\Delta m^*(t+1)$ the expected change in future military threat; θ is other exogenous factors.

5. The case of the nonseparable utility function

Our analysis so far has been focusing on the utility function separable between consumption and the weapon stocks. In this section, we present our analysis for the utility function nonseparable between consumption and the weapon stocks. Recall that the general utility function $U(c, m, m^*)$ is assumed to have the following properties in Section 2 [Eqs. (1) and (2)]:

$$U_1 > 0, \quad U_2 > 0, \quad U_3 < 0, \quad U_{11} < 0, \quad U_{22} < 0,$$

$$U_{12} = U_{21} \geq 0, \quad U_{13} = U_{31} \leq 0, \quad U_{23} = U_{32} > 0.$$

With this general utility function, the current-value Hamiltonian is

$$H = U(c, m, m^*) + \lambda(f(k) - c - \delta w) + \gamma(w - k - m).$$

The first-order conditions for optimality are

$$U_1(c, m, m^*) = \lambda, \tag{9'}$$

$$U_2(c, m, m^*) = \lambda f'(k), \tag{10'}$$

$$w = k + m, \tag{11'}$$

$$\dot{\lambda}/\lambda = \delta + \rho - f'(k), \tag{12'}$$

$$\dot{w} = f(k) - c - \delta w, \tag{13'}$$

$$\lim \lambda w e^{-\rho t} = 0. \tag{14'}$$

These conditions are similar to, or the same as, conditions (9) to (14) in Section 2. Their explanations are more or less the same and we omit them here.

We first note that in the steady state, namely, $\dot{\lambda} = 0$ and $\dot{w} = 0$, condition (12′) implies that the steady state capital is again independent of military spending and military shocks: $f'(k) = \delta + \rho$. That is to say, even when the utility function is nonseparable, the optimal steady state capital stock is determined again by the modified golden rule. Hence we have verified the superneutrality result for both separable and nonseparable utility functions.

To analyze the short-run effects, as in the case of the separable utility function, we first solve c, m, and k as functions of λ, w, and m^* and substitute the solutions $c(\lambda, w, m^*)$, $m(\lambda, w, m^*)$, and $k(\lambda, w, m^*)$ into (12′) and (13′), and again denote

$$h(\lambda, w, m^*) = \delta + \rho - f'(k(\lambda, w, m^*)),$$

$$g(\lambda, w, m^*) = f(k(\lambda, w, m^*)) - c(\lambda, w, m^*) - \delta w,$$

then we have

$$\dot{\lambda} = \lambda h(\lambda, w, m^*), \tag{15a′}$$

$$\dot{w} = g(\lambda, w, m^*). \tag{15b′}$$

The properties of these functions – $c(\lambda, w, m^*)$, $m(\lambda, w, m^*)$, $k(\lambda, w, m^*)$, $h(\lambda, w, m^*)$, and $g(\lambda, w, m^*)$ – are presented in Appendix 2. In particular, we note that, for the separable utility function, only the sign g_w is not determined; but now with a nonseparable utility function, in addition to the sign of g_w, we cannot determine the signs of h_{m^*} and g_{m^*}. These ambiguous signs will prevent us from drawing clear conclusions from the short-run analysis. Recall that

$$\lambda_\varepsilon(0) = \{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda}\mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} Z(\mu_1), \tag{26}$$

$$\dot{w}_\varepsilon(0) = g_\lambda \lambda_\varepsilon(0) + g_{m^*} z(0)$$

$$= g_\lambda\{[\bar{\lambda} g_w h_{m^*} - \bar{\lambda} h_w g_{m^*}] - \bar{\lambda}\mu_1 h_{m^*}\}(\mu_1 - g_w)^{-1} Z(\mu_1) + g_{m^*} z(0). \tag{28}$$

Now the coefficients for the future military threat $Z(\mu_1)$ and the current military threat $z(0)$ are all ambiguous because h_{m^*} and g_{m^*} do not have definite signs as a result of the nonseparability in the utility function between consumption and the weapon stocks.

With the nonseparable utility function, consumption is still negatively related to the shadow price of the total asset as given by Eq. (B.2a) in Appendix 2. But even if λ has a definite sign, from the change of λ alone we cannot derive the effect on consumption because, unlike in the case of a separable utility function, the simple relation between c and λ in Eq. (9), $u'(c) = \lambda$, does not hold any more; the new optimal condition is

$$U_1(c, m, m^*) = \lambda. \tag{9'}$$

From (9′), it is clear that even an unanticipated military threat rises today, today's marginal utility of consumption will be reduced since $U_{13}(c, m, m^*)$ is negative. On the other hand, an increase in the home military stock m will increase the marginal utility of consumption due to the assumption that $U_{12}(c, m, m^*)$ is positive. Therefore, facing a current increase in the foreign military threat, the home country will cut its current consumption and spend more on weapon accumulation.

What is the effect of the unanticipated current military threat on current investment? With the separability in the utility function, we know that current consumption is not affected and current investment is cut as a result of the unanticipated current military threat. When the utility function is nonseparable, current consumption is going to be reduced as we have argued above. The effect on current investment may be ambiguous. We can provide the following reason. If the marginal utility is reduced significantly as a result of the current increase of foreign military threat, namely, $U_{13}(c, m, m^*)$ is large, consumption will be reduced to a great extent and the short-run investment may not be affected. On the other hand, if the marginal utility of consumption is affected by the foreign military threat very weakly, current consumption will not be reduced very much as a result of an unanticipated current foreign military threat. In this case, current investment will be partly sacrificed. In the extreme or the limit case when the marginal utility of consumption is independent of the current military threat and the weapon stocks, namely, $U_{13}(c, m, m^*) = 0$ and $U_{12}(c, m, m^*) = 0$, we return to the separable utility case where current consumption is not affected and all increased military spending will come at the cost of current investment as shown in Proposition 4 in Section 4.

Similar reasoning applies to the effects of an anticipated military threat in the future. But we need to emphasize that, when the marginal utility of consumption is severely affected by the military threat, the short-run investment may even be sacrificed in order to build up the defense as soon as possible.

6. Summary

This paper has made an attempt to answer the important question underlying many policy discussions and empirical studies: How does military spending affect investment and output growth? Our answer consists of the following: (1) for the most general utility function, superneutrality holds in the long run; capital accumulation is independent of the military threat; (2) when the utility function is separable in consumption and the weapon stocks, any anticipating military tension in the future stimulates current investment and any unanticipated current military threat reduces current investment; (3) when the utility function is nonseparable between consumption and the weapon stocks, a current increase in the foreign military threat will directly reduce current consumption, and current investment may be sacrificed as well.

These theoretical results are helpful for us in explaining the empirical findings about the ambiguous effects of military spending on growth rates and the negative impact of defense spending on national saving–income ratios, they also indicate new directions about how to statistically test both military spending and investment as functions of exogenous military shocks; in particular, our theoretical conclusions point out the importance of treating current military threat and future military threat differently in the regression analysis.

Appendix 1

In this appendix, we essentially undertake an analysis along the line of Arrow and Kurz (1970) and Mankiw (1987). Suppose that the total asset, the shadow price of the asset, and the foreign military threat are given. How does the home country choose its consumption, capital, and arms? Totally differentiating (9), (10), and (11), we have

$$\begin{bmatrix} u''(c) & 0 & 0 \\ 0 & v_{11}(m, m^*) & -\lambda f''(k) \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{d}c \\ \mathrm{d}m \\ \mathrm{d}k \end{bmatrix} = \begin{bmatrix} \mathrm{d}\lambda \\ f'(k)\mathrm{d}\lambda - v_{12}(m, m^*)\mathrm{d}m^* \\ \mathrm{d}w \end{bmatrix}. \tag{A.1}$$

It is easy to show that the determinant of the 3×3 matrix in (A.1), denoted as Δ, is positive:

$$\Delta = u''(c)v_{11}(m, m^*) + \lambda f''(k)u''(c) > 0.$$

By Cramer's rule,

$$\mathrm{d}c/\mathrm{d}\lambda = (v_{11}(m, m^*) + \lambda f''(k))/\Delta < 0, \tag{A.2a}$$

$$\mathrm{d}c/\mathrm{d}m^* = 0, \tag{A.2b}$$

$$\mathrm{d}c/\mathrm{d}w = 0, \tag{A.2c}$$

$$\mathrm{d}m/\mathrm{d}\lambda = f'(k)u''(c)/\Delta < 0, \tag{A.2d}$$

$$\mathrm{d}m/\mathrm{d}m^* = -v_{12}(m, m^*)u''(c) > 0, \tag{A.2e}$$

$$\mathrm{d}m/\mathrm{d}w = \lambda f''(k)u''(c)/\Delta > 0, \tag{A.2f}$$

$$\mathrm{d}k/\mathrm{d}\lambda = -f'(k)u''(c)/\Delta > 0, \tag{A.2g}$$

$$\mathrm{d}k/\mathrm{d}m^* = v_{12}(m, m^*)u''(c)/\Delta < 0, \tag{A.2h}$$

$$\mathrm{d}k/\mathrm{d}w = v_{11}(m, m^*)u''(c)/\Delta > 0. \tag{A.2i}$$

Substituting $c(\lambda, w, m^*)$, $m(\lambda, w, m^*)$, and $k(\lambda, w, m^*)$ into (12) and (13), and denoting

$$h(\lambda, w, m^*) = \delta + \rho - f'(k(\lambda, w, m^*)),$$

$$g(\lambda, w, m^*) = f(k(\lambda, w, m^*)) - c(\lambda, w, m^*) - \delta w.$$

Then, we have

$$\dot{\lambda} = \lambda h(\lambda, w, m^*), \tag{A.3a}$$

$$\dot{w} = g(\lambda, w, m^*). \tag{A.3b}$$

With (A.1), the functions (A.3) have the following properties:

$$h_\lambda(\lambda, w, m^*) = -f''(k)\mathrm{d}k/\mathrm{d}\lambda > 0, \tag{A.4a}$$

$$h_w(\lambda, w, m^*) = -f''(k)\mathrm{d}k/\mathrm{d}w > 0, \tag{A.4b}$$

$$h_{m^*}(\lambda, w, m^*) = -f''(k)\mathrm{d}k/\mathrm{d}m^* < 0, \tag{A.4c}$$

$$g_\lambda(\lambda, w, m^*) = f'(k)\mathrm{d}k/\mathrm{d}\lambda - \mathrm{d}c/\mathrm{d}\lambda > 0, \tag{A.4d}$$

$$g_w(\lambda, w, m^*) = f'(k)\mathrm{d}k/\mathrm{d}w - \delta, \tag{A.4e}$$

$$g_{m^*}(\lambda, w, m^*) = f'(k)\mathrm{d}k/\mathrm{d}m^* < 0. \tag{A.4f}$$

Note that the sign for g_w is not determined.

Appendix 2

As in Appendix 1, we suppose that the total asset, the shadow price of the asset, and the foreign military threat are given. We want to find out how the home country choose its consumption, capital, and arms. Totally differentiating (9′), (10′), and (11′), we have

$$\begin{bmatrix} U_{11} & U_{12} & 0 \\ U_{21} & U_{22} & -\lambda f''(k) \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{d}c \\ \mathrm{d}m \\ \mathrm{d}k \end{bmatrix} = \begin{bmatrix} \mathrm{d}\lambda - U_{13}\mathrm{d}m^* \\ f'(k)\mathrm{d}\lambda - U_{23}\mathrm{d}m^* \\ \mathrm{d}w \end{bmatrix}. \tag{B.1}$$

Since $U(c, m, m^*)$ is assumed to be concave in c and m, it is easy to show that the determinant of the 3×3 matrix in (B.1), denoted as Δ'', is positive:

$$\Delta'' = [U_{11}U_{22} - (U_{12})^2] + \lambda f''(k)U_{11} > 0.$$

By Cramer's rule,

$$\mathrm{d}c/\mathrm{d}\lambda = (U_{22} + \lambda f''(k) - U_{12}f'(k))/\Delta'' < 0, \tag{B.2a}$$

$$\mathrm{d}c/\mathrm{d}w = -U_{12}\lambda f''(k)/\Delta'' > 0, \tag{B.2b}$$

$$\mathrm{d}c/\mathrm{d}m^* = [-U_{22}U_{13} + U_{12}U_{23} - U_{13}\lambda f''(k)]/\Delta'', \tag{B.2c}$$

$$\mathrm{d}m/\mathrm{d}\lambda = [U_{11}f'(k) - U_{12}]/\Delta'' < 0, \tag{B.2d}$$

$$\mathrm{d}m/\mathrm{d}m^* = [-U_{11}U_{23} + U_{12}U_{13}]/\Delta'', \tag{B.2e}$$

$$\mathrm{d}m/\mathrm{d}w = \lambda f''(k)U_{11}/\Delta'' > 0, \tag{B.2f}$$

$$\mathrm{d}k/\mathrm{d}\lambda = [U_{12} - U_{11}f'(k)]/\Delta'' > 0, \tag{B.2g}$$

$$\mathrm{d}k/\mathrm{d}m^* = [-U_{12}U_{13} + U_{11}U_{23}]/\Delta'', \tag{B.2h}$$

$$\mathrm{d}k/\mathrm{d}w = [U_{11}U_{22} - (U_{12})^2]/\Delta'' > 0. \tag{B.2i}$$

We note that, due to the nonseparability in the utility function, three derivatives, $\mathrm{d}c/\mathrm{d}w$, $\mathrm{d}m/\mathrm{d}m^*$, and $\mathrm{d}k/\mathrm{d}m^*$, do not have definite signs in the expressions above. We can substitute $c(\lambda, w, m^*)$, $m(\lambda, w, m^*)$, and $k(\lambda, w, m^*)$ into (12′) and (13′), and again denote

$$h(\lambda, w, m^*) = \delta + \rho - f'(k(\lambda, w, m^*)),$$

$$g(\lambda, w, m^*) = f(k(\lambda, w, m^*)) - c(\lambda, w, m^*) - \delta w.$$

Then, we have

$$\dot{\lambda} = \lambda h(\lambda, w, m^*), \tag{B.3a}$$

$$\dot{w} = g(\lambda, w, m^*). \tag{B.3b}$$

With (B.1), the functions (B.3) have the following properties:

$$h_\lambda(\lambda, w, m^*) = -f''(k)\mathrm{d}k/\mathrm{d}\lambda > 0, \tag{B.4a}$$

$$h_w(\lambda, w, m^*) = -f''(k)\mathrm{d}k/\mathrm{d}w > 0, \tag{B.4b}$$

$$h_{m^*}(\lambda, w, m^*) = -f''(k)\mathrm{d}k/\mathrm{d}m^*, \tag{B.4c}$$

$$g_\lambda(\lambda, w, m^*) = f'(k)\mathrm{d}k/\mathrm{d}\lambda - \mathrm{d}c/\mathrm{d}\lambda > 0, \tag{B.4d}$$

$$g_w(\lambda, w, m^*) = f'(k)[\mathrm{d}k/\mathrm{d}w] - [\mathrm{d}c/\mathrm{d}w] - \delta, \tag{B.4e}$$

$$g_{m^*}(\lambda, w, m^*) = f'(k)[\mathrm{d}k/\mathrm{d}m^*] - [\mathrm{d}c/\mathrm{d}m^*]. \tag{B.4f}$$

For the separable utility function, only the sign g_w is not determined. Now with the nonseparable utility function, in addition to the sign of g_w, we cannot determine the signs of h_{m^*} and g_{m^*}.

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